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Fuzzy Inventory Model without Shortages Using Signed Distance Method

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In this paper an inventory model without shortage is considered under fuzzy environment. Costs involved; ordering cost and holding cost, imprecise in nature, considered as fuzzy parameters. Representing both the costs by fuzzy triangular number, the optimum order quantity is calculated using Signed Distance Method for defuzzification. Method is illustrated numerically along with sensitivity analysis.

Keywords: Fuzzy triangular number, signed distance method, defuzzification.

1 Introduction

Various types of uncertainties and imprecision is inherent in real inventory problems. They are classically modeled using the approaches from the probability theory. However, there are uncertainties that cannot be appropriately treated by usual probabilistic models. The question arise how to define inventory optimization tasks in such environment and how to interpret optimal solutions ([1], [2]). Therefore it becomes more convenient to deal such problems with fuzzy set theory rather than probability theory [3].

Park [4] and Vujosevic et al [5] developed the inventory models in fuzzy sense where ordering cost and holding cost are represented by fuzzy numbers. Park has represented costs as trapezoidal fuzzy numbers. Wherein Vujosevic et al represented ordering cost by triangular fuzzy number and holding cost by trapezoidal fuzzy number. Centroid of fuzzy total cost was taken as the estimate for fuzzy total cost.

But study shows that, in spite of centroid method, the signed distance method is better for defuzzification [6]. Therefore, an inventory model without shortages is considered in fuzzy sense and the signed distance method is used for defuzzification.

1- AN INVENTORY MODEL WITHOUT SHORTAGES

As mentioned earlier, the sign distance method is more suitable and appropriate for defuzzification. Therefore, in this chapter an inventory model without shortages is developed with fuzzy parameters ([7], [8]). Ordering cost and holding cost are represented by fuzzy triangular membership function and fuzzy total cost is obtained by applying signed distance method.

2- ASSUMPTION AND NOTATIONS

Assumptions:
Following assumptions are made for the development of model under study [9]:
1. Shortages are not allowed.
2. Instantaneous replenishment.
3. Lead- time is zero.
4. Demand is constant.

Notations:

T – Length of plan;
c – Cost of placing an order;
a – Cost of storage a unit;
r – Demand in $[0.T]$;
t_q – Length of a cycle;
q – Order quantity;
$\widehat{c}$ – Fuzzy cost of placing an order;
$\widehat{a}$ –Fuzzy cost of storage a unit.

2 Crisp Mathematical Model

In scientific inventory management, a formula for economic order quantity (EOQ) given by Harris was considered as a point of departure, which is given as:

$$q^* = \sqrt{\frac{2cr}{aT}}, \tag{2.1}$$

which minimizes the total inventory cost derived from the sum of ordering cost and inventory holding cost.

$$\text{Total inventory cost=holding cost+ordering cost} \tag{2.2}$$

$$TC = \frac{aTq}{2} + \frac{cr}{q} \tag{2.3}$$

Where a, T, q and r are as explained in notations.

Model can be represented diagrammatically in figure 2.1.

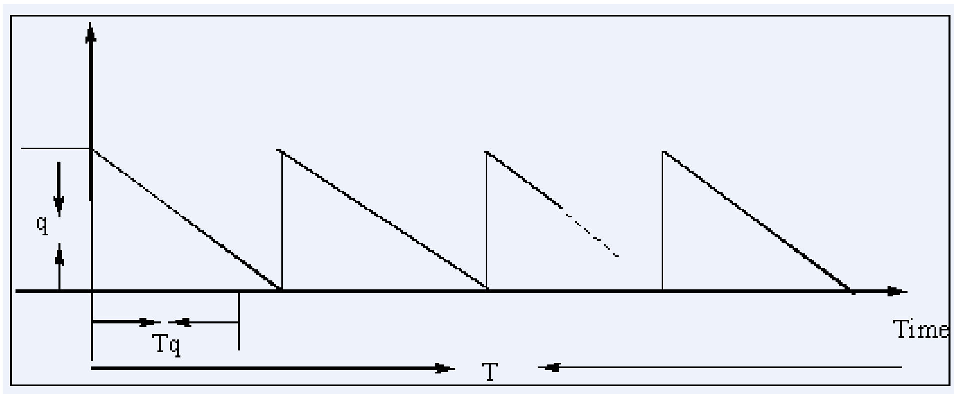

Figure 2.1: Inventory model without shortages

3 Fuzzy Mathematical Model

Fuzzy mathematical model considers the modification of EOQ formula in the presence of imprecise parameters [10]. In practice, holding and ordering costs are often not precisely known and usually expressed by linguistic terms. Fuzzy numbers represents these imprecise parameters [11].

Rewriting (2.3) we get

$$\widehat{T}C = \frac{\widehat{a}Tq}{2} + \frac{\widehat{c}r}{q}, \tag{3.1}$$

where wavy bar (˜) indicates fuzzification of parameters. Using Signed Distance Method, optimum solution for (2.1) is determined.

In practical problems, it is not easy to decide the ordering cost for long period due to some uncountable factors. Therefore it becomes reasonable to locate an ordering cost in an interval $[c - \Delta_1, c - \Delta_2]$ where $0 < \Delta_1 < c$ and $\Delta_1\Delta_2 > 0$ such as Δ_1, Δ_2 are chosen appropriately.

Once interval is chosen then there is need to find an approperiate value in the interval $[c - \Delta_1, c - \Delta_2]$. If "$c$" is chosen then there is need to find an appropriate value in the interval . If Δ is chosen then it is coincident with the ordering cost. In crisp case error is considered as 0. If the value is within interval, then the error is larger when the value deviates from ' c' farther. Obviously error will attain its maximum at $c - \Delta_1$ and $c + \Delta_2$.

From fuzzy point of view, error can be changed to confidence level such as: if error is zero then confidence level to be 1. If error is maximum which attains at $c - \Delta_1$ and $c + \Delta_2$, then confidence level be zero. If value is chosen from an interval $[c-\Delta_1, c-\Delta_2]$, then confidence level is any real value between 0 and 1.

Hence, above condition can be suitably represented by fuzzy triangular number,

$$\widehat{c} = (c-\Delta_1, c, c+\Delta_2) \tag{3.2}$$

and $0<\Delta_1<c$ and $\Delta_1\Delta_2>0$.

Where the membership grade of c for $\widehat{c}$ is 1. For the points in the interval $[c-\Delta_1, c-\Delta_2]$, as value is far from 'c ' membership grade is less, and at the points $0<\Delta_1<c$ and $\Delta_1\Delta_2>0$ membership grade is zero. Therefore, it is natural and reasonable to correspond the interval $[c-\Delta_1, c-\Delta_2]$ to the fuzzy number $\widehat{c}$ in equation (3.2), when we respond membership grade to confidence level [12].

The signed distance of $\widehat{c}$ is given by,

$$d(c,0) = \frac{3}{4}c + \frac{1}{4}(c-\Delta_1), \tag{3.3}$$

$$d(\widetilde{c},\widetilde{0}) > 0 \quad \text{and} \quad d(\widetilde{c},\widetilde{0}) \in [c-\Delta_1, c+\Delta_2] \tag{3.4}$$

$d(\widetilde{c},\widetilde{0})$ can be taken as the estimate of total ordering cost during the plan period [0, T] in the fuzzy sense based on the singed distance.

As a fact, we know that in a perfect competitive market, the cost of storing a unit per day in a plan period T may fluctuate a little from its actual value. Suppose it lies in the interval $[a-\Delta_3, a+\Delta_4]$. Similarly, as discussed above in the case of ordering cost, we can find a fuzzy triangular number to represent the vagueness in holding cost as:

$$\begin{aligned} \widetilde{a} &= (a-\Delta_3, a+\Delta_4), \\ 0 &< \Delta_3, \qquad \Delta_3\,\Delta_4 > 0, \end{aligned}$$

where the membership grade of a for $\widetilde{a}$ is 1. For the points in the interval $[a-\Delta_3, a+\Delta_4]$ membership grade is less as values within interval farther from 'a'. For the points $a-\Delta_3$ and $a-\Delta_4$ membership grade is 0. Then the signed distance of $\widetilde{a}$ is given by

$$\begin{aligned} d(\widetilde{a},\widetilde{0}) &= a + \frac{1}{4}(\Delta_4-\Delta_3) \\ &= \frac{3}{4}a + \frac{1}{3}(a-\Delta_3) \end{aligned}$$

where $d(\widetilde{a},\widetilde{0}) > 0$ and $d(\widetilde{a},\widetilde{0}) \in (a-\Delta_3, a+\Delta_4)$. Also, $d(\widetilde{a},\widetilde{0})$ can be taken as an estimate of storing cost of a unit per day in the fuzzy sense based on the signed distance.

From Equation (1.3), we have, for any q>0,

$$Fq(c,a) = TC = \frac{1}{2}aTq + \frac{cr}{q}. \tag{3.5}$$

Using equation (3.1), (3.2) and (3.5), we get the fuzzy total cost as:

$$Fq(\widetilde{c},\widetilde{a}) = TC = \left(\left(\frac{T}{2}q\right)\otimes\widetilde{a}\right)\oplus\left(\left(\frac{\widetilde{r}}{q}\right)\otimes\widetilde{c}\right), \tag{3.6}$$

where $\frac{T}{2}q$ and $\frac{\widetilde{r}}{q}$ are fuzzy points. Now $q > 0, T > 0$ and $c > 0$. Equation (3.1), (3.2) and (3.6) simultaneously gives,

$$Fq(\widetilde{c},\widetilde{a}) = (F_1, F_2, F_3), \tag{3.7}$$

where $Fq(\widetilde{c},\widetilde{a})$ is an estimate of fuzzy total cost. And F_1, F_2, F_3 can be calculated as:

$$\begin{aligned} F_1 &= \frac{Tq}{2}(a-\Delta_3)+\frac{r}{q}(c-\Delta_3) \\ &= \frac{aTq}{2}+\frac{cr}{q}-\left(\frac{Tq\Delta_3}{2}+\frac{r\Delta_1}{q}\right) \\ &= Fq(c,a)+\left(\frac{Tq\Delta_3}{2}+\frac{r\Delta_1}{q}\right) \end{aligned} \tag{3.8}$$

$$F_2 = Fq(c,a) = \left(\frac{aTq}{2}+\frac{cr}{q}\right) \tag{3.9}$$

$$\begin{aligned} F_3 &= \frac{Tq}{2}(a+\Delta_4)+\frac{r}{q}(c+\Delta_2) \\ &= \frac{aTq}{2}+\frac{cr}{q}+\left(\frac{Tq\Delta_4}{2}+\frac{r\Delta_2}{q}\right) \\ &= Fq(c,a)+\left(\frac{Tq\Delta_4}{2}+\frac{r\Delta_2}{q}\right) \end{aligned} \tag{3.10}$$

Defuzzication of fuzzy number $Fq(\widetilde{c},\widetilde{a})$ by using singed distance method is given as:

$$d(Fq(\widetilde{c},\widetilde{a}),0) = Fq(c,a)+\frac{1}{4}\left[\frac{T}{2}\left(\Delta_4-\Delta_3\right)q+\frac{r}{q}\left(\Delta_2-\Delta_1\right)\right] = F_d(q) \tag{3.11}$$

Equation (3.11) gives an estimate of the total cost in the fuzzy sense based on singed distance. Differentiating equation (3.11) partially w. r. to q and equating it to zero, we get

$$\begin{aligned} \frac{d}{dq}(F_d(q) &= \frac{1}{2}at-\frac{cr}{q^2}+\left[\frac{T}{8}\left(\Delta_4-\Delta_3\right)q+\frac{r}{4q^2}\left(\Delta_2-\Delta_1\right)\right] \\ &\Rightarrow \left(4a+\Delta_4-\Delta_3\right)Tq^2 = 8r(c+\Delta_2-\Delta_1) \end{aligned}$$

Then

$$q_d = \sqrt{\frac{2r(c+\Delta_2-\Delta_1)}{\left(a+\Delta_4-\Delta_3\right)T}} \tag{3.12}$$

Also,

$$\frac{d^2}{dq^2}(F_d(q) = \frac{2c}{q^3}\left(r + \frac{1}{4}(\Delta_2 - \Delta_1)\right) > 0 \tag{3.13}$$

$Fd(q, d)$ is minimum by the principle of minima and maxima.

4 Numerical Example and Sensitivity Analysis

(i) Crisp Model

Let, Ordering Cost,$c = Rs.20$ per unit, Holding Cost, $a = Rs.12$ per unit, Demand, $r = 500$ units and Length of Plan, $T = 6$ days. Then we obtain Economic Order Quantity, $q* = 16.66$ units and Total Cost $= Rs.1200$.

(ii) Fuzzy Model

Table-A: Calculation of Optimum Order Quantity and Total Cost

input
Signed Distance Method
$c = 20, r = 500, a = 12, T = 6$
$\Delta_1 = 4, \Delta_2 = 5, \Delta_3 = 2, \quad \Delta_4 = 1$

output
$q_d = 16.73, TC = 1279$

Table-B: Sensitivity Analysis

Sr. No.	Demand Change%	Signed Distance Method qd*	TC
1	450	15.56	1214.16
2	475	15.99	1247.43
3	525	16.81	1311.41
4	550	17.20	1342.31

Observations:

Table-A shows the calculations for Economic Order Quantity and Total Cost. It is observed that:

(i) The Economic Order Quantity obtained by Signed Distance Method is closer to crisp Economic Order Quantity.

(ii) Total cost obtained by Signed distance Method is more than crisp total cost.

From Table-B, it is observed that:

(i) Economic Order Quantity is more sensitive towards demand.

(ii) Total cost increases as the demand increases.

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