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# Estimating the Problem of Non-Response and Measurement Error in Sample Survey

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**Abstract:** In this paper, the problem of estimation of finite population mean of the study variable is discussed in the presence of non-response and measurement error using the auxiliary variable. Some realistic conditions have been obtained under which the proposed estimator is more efficient than usual unbiased estimator, ratio estimators and product estimators. An empirical study is also conducted to support the theoretical findings in different situations.

**Keywords:** Study Variable, Auxiliary Variable, Non-response, Measurement error

## 1 Introduction

Sample surveys are conducted to obtain data on a variety of matters in many fields of life. The use of auxiliary information which is correlated with the variable of interest may improve the efficiency of the estimators. If there is a positive correlation between the auxiliary variable in the study and the variable under study, the ratio method of estimation is used Cochran [1,2]. The product estimation method is used in situations where the correlation coefficient between the variable of interest and the auxiliary variable is negative Robson [3] and Murthy [4]. A survey usually encounters various technical difficulties. No survey is perfect in all regards. Generally errors are of two categories sampling and non-sampling errors. Sampling errors comprise of the differences between the sample and the population due solely to the particular units that have been selected. Non-sampling errors encompass all other things that contribute to survey error. Non-sampling errors are said to arise from wrongly conceived definitions, imperfections in the tabulation plans, failure to obtain response from all sample members, and so on (see, Ilves [5], Groves, [6]).
In practice, this ideal is not met and the researcher faces the problem of measurement error while collecting information from the individuals. Measurement error is the difference between the value which is recorded and the true value of a variable in the study. Many researchers, such as Cochran [7,8,9], Fuller [10], Shalabh [11], Manisha and Singh [12,13], Wang [14], Allen et al. [15], Singh and Karpe [16,17,18,19], Salas and Gregoire [20], Kumar et al. [21] and Shukla et al. [22] etc., have studied measurement errors.
In sample surveys, the term non-response refers to the failure to collect information from one or more respondents on one or more variables. The reasons why non-response occurs include non-availability of the respondents at home, refusal to answer the questionnaire, lack of information, etc. The problem of non-response was first studied by Hansen and Hurwitz [23]. They addressed incomplete samples in mail sample survey and estimated the sample mean of the responding individuals and the sample mean of the sub-sample drawn from the non-respondents. Other researchers who have studied non-response include El-Badry [24], Foradari [25], Srinath [26], Cochran [9], Rao [27,28,29], Khare and Srivastava [30,31,32,33], Sodipo and Obisesan [34], Singh and Kumar [35,36,37,38,39], Ismail et al. [40], Khare et al. [41,42], Kumar and Bhougal [43], Khare and Kumar [44], Singh et al. [45], Shabbir and Khan [46], Kumar [47], Kumar and Chatterjee [48], Sharma and Kumar [49] and many others.

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In practice, the researchers face the problem of measurement error and non-response while collecting information from individuals. Researchers who studied non-response have ignored the presence of possible measurement errors and vice − versa. Jackman [50] dealt with both non-response and measurement error simultaneously in the case of voter turnout. Furthermore, Dixon [51] studied the estimation of non-response bias and measurement error on the data from Consumer Expenditure Quarterly Interview Survey (CEQ), Current Population Survey (CPS) and National Health Interview Survey (NHIS), as an attempt to measure the differences in employment status of Washington. Azeem [52] suggested estimators for estimating the population mean of study variable in the presence of non-response and measurement error. Moreover, Kumar et al. [53] and Kumar and Choudhary [54] have suggested estimator for estimating the population mean of the study variable in the presence of non-response and measurement error in the study as well as the auxiliary variable.
In this article, we have suggested a dual to Sahai's [55] estimator for estimating the population mean of the study variable in the presence of non-response and measurement error as well as auxiliary variable. An empirical study is conducted to judge the performance of the proposed estimator over other estimators of the population mean of the study variable.

## 2 Notations and Sampling Procedure

Most of the surveys yield estimates that are suspected to be biased, because of the presence of non-response and/or measurement bias. Numerous literature exist for eliminating the effect of non-response and measurement error independently in the process of estimating the population mean of the study variable $Y$. Cochran [2] established the use of auxiliary information to improve the precision of estimators of the population parameters.
Let us consider a population $U = U_1, U_2, ...., U_N$ of N units. A simple random sample of size $n$ is selected from the population without replacement on study variable $Y$ and auxiliary variable $X$. In the situation, when there is presence of non-response and measurement error associated with the study variable $U_i^* = y_i^* - Y_i^*$, and in the presence of non-response on the auxiliary variable, let the measurement error associated with auxiliary variable be $V_i^* = x_i^* - X_i^*$. The measurement errors are random in nature and have mean zero and variances $\sigma_U^2$ and $\sigma_V^2$ respectively for the responding units and $\sigma_{U(2)}^2$ and $\sigma_{V(2)}^2$ respectively for the non-responding units of the population.
The classical ratio and product estimators for the population mean $\mu_y$ of the study variable $y$ in the presence of non-response and measurement error are defined as

$$t_R = \hat{\mu}_y^* \left( \frac{\mu_x}{\hat{\mu}_x^*} \right) \tag{1}$$

and

$$t_P = \hat{\mu}_y^* \left( \frac{\hat{\mu}_x^*}{\mu_x} \right) \tag{2}$$

where $\hat{\mu}_y^*$ and $\hat{\mu}_x^*$ are the sample means of study and auxiliary variable in the presence of non-response and measurement error respectively, and $\mu_x$ is the population mean of the auxiliary variables.
Consider the transformation

$$x_i' = (1+g)\mu_x - g x_i; i = 1, 2, 3, ..., N, \tag{3}$$

where $g = \frac{n}{(N-n)}$.
Then

$$\hat{\mu}_x' = (1+g)\mu_x - g\hat{\mu}_x^*, \tag{4}$$

is an unbiased estimator for the population mean $\mu_x$ of the auxiliary variable $X$ and the correlation between $\mu_y$ and $\hat{\mu}_x'$ is negative. Using the transformation given in equation(3), Srivenkataramana [56] obtained dual to ratio estimator for the population mean $\mu_y$ as

$$t_R' = \hat{\mu}_y^* \left( \frac{\hat{\mu}_x'}{\mu_x} \right) \tag{5}$$

and

$$t_P' = \hat{\mu}_y^* \left( \frac{\mu_x}{\hat{\mu}_x'} \right). \tag{6}$$

In the present study, we have suggested a dual to Sahai$'$s [34] estimator for estimating the population mean $\mu_y$ of the study variable $y$ in the presence of non-response and measurement error in study as well as auxiliary variable. To the first degree of approximation, we have obtained the bias and mean squared error (MSE) of the proposed estimator. Furthermore, the conditions in which the proposed estimator is more efficient than the other existing estimators are obtained.

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## 3 Proposed Dual to Sahai's Estimator

The following is the suggested dual to *Sahai's* [34] estimator for the population mean $\mu_y$ as

$$\hat{t}_s = \hat{\mu}_y^* \left( \frac{\mu_x + \delta \hat{\mu}_x}{\hat{\mu}_x + \delta \mu_x} \right), \tag{7}$$

where $\mu_x = \frac{N\mu_x - n\hat{\mu}_x^*}{N-n}$, and $\delta$ is a scalar used as the design parameter. It is interesting to note that
**a)** For $\delta = 1, \hat{t}_s = \hat{\mu}_y^*$, usual unbiased estimator.
**b)** For $\delta = 0, \hat{t}_s = \hat{\mu}_y^* (\frac{\mu_x}{\hat{\mu}_x}) = \hat{t}_P$, dual to product estimator.
**c)** For $\delta$ very large, $\hat{t}_s = \hat{\mu}_y^* (\frac{\hat{\mu}_x}{\mu_x}) = \hat{t}_R$, dual to ratio estimator,
i.e.

$\lim_{\delta \to \infty} \hat{t}_s = \lim_{\delta \to \infty} \hat{\mu}_y^* \left( \frac{\mu_x + \delta \hat{\mu}_x}{\hat{\mu}_x + \delta \mu_x} \right) = \hat{\mu}_y^* \lim_{\delta \to \infty} \left( \frac{\mu_x + \delta \hat{\mu}_x}{\hat{\mu}_x + \delta \mu_x} \right) \cong \hat{\mu}_y^* \left( \frac{\mu_x}{\hat{\mu}_x} \right) = \hat{t}_R.$

To obtain the expressions of bias and MSE of the proposed estimator, let us assume the following

$\omega_y^* = \frac{1}{\sqrt{n}} \Sigma_{i=1}^n (y_i^* - \mu_Y)$ , $\omega_U^* = \frac{1}{\sqrt{n}} \Sigma_{i=1}^n U_i^*$ , $\omega_x^* = \frac{1}{\sqrt{n}} \Sigma_{i=1}^n (x_i^* - \mu_X)$ and $\omega_V^* = \frac{1}{\sqrt{n}} \Sigma_{i=1}^n V_i^*$.

Adding $\omega_y^*$ and $\omega_U^*$, we have $\omega_y^* + \omega_U^* = \frac{1}{\sqrt{n}} [\Sigma_{i=1}^n (y_i^* - \mu_Y) + \Sigma_{i=1}^n U_i^*]$.
Multiplying both sides by $\frac{1}{\sqrt{n}}$, we have

$$\frac{1}{\sqrt{n}} (\omega_y^* + \omega_U^*) = \left[ \frac{1}{n} \Sigma_{i=1}^n (y_i^* - \mu_Y) + \frac{1}{n} \Sigma_{i=1}^n (y_i^* - Y_i^*) \right]$$

or

$$\frac{1}{\sqrt{n}} (\omega_y^* + \omega_U^*) = \hat{\mu}_y^* - \mu_Y,$$

or

$$\hat{\mu}_y^* = \mu_Y + \frac{1}{\sqrt{n}} (\omega_y^* + \omega_U^*) = \mu_Y + \omega_Y.$$

Similarly, one can obtain

$$\hat{\mu}_x^* = \mu_X + \frac{1}{\sqrt{n}} (\omega_x^* + \omega_V^*) = \mu_x + \omega_x.$$

Further

$$E \left( \frac{\omega_y^* + \omega_U^*}{\sqrt{n}} \right)^2 = \lambda_2 (\sigma_y^2 + \sigma_U^2) + \theta (\sigma_{y(2)}^2 + \sigma_{U(2)}^2)$$

$$E \left( \frac{\omega_x^* + \omega_V^*}{\sqrt{n}} \right)^2 = \lambda_2 (\sigma_x^2 + \sigma_V^2) + \theta (\sigma_{x(2)}^2 + \sigma_{V(2)}^2) = B_o(say)$$

$$E \left[ \left( \frac{\omega_y^* + \omega_U^*}{\sqrt{n}} \right) \left( \frac{\omega_x^* + \omega_V^*}{\sqrt{n}} \right) \right] = \lambda_2 \rho_{yx} \sigma_y \sigma_x + \theta \rho_{yx(2)} \sigma_{y(2)} \sigma_{x(2)} = A_o(say),$$

where $\mu_Y = \frac{1}{N} \Sigma_{i=1}^N y_i$ , $\mu_X = \frac{1}{N} \Sigma_{i=1}^N x_i$ , $\sigma_y^2 = \frac{1}{N} \Sigma_{i=1}^N (y_i - \mu_Y)^2$ and $\sigma_x^2 = \frac{1}{N-1} \Sigma_{i=1}^N (x_i - \mu_X)^2$ denote the population mean and the population variance of the study variable y and auxiliary variable x. Let $\mu_{Y_1} = \frac{1}{N_1} \Sigma_{i=1}^{N_1} y_i$ and $\sigma_{y(1)}^2 = \frac{1}{(N_1 - 1)} \Sigma_{i=1}^{N_1} (y_i - \mu_{Y_1})^2$ denote the mean and variance of the response group. Similarly, let $\mu_{Y_2} = \frac{1}{N_2} \Sigma_{i=1}^{N_2} y_i$ and $\sigma_{y(2)}^2 = \frac{1}{N_2 - 1} \Sigma_{i=1}^{N_2} (y_i - \mu_{Y_2})^2$ denote the mean and variance of the non-response group of the study variable. Similarly, for auxiliary variable, $\mu_{x_1} = \frac{1}{N_1} \Sigma_{i=1}^{N_1} x_i$ , $\sigma_{x(1)}^2 = \frac{1}{N_1 - 1} \Sigma_{i=1}^{N_1} (x_i - \mu_{x_1})^2$ , $\mu_{x_2} = \frac{1}{N_2} \Sigma_{i=1}^{N_2} x_i$ and $\sigma_{x(2)}^2 = \frac{1}{N_2 - 1} \Sigma_{i=1}^{N_2} (x_i - \mu_{x_2})^2$ are the mean and variance of response and non-response group of auxiliary variable, respectively. Let $\rho_{yx} = \frac{1}{N-1} \Sigma_{i=1}^N (y_i - \mu_y)(x_i - \mu_x)$ and $\rho_{yx(2)} = \frac{1}{N_2 - 1} \Sigma_{i=1}^{N_2} (y_i - \mu_y)(x_i - \mu_x)$ are the coefficient of correlation between study and auxiliary variables for response and non-response, respectively; $\lambda_2 = (\frac{1}{n} - \frac{1}{N})$; $\theta = \frac{W_2(k-1)}{n}$; $W_1 = \frac{N_1}{N}$ and $W_2 = \frac{N_2}{N}$; k is the inverse sampling ratio, $k > 1$.
For bias of the proposed estimator $\hat{t}_s$, we have

$$\hat{\mu}_x = \frac{N\mu_x - n\hat{\mu}_x^*}{N-n} = \frac{N}{N-n} \mu_x - \frac{n}{N-n} \hat{\mu}_x^*$$

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Let $g = \frac{n}{N-n} \Rightarrow g+1 = \frac{N}{N-n}$, then

$$\hat{\mu}_x = (g+1)\mu_x - g\hat{\mu}_x^*.$$

Thus,the proposed estimator $\hat{t}_s$ under the transformation becomes

$$\hat{t}_s = \hat{\mu}_y^* \left[\frac{\mu_x + \delta\{(1+g)\mu_x - g\hat{\mu}_x^*\}}{(1+g)\mu_x - g\hat{\mu}_x^* + \delta\mu_x}\right]. \tag{8}$$

Now, expressing equation (8) in terms of $\omega_i; i = x, y, U, V$; we have

$$\hat{t}_s = (\mu_y + \omega_y)\left[1 - \left(\frac{g\delta}{1+\delta}\right)\frac{\omega_x}{\mu_x}\right]\left[1 - \left(\frac{g}{1+\delta}\right)\frac{\omega_x}{\mu_x}\right]^{-1}. \tag{9}$$

Assume that $|(\frac{g\delta}{1+\delta})\frac{\omega_x}{\mu_x}| < 1$ so that $[1 - (\frac{g}{1+\delta})\frac{\omega_x}{\mu_x}]^{-1}$ is expandable.
Expanding the right hand side of equation (9), one can obtain

$$\hat{t}_s = (\mu_y + \omega_y)\left\{1 - \left(\frac{g\delta}{1+\delta}\right)\frac{\omega_x}{\mu_x}\right\}\left\{1 + \left(\frac{g}{1+\delta}\right)\frac{\omega_x}{\mu_x} - \left(\frac{g}{1+\delta}\right)^2\frac{{\omega_x}^2}{{\mu_x}^2} + \cdots\right\}$$

Neglecting terms of $\omega_i; i = x, y$, having power greater than two, we have

$$\hat{t}_s - \mu_y \cong \omega_y + \left(\frac{g}{1+\delta}\right)(1-\delta)\frac{\omega_x}{\mu_x} - \left(\frac{g}{1+\delta}\right)^2\frac{\omega_x^2}{\mu_x^2}(\mu_y - \delta) + \left(\frac{g}{1+\delta}\right)(1-\delta)\frac{\omega_x\omega_y}{\mu_x}. \tag{10}$$

Assuming that $\Delta = \frac{1-\delta}{1+\delta} \Rightarrow \frac{1}{1+\delta} = \frac{1+\Delta}{2}$ in equation (10), we have

$$\hat{t}_s - \mu_y \cong \omega_y + g\Delta\frac{\omega_x}{\mu_x} - \frac{g^2(1+\Delta)^2}{4}\frac{\omega_x^2}{\mu_x^2}(\mu_y - \delta) + g\Delta\frac{\omega_x\omega_y}{\mu_x}. \tag{11}$$

Taking expectation of both sides of equation (11), one can obtain the bias of $\hat{t}_s$ to the first degree of approximation

$$B(\hat{t}_s) = E(\hat{t}_s - \mu_y)$$

$$B(\hat{t}_s) = -\frac{g^2(1+\Delta)^2}{4}\frac{(\mu_y - \delta)}{\mu_x^2}B_o + \frac{g\Delta}{\mu_x}A_o. \tag{12}$$

Squaring both sides of equation (11) to the first degree of approximation, the MSE of $\hat{t}_s$ is given by

$$(\hat{t}_s - \mu_y)^2 = \left(\omega_y + g\Delta\frac{\omega_x}{\mu_x}\right)^2 = \omega_y^2 + \frac{g^2\Delta^2}{\mu_x^2}\omega_x^2 + 2\frac{g\Delta}{\mu_x}\omega_y\omega_x. \tag{13}$$

Taking expectation on both sides of equation (13), we get the MSE of $\hat{t}_s$ to the first degree of approximation as

$$\begin{aligned} MSE(\hat{t}_s) = \Bigg[\lambda_2\Bigg\{\sigma_y^2 + \frac{g^2\Delta^2}{\mu_x^2}\sigma_x^2 + 2\frac{g\Delta}{\mu_x}\rho_{yx}\sigma_y\sigma_x + \left(\sigma_U^2 + \frac{g^2\Delta^2}{\mu_x^2}\sigma_V^2\right)\Bigg\} + \theta\Bigg\{\sigma_{y(2)}^2 \\ + \frac{g^2\Delta^2}{\mu_x^2}\sigma_{x(2)}^2 + 2\frac{g\Delta}{\mu_x}\rho_{yx(2)}\sigma_{y(2)}\sigma_{x(2)} + \left(\sigma_{U(2)}^2 + g^2\Delta^2\mu_x^2\sigma_{V(2)}^2\right)\Bigg\}\Bigg]. \end{aligned} \tag{14}$$

The MSE of $\hat{t}_s$ is minimized when

$$\Delta = -\frac{\mu_x A_o}{gB_o} = \Delta_o(say). \tag{15}$$

Substitute the optimum value of $\Delta$ from (15) in (14) to obtain the optimum MSE of $\hat{t}_s$ as

$$\begin{aligned} min.MSE(\hat{t}_s) = \Bigg[\lambda_2\Bigg\{\sigma_y^2 + \frac{A_o^2}{B_o^2}\sigma_x^2 - 2\frac{A_o}{B_o}\rho_{yx}\sigma_y\sigma_x + \left(\sigma_U^2 + \frac{A_o^2}{B_o^2}\sigma_V^2\right)\Bigg\} + \theta\Bigg\{\sigma_{y(2)}^2 \\ + \frac{A_o^2}{B_o^2}\sigma_{x(2)}^2 - 2\frac{A_o}{B_o}\rho_{yx(2)}\sigma_{y(2)}\sigma_{x(2)} + \left(\sigma_{U(2)}^2 + \frac{A_o^2}{B_o^2}\sigma_{V(2)}^2\right)\Bigg\}\Bigg]. \end{aligned} \tag{16}$$

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## 4 Efficiency Comparison

The MSE of the mentioned estimator are as follows

$$MSE(\hat{\mu}_y^*) = \lambda_2(\sigma_y^2+\sigma_U^2)+\theta(\sigma_{y(2)}^2+\sigma_{U(2)}^2), \tag{17}$$

$$\begin{aligned}MSE(t_R) = \Bigg[\lambda_2\Bigg\{\sigma_y^2+\frac{\mu_y^2}{\mu_x^2}\sigma_x^2-2\frac{\mu_y}{\mu_x}\rho_{yx}\sigma_y\sigma_x+\Bigg(\sigma_U^2+\frac{\mu_y^2}{\mu_x^2}\sigma_V^2\Bigg)\Bigg\}+\\ \theta\Bigg\{\sigma_{y(2)}^2+\frac{\mu_y^2}{\mu_x^2}\sigma_{x(2)}^2-2\frac{\mu_y}{\mu_x}\rho_{yx(2)}\sigma_{y(2)}\sigma_{x(2)}+\Bigg(\sigma_{U(2)}^2+\frac{\mu_y^2}{\mu_x^2}\sigma_{V(2)}^2\Bigg)\Bigg\}\Bigg],\end{aligned} \tag{18}$$

$$\begin{aligned}MSE(t_P) = \Bigg[\lambda_2\Bigg\{\sigma_y^2+\frac{\mu_y^2}{\mu_x^2}\sigma_x^2+2\frac{\mu_y}{\mu_x}\rho_{yx}\sigma_y\sigma_x+\Bigg(\sigma_U^2+\frac{\mu_y^2}{\mu_x^2}\sigma_V^2\Bigg)\Bigg\}+\\ \theta\Bigg\{\sigma_{y(2)}^2+\frac{\mu_y^2}{\mu_x^2}\sigma_{x(2)}^2+2\frac{\mu_y}{\mu_x}\sigma_{yx(2)}\sigma_{y(2)}\sigma_{x(2)}+\Bigg(\sigma_{U(2)}^2+\frac{\mu_y^2}{\mu_x^2}\sigma_{V(2)}^2\Bigg)\Bigg\}\Bigg],\end{aligned} \tag{19}$$

$$\begin{aligned}MSE(t_R') = \Bigg[\lambda_2\Bigg\{\sigma_y^2+g^2\frac{\mu_y^2}{\mu_x^2}\sigma_x^2-2g\frac{\mu_y}{\mu_x}\rho_{yx}\sigma_y\sigma_x+\Bigg(\sigma_U^2+g^2\frac{\mu_y^2}{\mu_x^2}\sigma_V^2\Bigg)\Bigg\}+\theta\Bigg\{\sigma_{y(2)}^2\\ +g^2\frac{\mu_y^2}{\mu_x^2}\sigma_{x(2)}^2-2g\frac{\mu_y}{\mu_x}\sigma_{yx(2)}\sigma_{y(2)}\sigma_{x(2)}+\Bigg(\sigma_{U(2)}^2+g^2\frac{\mu_y^2}{\mu_x^2}\sigma_{V(2)}^2\Bigg)\Bigg\}\Bigg],\end{aligned} \tag{20}$$

$$\begin{aligned}MSE(t_P') = \Bigg[\lambda_2\Bigg\{\sigma_y^2+g^2\frac{\mu_y^2}{\mu_x^2}\sigma_x^2+2g\frac{\mu_y}{\mu_x}\rho_{yx}\sigma_y\sigma_x+\Bigg(\sigma_U^2+g^2\frac{\mu_y^2}{\mu_x^2}\sigma_V^2\Bigg)\Bigg\}+\theta\Bigg\{\sigma_{y(2)}^2\\ +g^2\frac{\mu_y^2}{\mu_x^2}\sigma_{x(2)}^2+2g\frac{\mu_y}{\mu_x}\rho_{yx(2)}\sigma_{y(2)}\sigma_{x(2)}+\Bigg(\sigma_{U(2)}^2+g^2\frac{\mu_y^2}{\mu_x^2}\sigma_{V(2)}^2\Bigg)\Bigg\}\Bigg],\end{aligned} \tag{21}$$

$$\begin{aligned}MSE(\hat{t}_R) = \Bigg[\lambda_2\Bigg\{\sigma_y^2+\frac{g^2}{\mu_x^2}\sigma_x^2-2\frac{g}{\mu_x}\rho_{yx}\sigma_y\sigma_x+\Bigg(\sigma_U^2+\frac{g^2}{\mu_x^2}\sigma_V^2\Bigg)\Bigg\}+\\ \theta\Bigg\{\sigma_{y(2)}^2+\frac{g^2}{\mu_x^2}\sigma_{x(2)}^2-2\frac{g}{\mu_x}\rho_{yx(2)}\sigma_{y(2)}\sigma_{x(2)}+\Bigg(\sigma_{U(2)}^2+\frac{g^2}{\mu_x^2}\sigma_{V(2)}^2\Bigg)\Bigg\}\Bigg],\end{aligned} \tag{22}$$

$$\begin{aligned}MSE(\hat{t}_P) = \Bigg[\lambda_2\Bigg\{\sigma_y^2+\frac{g^2}{\mu_x^2}\sigma_x^2+2\frac{g}{\mu_x}\rho_{yx}\sigma_y\sigma_x+\Bigg(\sigma_U^2+\frac{g^2}{\mu_x^2}\sigma_V^2\Bigg)\Bigg\}+\\ \theta\Bigg\{\sigma_{y(2)}^2+\frac{g^2}{\mu_x^2}\sigma_{x(2)}^2+2\frac{g}{\mu_x}\rho_{yx(2)}\sigma_{y(2)}\sigma_{x(2)}+\Bigg(\sigma_{U(2)}^2+\frac{g^2}{\mu_x^2}\sigma_{V(2)}^2\Bigg)\Bigg\}\Bigg].\end{aligned} \tag{23}$$

From (17-23) and equation (14), one can obtain the following

$$\begin{aligned}MSE(\hat{\mu}_y^*)-MSE(\hat{t}_s)\geq 0\\ if\,\Delta \leq -\frac{2\mu_x A_o}{gB_o},\end{aligned} \tag{24}$$

$$\begin{aligned}MSE(t_R)-MSE(\hat{t}_s)\geq 0\\ if\,\frac{-2A_o+\mu_x B_o}{gB_o}\leq \Delta \leq -\frac{2\mu_y}{g},\end{aligned} \tag{25}$$

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$$MSE(t_P) - MSE(\hat{t}_s) \geq 0$$
$$if \frac{-2A_o - \mu_x B_o}{gB_o} \leq \Delta \leq \frac{\mu_y}{g}, \tag{26}$$

$$MSE(t'_R) - MSE(\hat{t}_s) \geq 0$$
$$if \Delta \leq \frac{gB_o\mu_x - 2A_o\mu_x}{gB_o}, \tag{27}$$

$$MSE(t'_P) - MSE(\hat{t}_s) \geq 0$$
$$if \Delta \geq \frac{2A_o\mu_x - gB_o\mu_x}{gB_o}, \tag{28}$$

$$MSE(\hat{t}_R) - MSE(\hat{t}_s) \geq 0$$
$$if \Delta \geq \frac{gB_o - 2A_o\mu_x}{gB_o}, \tag{29}$$

$$MSE(\hat{t}_P) - MSE(\hat{t}_s) \geq 0$$
$$if \Delta \geq \frac{2A_o\mu_x - gB_o}{gB_o}. \tag{30}$$

If the conditions (24-30) holds true, the proposed estimator $\hat{t}_s$ is more efficient than other mentioned estimators when there is non-response and measurement error on the study as well as auxiliary variables.

## 5 Empirical Study

In this section, we demonstrate the performance of different estimators over the usual unbiased estimators by generating four populations from normal distribution with different choices of parameters using R language program. The auxiliary information on variable $'X'$ has been generated from N (5, 10) population. This type of population is very relevant in mostly all socio-economic situations with one study and one auxiliary variable.

**Population I** $X = N(5,10); Y = X + N(0,1); y = Y + N(1,3); X = X + N(1,3); N = 5000; \mu_Y = 4.927167; \mu_X = 4.924306; \sigma_y^2 = 102.0075; \sigma_x^2 = 101.4117; \sigma_U^2 = 8.862114; \sigma_V^2 = 9.001304; \rho_{yx} = 0.995059$

| $N_1$ | $N_2$ | $\sigma^2_{y(2)}$ | $\sigma^2_{x(2)}$ | $\sigma^2_{U(2)}$ | $\sigma^2_{V(2)}$ | $\rho_{yx(2)}$ |
|---|---|---|---|---|---|---|
| 4500 | 500 | 99.99174 | 99.87471 | 9.150544 | 8.756592 | 0.994916 |
| 4250 | 750 | 100.9428 | 100.8224 | 9.053862 | 8.766538 | 0.994916 |
| 4000 | 1000 | 104.2711 | 103.2349 | 8.821278 | 8.339179 | 0.995472 |

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**Population II** $X = N(5,10); Y = X + N(0,1); y = Y + N(1,5); X = X + N(1,5); N = 5000; \mu_Y = 4.996681; \mu_X = 5.013507; \sigma_y^2 = 97.12064; \sigma_x^2 = 95.95803; \sigma_U^2 = 23.96055; \sigma_V^2 = 24.19283; \rho_{yx} = 0.994822$

| $N_1$ | $N_2$ | $\sigma^2_{y(2)}$ | $\sigma^2_{x(2)}$ | $\sigma^2_{U(2)}$ | $\sigma^2_{V(2)}$ | $\rho_{yx(2)}$ |
|---|---|---|---|---|---|---|
| 4500 | 500 | 97.02783 | 94.54578 | 22.80557 | 25.43263 | 0.994546 |
| 4250 | 750 | 98.27616 | 97.42674 | 23.27837 | 24.13829 | 0.994992 |
| 4000 | 1000 | 96.09359 | 94.71923 | 24.42978 | 23.03076 | 0.99467 |

**Population III** $X = N(5,10); Y = X + N(0,1); y = Y + N(2,3); X = X + N(2,3); N = 5000; \mu_Y = 4.730993; \mu_X = 4.741928; \sigma_y^2 = 101.2633; \sigma_x^2 = 100.2288; \sigma_U^2 = 9.1025; \sigma_V^2 = 9.052019; \rho_{yx} = 0.995187$

| $N_1$ | $N_2$ | $\sigma^2_{y(2)}$ | $\sigma^2_{x(2)}$ | $\sigma^2_{U(2)}$ | $\sigma^2_{V(2)}$ | $\rho_{yx(2)}$ |
|---|---|---|---|---|---|---|
| 4500 | 500 | 102.7504 | 101.2097 | 9.095136 | 8.8123 | 0.995045 |
| 4250 | 750 | 99.55993 | 99.49764 | 9.233619 | 8.805872 | 0.995314 |
| 4000 | 1000 | 105.4334 | 103.8947 | 9.277715 | 9.072151 | 0.995105 |

**Population IV** $X = N(5,10); Y = X + N(0,1); y = Y + N(2,5); X = X + N(2,5); N = 5000; \mu_Y = 4.961081; \mu_X = 4.96178; \sigma_y^2 = 102.2408; \sigma_x^2 = 100.8680; \sigma_U^2 = 25.94111; \sigma_V^2 = 25.03951; \rho_{yx} = 0.394221$

| $N_1$ | $N_2$ | $\sigma^2_{y(2)}$ | $\sigma^2_{x(2)}$ | $\sigma^2_{U(2)}$ | $\sigma^2_{V(2)}$ | $\rho_{yx(2)}$ |
|---|---|---|---|---|---|---|
| 4500 | 500 | 103.5361 | 102.1031 | 25.31099 | 22.84483 | 0.394622 |
| 4250 | 750 | 103.6790 | 102.7446 | 24.6859 | 26.12337 | 0.395036 |
| 4000 | 1000 | 100.1031 | 99.31665 | 25.80394 | 24.50468 | 0.394778 |

**Table 1:** Percent relative efficiencies of the estimators with respect to the usual unbiased estimator for Population I

| $N1$ | $N_2$ | k | **PRE($\hat{t}_s$)** | PRE($\hat{\mu}_y^*$) | PRE($t_R$) | PRE($t_P$) | PRE($t_R'$) | PRE($t_P'$) | PRE($\hat{t}_R$) | PRE($\hat{t}_P$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 4500 | 500 | 2 | **611.3859** | 100 | 586.1473 | 26.155 | 123.5517 | 82.28832 | 104.2404 | 95.99756 |
| 4250 | 750 | 2 | **612.4949** | 100 | 587.2272 | 26.15129 | 123.5588 | 82.285 | 104.2414 | 95.99667 |
| 4000 | 1000 | 2 | **618.4895** | 100 | 593.7605 | 26.16283 | 123.5637 | 82.28579 | 104.2418 | 95.99656 |
| 4500 | 500 | 3 | **610.4908** | 100 | 585.2622 | 26.15738 | 123.5467 | 82.29064 | 104.2397 | 95.9982 |
| 4250 | 750 | 3 | **612.5058** | 100 | 587.2254 | 26.15069 | 123.5594 | 82.28464 | 104.2416 | 95.99658 |
| 4000 | 1000 | 3 | **622.7055** | 100 | 598.3392 | 26.117023 | 123.5677 | 82.286 | 104.2421 | 95.9964 |
| 4500 | 500 | 4 | **609.7453** | 100 | 584.525 | 26.15938 | 123.5423 | 82.29258 | 104.239 | 95.99872 |
| 4250 | 750 | 4 | **612.5143** | 100 | 587.224 | 26.15023 | 123.5599 | 82.28436 | 104.2416 | 95.9965 |
| 4000 | 1000 | 4 | **625.8268** | 100 | 601.7272 | 26.17563 | 123.5706 | 82.28615 | 104.2423 | 95.99628 |
| 4500 | 500 | 5 | **609.1146** | 100 | 583.9015 | 26.16107 | 123.5389 | 82.29423 | 104.2385 | 95.99917 |
| 4250 | 750 | 5 | **612.5211** | 100 | 587.2229 | 26.14985 | 123.5603 | 82.28413 | 104.2417 | 95.99645 |
| 4000 | 1000 | 5 | **628.2307** | 100 | 593.3356 | 26.17976 | 123.5729 | 82.28626 | 104.2425 | 95.99619 |

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**Table 2:** Percent relative efficiencies of the estimators with respect to the usual unbiased estimator for Population II

| $N1$ | $N_2$ | k | **PRE($\hat{t}_s$)** | PRE($\hat{\mu}_y^*$) | PRE($t_R$) | PRE($t_P$) | PRE($t_R'$) | PRE($t_P'$) | PRE($\hat{t}_R$) | PRE($\hat{t}_P$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 4500 | 500 | 2 | **272.9998** | 100 | 246.7997 | 28.02771 | 119.5495 | 84.18319 | 103.5922 | 96.55749 |
| 4250 | 750 | 2 | **274.0183** | 100 | 247.8102 | 28.01111 | 119.579 | 84.16816 | 103.5967 | 96.55358 |
| 4000 | 1000 | 2 | **273.284** | 100 | 247.5285 | 28.0755 | 119.5172 | 84.20558 | 103.5866 | 96.56272 |
| 4500 | 500 | 3 | **272.8247** | 100 | 246.5344 | 28.01987 | 119.5527 | 84.18034 | 103.5929 | 96.55688 |
| 4250 | 750 | 3 | **274.6241** | 100 | 248.3268 | 27.99153 | 119.6041 | 84.1543 | 103.6007 | 96.55008 |
| 4000 | 1000 | 3 | **273.3338** | 100 | 247.8097 | 28.10205 | 119.4975 | 84.21871 | 103.5832 | 96.56582 |
| 4500 | 500 | 4 | **272.679** | 100 | 246.3135 | 28.01332 | 119.5554 | 84.177978 | 103.5934 | 96.55636 |
| 4250 | 750 | 4 | **275.0968** | 100 | 248.7298 | 27.97633 | 119.6236 | 84.14354 | 103.6037 | 96.54736 |
| 4000 | 1000 | 4 | **273.371** | 100 | 248.0165 | 28.12157 | 119.4831 | 84.22835 | 103.5808 | 96.5681 |
| 4500 | 500 | 5 | **272.5558** | 100 | 246.1266 | 28.00778 | 119.5577 | 84.17596 | 103.5939 | 96.55593 |
| 4250 | 750 | 5 | **275.4759** | 100 | 249.0539 | 27.96419 | 119.6391 | 84.13493 | 103.6062 | 96.54519 |
| 4000 | 1000 | 5 | **273.3998** | 100 | 248.1751 | 28.13653 | 119.472 | 84.23573 | 103.5789 | 96.56984 |

**Table 3:** Percent relative efficiencies of the estimators with respect to the usual unbiased estimator for Population III

| $N1$ | $N_2$ | k | **PRE($\hat{t}_s$)** | PRE($\hat{\mu}_y^*$) | PRE($t_R$) | PRE($t_P$) | PRE($t_R'$) | PRE($t_P'$) | PRE($\hat{t}_R$) | PRE($\hat{t}_P$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 4500 | 500 | 2 | **601.5921** | 100 | 579.6432 | 26.33387 | 123.3383 | 82.40343 | 104.3867 | 95.86719 |
| 4250 | 750 | 2 | **600.0930** | 100 | 577.75412 | 26.31837 | 123.3498 | 82.39595 | 104.3887 | 95.86531 |
| 4000 | 1000 | 2 | **602.8336** | 100 | 580.9905 | 26.33597 | 123.3398 | 82.40336 | 104.3868 | 95.86710 |
| 4500 | 500 | 3 | **602.5760** | 100 | 580.7790 | 26.33891 | 123.3360 | 82.40535 | 104.3862 | 95.86764 |
| 4250 | 750 | 3 | **599.8685** | 100 | 577.3757 | 26.31135 | 123.3563 | 82.39209 | 104.3898 | 95.86431 |
| 4000 | 1000 | 3 | **604.5177** | 100 | 582.8746 | 26.34164 | 123.3389 | 82.40492 | 104.3865 | 95.86742 |
| 4500 | 500 | 4 | **603.3976** | 100 | 581.7269 | 26.34311 | 123.3341 | 82.40694 | 104.3858 | 95.86801 |
| 4250 | 750 | 4 | **599.6940** | 100 | 577.0808 | 26.30589 | 123.3614 | 82.38907 | 104.3907 | 95.86353 |
| 4000 | 1000 | 4 | **605.7497** | 100 | 584.2524 | 26.34576 | 123.3382 | 82.40605 | 104.3863 | 95.86765 |
| 4500 | 500 | 5 | **604.0940** | 100 | 582.5301 | 26.34665 | 123.3325 | 82.40829 | 104.3855 | 95.86832 |
| 4250 | 750 | 5 | **599.5544** | 100 | 576.8447 | 26.30151 | 123.3655 | 82.38666 | 104.3914 | 95.86291 |
| 4000 | 1000 | 5 | **606.6901** | 100 | 585.3037 | 26.34889 | 123.3377 | 82.40692 | 104.3861 | 95.86782 |

**Table 4:** Percent relative efficiencies of the estimators with respect to the usual unbiased estimator for Population IV

| $N1$ | $N_2$ | k | **PRE($\hat{t}_s$)** | PRE($\hat{\mu}_y^*$) | PRE($t_R$) | PRE($t_P$) | PRE($t_R'$) | PRE($t_P'$) | PRE($\hat{t}_R$) | PRE($\hat{t}_P$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 4500 | 500 | 2 | **354.8893** | 100 | 328.7961 | 27.14476 | 121.2849 | 83.33059 | 103.9072 | 96.28215 |
| 4250 | 750 | 2 | **301.8073** | 100 | 274.5276 | 27.52986 | 120.3716 | 83.75985 | 103.7712 | 96.39880 |
| 4000 | 1000 | 2 | **271.2105** | 100 | 244.0121 | 27.93889 | 119.5888 | 84.14978 | 103.6517 | 96.50254 |
| 4500 | 500 | 3 | **270.4614** | 100 | 243.5957 | 27.98911 | 119.5372 | 84.18031 | 103.6432 | 96.51016 |
| 4250 | 750 | 3 | **226.8626** | 100 | 198.3265 | 28.61770 | 118.1727 | 84.85689 | 103.4347 | 96.69096 |
| 4000 | 1000 | 3 | **204.2863** | 100 | 175.7478 | 29.26783 | 117.0891 | 85.43617 | 103.2636 | 96.84161 |
| 4500 | 500 | 4 | **228.7139** | 100 | 201.1703 | 29.71607 | 118.1516 | 84.88476 | 103.4294 | 96.69652 |
| 4250 | 750 | 4 | **193.4410** | 100 | 163.9436 | 29.49841 | 116.5639 | 85.70503 | 103.1819 | 96.91288 |
| 4000 | 1000 | 4 | **176.0908** | 100 | 146.5301 | 30.29490 | 115.3703 | 86.37732 | 103.9887 | 97.08473 |
| 4500 | 500 | 5 | **203.9089** | 100 | 175.7708 | 29.34852 | 117.0261 | 85.47826 | 103.2526 | 96.85169 |
| 4250 | 750 | 5 | **174.6404** | 100 | 144.3668 | 30.22601 | 115.3357 | 86.38035 | 103.9850 | 96.08718 |
| 4000 | 1000 | 5 | **160.6783** | 100 | 130.3080 | 31.11247 | 114.1159 | 87.09577 | 103.7838 | 96.26760 |

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Tables 1 to 4 demonstrate that the proposed estimator $\hat{t}_s$ performs better compared to the other estimators considered in the study in terms of gaining efficiency. according to the Tables 1 and 2, it is envisaged that the percent relative efficiency (PRE) of the proposed estimator $\hat{t}_s$ decreases with the increase in the value k, when $N_1 = 4500; N_2 = 500$. However, PRE of $\hat{t}_s$ increases with the increase in the value of k, when $N_1 = 4250; N_2 = 750$ and $N_1 = 4000; N_2 = 1000$ for population I and II. Table 3 shows that the PRE of the proposed estimator $\hat{t}_s$ decreases with the increase in the value of k when $N_1 = 4250; N_2 = 750$, while it increases when $N_1 = 4500; N_2 = 500$ and $N_1 = 4000; N_2 = 1000$. Furthermore, Table 4 indicates that the PRE of $\hat{t}_s$ decreases with the increase in the value of k in all cases.

## 6 Conclusion

In the present paper, we have proposed an estimator for estimating the population mean of the study variable. The suggested estimator used auxiliary information to improve efficiencies in the situation when there are non-response and measurement errors on study variable and auxiliary variable. The relative performance of the proposed estimators was compared with the conventional estimators. The proposed estimator performs better than the usual unbiased estimator, ratio estimator and product estimators, transformed ratio and product estimators and dual to ratio and product estimators in the presence of measurement and non-response error. The study was supported by empirical study based on four populations. We recommend a proposed estimator for future study to investigate the characteristics of the variable of interest when there is measurement and non-response occurs in the survey.

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**Conflict of Interest**
The authors declare that they have no conflict of interest.

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