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Testing Exponentiality Against RNBUL Class of Life Distribution Based on Goodness of Fit

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Abstract: In this paper a new test statistic for testing exponentiality against renewal new better than used in Laplace transform order (RNBUL) based on goodness of fit is studied. selected critical values are tabulated for sample size 5(5)50. Pitman's asymptotic efficiencies of the test and Pitman's asymptotic relative efficiencies (PARE) are calculated . The power of this test are estimated for some famous alternatives distributions in reliability such as Wiebull, linear failure rate (LFR) and Gamma distributions. The problem in case of right censored data is also handled. Finally, some applications to elucidate the usefulness of the proposed test in reliability analysis are discussed.

Keywords: Classes of life distributions, RNBUL, Testing Exponentiality, goodness of fit, U-statistic, Pitman asymptotic efficiency,Censored data.

1 Introduction

Over the last few decades, classes of life distributions is a new branche of reliability which is established by developing the measurments of success or failure concerning reliability. Many experts in statistics and reliability analysts have shown great intrest in displying survival data using classification of life distributions based on different aspects of aging cocepts that describe how population of units or systems improved or depreciates with age. Engineering,biological science, maintenance and biometrics are some important applications of classes of life distributions that can be seen in reliability.

It was noticed that the exponential distribution is a fundamental distribution of statistical reliability theory, see for example [8, 32]. As stated for the common classes of life distributions that contains most of previously known classes like increasing failure rate (IFR), increasing failure rate average (IFRA), new better than used (NBU), new better than used in average (NBUA), new better than used in expectation (NBUE) and harmonic new better than used in expectation (HNBUE). The implications between these classes are

$$IFR \Rightarrow IFRA \Rightarrow NBU \Rightarrow NBUA \Rightarrow NBUE \Rightarrow HNBUE$$

see [8].

Testing exponentiality against the above classes of life distributions has seen a good deal of attention. We refer to works found in [3,7,9,29] for testing against IFR, among others. References is made to [4,6,10,21] for testing exponentiality against IFRA. Testing exponentiality against NBU is discussed by [5,15,19], among others. However, testing against NBUE is considered by [15,19], among others. Finally, testing against HNBUE can be found in work of [18,30]. Other relevent aging criteria have been introduced by different authors.

Some authors took up testing exponentiality based on goodness of fit technique versus many classes of life distributions; see [2,11,16,22,23,26].

Let X be a nonnegative random variable with distribution function F and survival function $\bar{F} = 1 - F$. Assume that X is continuous with probability density function f and has mean μ and variance σ^2.

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Consider a device with life lenght X and life distribution F. The device is replaced instantly upon failure by sequance of mutually independent devices. These devices are independent of the first unit and identically distributed with the same life distribution F. When the renewal of the system is continued indefinitly, the stationary life distribution of a device in operation at time t is

$$W_f(t)=\mu_F^{-1}\int_0^t \bar{F}(u)du, \qquad 0\le t<\infty,$$

the corresponding renewal survival function is

$$\bar{W}_f(t)=\mu_F^{-1}\int_t^\infty \bar{F}(u)du, \qquad 0\le t<\infty,$$

where $\mu_F=\mu=\int_0^\infty \bar{F}(u)du$.

For details, see [1,8].

Testing exponentiality against NBRUL is stadied by [24,25]. The class of life distribution RNBU is defined as follows:

Definition 1. *(Abouammoh et al. [1]) If X is a random variable with survival function $\bar{F}(x)$; then X is said to have renewal new better (worth) than used property, denoted by $RNBU(RNWU)$; if:*

$$\bar{W}_F(x+t)\le(\ge)\bar{W}_F(x)\bar{W}_F(t), \qquad x\ge 0, t\ge 0.$$

Now, depending on Defination 1 we define a new class of life distributions called renewal new better (worth) than used in Laplace transform order.

Definition 2. *If X is a random variable with survival function $\bar{F}$; then X is said to be renewal new better (worth) than used in Laplace transform order, denoted by $RNBUL$ $(RNWUL)$; if:*

$$\int_0^\infty e^{-sx}\overline{W}_F(x+t)dx\le(\ge)\overline{W}_F(t)\int_0^\infty e^{-sx}\overline{W}_F(x)dx, \qquad x\ge 0, t\ge 0. \tag{1}$$

2 Hypothesis Testing Against RNBUL Class for Non-censored Data

In this section, we test the null hypotheses $H_0: F$ is exponential with mean μ against $H_1: F$ is $RNBUL$ and not exponential.

The following lemma is needed.

Lemma 1. *If F belongs to RNBUL class and X is a random varialble with distribution function F then*

$$\delta(\gamma)=\frac{\mu}{\gamma(\gamma-1)}\zeta(\gamma)+\frac{\mu}{\gamma-\gamma^2}\zeta(1)-\frac{1}{\gamma^2}\zeta(\gamma)-\frac{1}{\gamma^2}\zeta(1)+\frac{1}{\gamma^2}\zeta(1)\zeta(\gamma)+\frac{1}{\gamma^2}, \tag{2}$$

where $\zeta(\gamma)=\int_0^\infty e^{-\gamma x}\,dF(x)$.

Proof. Since F is RNBUL, recall Def.2 and multiplying both sides by e^{-t}, and integrating over $[0,\infty)$ with respect to t, we get

$$\int_0^\infty\int_0^\infty e^{-t}e^{-\gamma x}\overline{W}_F(x+t)dxdt\le\int_0^\infty e^{-t}\overline{W}_F(t)\int_0^\infty e^{-\gamma x}\overline{W}_F(x)dxdt. \tag{3}$$

Setting

$$I=\int_0^\infty\int_0^\infty e^{-t}e^{-\gamma x}\overline{W}_F(x+t)dxdt,$$

take $u=x+t, v=t\Rightarrow|J|=1$, where J is called the Jacobian of the transformation. Hence

$$I=\int_0^\infty\int_0^v e^{-v}e^{-\gamma(v-u)}\overline{W}_F(v)dudv,$$

then

$$I=\frac{1}{1-\gamma}[\int_0^\infty e^{-\gamma v}\overline{W}_F(v)dv-\int_0^\infty e^{-v}\overline{W}_F(v)dv],$$

where

$$\int_0^\infty e^{-\gamma v}\overline{W}_F(v)dv=\frac{1}{\gamma\mu}[\mu-\frac{1}{\gamma}(1-\zeta(\gamma))],$$

then, we get

$$I=\frac{1}{1-\gamma}[\frac{1}{\gamma}-\frac{1}{\gamma^2\mu}(1-\zeta(\gamma))+\frac{1}{\mu}(1-\zeta(1))-1]. \tag{4}$$

Put

$$II=\int_0^\infty e^{-t}\overline{W}_F(t)\int_0^\infty e^{-\gamma x}\overline{W}_F(x)dxdt=\frac{1}{\gamma\mu^2}[\mu^2-\frac{\mu}{\gamma}(1-\zeta(\gamma))-\mu(1-\zeta(1))+\frac{1}{\gamma}(1-\zeta(\gamma))(1-\zeta(1))]. \tag{5}$$

From (4) and (5), we get

$$\frac{1}{1-\gamma}[\frac{1}{\gamma}-\frac{1}{\gamma^2\mu}(1-\zeta(\gamma))+\frac{1}{\mu}(1-\zeta(1))-1]\leq\frac{1}{\gamma\mu^2}[\mu^2-\frac{\mu}{\gamma}(1-\zeta(\gamma))-\mu(1-\zeta(1))+\frac{1}{\gamma}(1-\zeta(\gamma))(1-\zeta(1))]. \tag{6}$$

To estimate the measure of departure from exponentiality $\delta(\gamma)$, let $X_{1,}X_{2,}...X_{n,}$ be a random sample from a population with distribution function $F\in RNBUL$ class. From (6) we get Eq. (2).

Not that under $H_0:\delta(\gamma)=0$, and $H_1:\delta(\gamma)$ is positive.

2.1 Empirical Test Statistic for RNBUL Alternative

The empirical estimate of $\delta(\gamma)$, can be rewrritten as

$$\Lambda(\gamma)=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}[\frac{X_i}{\gamma(\gamma-1)}e^{-\gamma X_j}+\frac{X_i}{\gamma-\gamma^2}e^{-X_j}-\frac{1}{\gamma^2}e^{-\gamma X_i}-\frac{1}{\gamma^2}e^{-X_i}+\frac{1}{\gamma^2}e^{-X_j}e^{-\gamma X_j}+\frac{1}{\gamma^2}].$$

To make the test scale invariant under H_0, we use $\hat{\Lambda}(\gamma)=\frac{\Lambda(\gamma)}{\overline{X}}$, where $\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ is the sample mean. Then

$$\hat{\Lambda}(\gamma)=\frac{1}{n^2\overline{X}}\sum_{i=1}^{n}\sum_{j=1}^{n}[\frac{X_i}{\gamma(\gamma-1)}e^{-\gamma X_j}+\frac{X_i}{\gamma-\gamma^2}e^{-X_j}-\frac{1}{\gamma^2}e^{-\gamma X_i}-\frac{1}{\gamma^2}e^{-X_i}+\frac{1}{\gamma^2}e^{-X_j}e^{-\gamma X_j}+\frac{1}{\gamma^2}]. \tag{7}$$

Setting

$$\phi(X_1,X_2)=\frac{X_1}{\gamma(\gamma-1)}e^{-\gamma X_2}+\frac{X_1}{\gamma-\gamma^2}e^{-X_2}-\frac{1}{\gamma^2}e^{\gamma X_1}-\frac{1}{\gamma^2}e^{-X_1}+\frac{1}{\gamma^2}e^{-X_1}e^{-\gamma X_2}+\frac{1}{\gamma^2}, \tag{8}$$

and defining symmetric kernel

$$\psi(X_1,X_2)=\frac{1}{2!}\sum\phi(X_1,X_2),$$

where the summation is over all arrangements of $X_{1,}X_{2,}...X_n$, then $\hat{\Lambda}(\gamma)$ is equivalent to U-statistic

$$U_n=\frac{1}{\binom{n}{2}}\sum\phi(X_1,X_2).$$

the following theorem summarize the asymptotic properties of the test.

Theorem 1. *As $n \to \infty$, $\sqrt{n}[\hat{\Lambda}(\gamma) - \Lambda(\gamma)]$ is asymptotically normal with mean zero and variance*

$$\sigma^2(s) = Var\{\zeta(\gamma)[\frac{X}{\gamma(\gamma-1)} + \frac{e^{-X}}{\gamma^2} - \frac{1}{\gamma^2}] + \zeta(1)[\frac{e^{-\gamma X}}{\gamma^2} + \frac{X}{\gamma-\gamma^2} - \frac{1}{\gamma^2}] + e^{-\gamma X}[\frac{\mu}{\gamma(\gamma-1)} - \frac{1}{\gamma^2}] + e^{-X}[\frac{\mu}{\gamma-\gamma^2} - \frac{1}{\gamma^2}] + \frac{2}{\gamma^2}\}, \tag{9}$$

under H_0 the variance tends to

$$\sigma_0^2(\gamma) = \frac{(\gamma-1)^2}{6(\gamma+2)(2\gamma+1)(\gamma^2-1)^2}. \tag{10}$$

Proof. Using standard U-statistics theory, see [20], and by direct calculations we can find the mean and the variance as follows

$$\sigma^2 = var\{E\left[\phi^{(1)}(X_1, X_2)\right] + E\left[\phi^{(2)}(X_1, X_2)\right]\}, \tag{11}$$

recall definition of $\phi(X_1, X_2)$ in Eq. (8), thus it is easy to show that

$$E\left[\phi^{(1)}(X_1, X_2)\right] = \frac{X}{\gamma(\gamma-1)}\zeta(\gamma) + \frac{X}{\gamma-\gamma^2}\zeta(1) - \frac{1}{\gamma^2}e^{-\gamma X} - \frac{1}{\gamma^2}e^{-X} + \frac{1}{\gamma^2}e^{-X}\zeta(\gamma) + \frac{1}{\gamma^2}, \tag{12}$$

and

$$E\left[\phi^{(2)}(X_1, X_2)\right] = \frac{\mu}{\gamma(\gamma-1)}e^{-\gamma X} + \frac{\mu}{\gamma-\gamma^2}e^{-X} - \frac{1}{\gamma^2}\zeta(\gamma) - \frac{1}{\gamma^2}\zeta(1) + \frac{1}{\gamma^2}\zeta(1)e^{-\gamma X} + \frac{1}{\gamma^2}. \tag{13}$$

Upon using (11),(12) and (13), Eq. (9) is obtained.
Under H_0, (10) is obtained.
When $\gamma = 0.2$, $\sigma_0 = 0.193851$.

2.2 The Pitman Asymptotic Relative Efficiency

To access the quality of the test, Pitman asymptotic efficiencies (PAEs) are computed and compared with some other tests for the following alternative:

(i) The Weibull Family:

$$\overline{F}_1(x) = e^{-x^\theta} \quad x \geq 0, \theta \geq 1$$

(ii) The Linear Failure Rate Family:

$$\overline{F}_2(x) = e^{-x-\frac{\theta}{2}x^2}, \quad x \geq 0, \theta \geq 0$$

(iii) The Makeham Family:

$$\overline{F}_3(x) = e^{[-x-\theta(x+e^{-x}+1)]}, \quad x \geq 0, \theta \geq 0$$

Note that for $\theta = 1, \overline{F}_1$ goes to exponential distribution and for $\theta = 0, \overline{F}_2$ and $\overline{F}_3$ reduce to the exponential distributions. The PAE is defined by

$$PAE(\hat{\Lambda}(\gamma)) = \frac{1}{\sigma_0}\left[\frac{d}{d\theta}\delta(\gamma)\right]_{\theta \to \theta_0},$$

Table 1. gives the efficiencies of our proposed test $\hat{\Lambda}(0.2)$ comparing with the tests; $\delta_{(3)}$ given by [27] and $\delta_{Fn}^{(2)}$ given by [22].

Table 1. comparison between The PAE of our test and some other tests

Distribution	$\hat{\Lambda}(0.2)$	$\delta_{(3)}$	$\delta_{Fn}^{(2)}$
Linear failure rate	0.89395	0.408	0.217
Makeham	3.48567	0.039	0.144
Weibull	0.5891	0.170	0.05

Also, the Pitman asymptotic relative efficiency (PARE) of our test $\hat{\Lambda}(0.2)$ comparing to $\delta_{(3)}$ and $\delta_{Fn}^{(2)}$ is calculated where

$$PARE(T_1, T_2) = \frac{PAE(T_1)}{PAE(T_2)}$$

Table 2. show that the asymptotic relative efficiences for our test

Distribution	PARE($\hat{\Lambda}(0.2),\ \delta_{(3)}$)	PARE($\hat{\Lambda}(0.2),\ \delta_{Fn}^{(2)}$)
Linear failure rate	2.191	4.1196
Makeham	89.376	24.206
Weibull	3.4653	11.782

We can see from Tables 1 and 2. that our test statistic $\hat{\Lambda}(\gamma)$ for RNBUL is more efficiency than the other cases.

3 Monte Carlo Null Distribution Critical Points

In this section, the upper percentile points of $\hat{\Lambda}(\gamma)$ for 90%, 95%, 98% and 99% are calculated based on 5000 simulated samples of sizes $n = 5(5)50$ and tabulated in Table 3.

Table 3. The upper percentile of $\hat{\Lambda}(\gamma)$ with 5000 replications at $\gamma = 0.2$

n	90%	95%	98%	99%
5	0.0937502	0.123805	0.164215	0.185003
10	0.0645106	0.0812811	0.102429	0.120486
15	0.0508777	0.0639841	0.0804773	0.0906853
20	0.0460578	0.0564048	0.0686507	0.0790545
25	0.0408935	0.0504019	0.061637	0.0679614
30	0.0377786	0.045507	0.0557467	0.0636555
35	0.0346345	0.0415154	0.0498707	0.0562549
40	0.032289	0.0390438	0.0457184	0.0521619
45	0.0317921	0.0377462	0.0457831	0.0504754
50	0.0307952	0.0358238	0.0422963	0.0469997

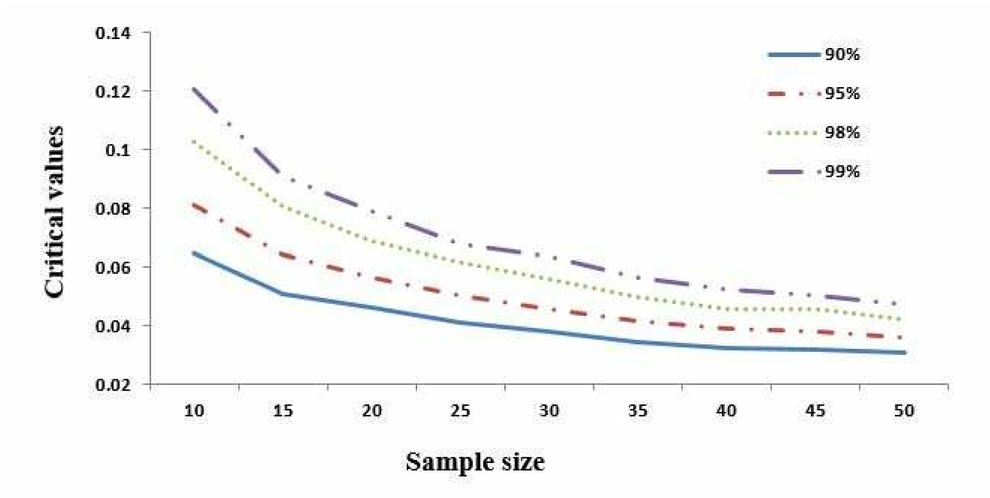

Fig. 1: Relation between critical values, sample size and confidence levels

It can be noticed from Table 3. and Fig.1 that the critical values are increasing as confidence level increasing and decreasing as the sample size increasing.

3.1 The Power Estimates

The power of proposed test will be estimated at (1-α)% confidence level $\alpha = 0.05$ with suitable parameters values of θ at $n = 10, 20$ and 30 for three commonly used distributions such as Weibull, linear failure rate and Gamma distributions based on 5000 simulated samples tabulated in Table 4.

Table 4. Power Estimates of the Statistic $\hat{\Lambda}(\gamma)$ at $\gamma = 0.2$

Distribution	Parameter θ	Sample size		
		n=10	n=20	n=30
LFR family	2	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000
Wiebull family	2	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000
Gamma family	2	0.9708	0.9818	0.989
	3	0.9928	0.9988	0.9998
	4	0.999	1.0000	1.0000

From Table 4., we see that our test $\hat{\Lambda}(\gamma)$ has a good power for all alternatives.

3.2 Applications Using Complete (Uncensored) Data

Here, we present some of a good real examples to illustrate the use of our test statistics $\hat{\Lambda}(\gamma)$ in the case of complete data at 95% confidence level.

Data-set #1.

Concider the data set given in [13]. These data gives the times between arrivals of 25 customers at a facility. It is easy to show that $\hat{\Lambda}(\gamma) = 0.630055$ which is greater than the critical value of Table 3. Then we accept H_1 the alternative hypotheses which show that the data set has RNBUL property but not exponential.

Data-set #2.

Consider the data-set given in [12] which represent failure times in hours, for a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress. In this case, we get $\hat{\Lambda}(\gamma) = 0.0378995$ which is less than the critical value of the Table 3. Hence we accept the null hypothesis H_0 and reject H_1. This means that this kind of data doesn't fit with RNBUL property.

Data-set #3.

Concider the data set given in [14]. These data gives the daily average wind speed from 1/3/2015 to 30/3/2015 for Cairo city in Egypt. It is easy to show that $\hat{\Lambda}(\gamma) = 0.75122$ which is greater than the critical value of Table 3. Then we reject the null hypotheses H_0 and data set has RNBUL property.

4 Testing Against RNBUL Class for Censored Data

A test statistic is proposed to test H_0 versus H_1 in case of randomly right-censored (RR-C) data in many practical expriments, the censored data are the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows: Suppose n units are put on test, and $X_1, X_2, ..., X_n$ denote there true life time which are independent, identically distributed (i.i.d.) according to continuous life distribution F. Let $Y_1, Y_2, ..., Y_n$ be (i.i.d.) according to continuous life distribution G. X's and Y's are assumedto be independent. In the RR-C model, we observe the pairs $(Z_j, \delta_j),\ j = 1, ..., n$, where $Z_j = \min(X_j, Y_j)$ and

$$\delta_j = \{.1\ if\ \ Z_j = X_j\ (\ j - \text{th observation is uncensored}).0\ if\ \ Z_j = Y_j\ (\ j - \text{th observation is censored}).$$

Let $Z_{(0)} = 0 < Z_{(1)} < Z_{(2)} < ... < Z_{(n)}$ denote the ordered Z's and $\delta_{(j)}$ is δ_j corresponding to $Z_{(j)}$. Using the censored data $(Z_j, \delta_j),\ j = 1, ..., n$.[17] proposed the product limit estimator,

$$\bar{F}_n(X) = \prod_{[j:Z_{(j)} \leq X]} \{(n-j)/(n-j+1)\}^{\delta_{(j)}},\ X \in [0, Z_{(n)}]$$

Now, for testing $H_0 : \delta(\gamma) = 0$ against $H_1 : \delta(\gamma) > 0$,using randomly right censored data, we propose the following test statistic

$$\hat{\Lambda}_c(\gamma) = \zeta(\gamma)[\frac{\mu}{\gamma(\gamma-1)} - \frac{1}{\gamma^2}] + \zeta(1)[\frac{\mu}{\gamma-\gamma^2} - \frac{1}{\gamma^2}] + \frac{1}{\gamma^2}\zeta(\gamma)\zeta(1) + \frac{1}{\gamma^2}.$$

For computational purposes, $\hat{\Lambda}_c(\gamma)$ may be rewritten as

$$\hat{\Lambda}_c(\gamma) = \Theta[\frac{\Phi}{\gamma(\gamma-1)} - \frac{1}{\gamma^2}] + \Omega[\frac{\Phi}{\gamma-\gamma^2} - \frac{1}{\gamma^2}] + \frac{1}{\gamma^2}\Theta\Omega + \frac{1}{\gamma^2},$$

where

$$\Phi = \sum_{k=1}^{n}[\prod_{m=1}^{k-1} C_m^{\delta(m)}(Z_{(k)} - Z_{(k-1)})],$$

$$\Theta = \sum_{j=1}^{n} e^{-\gamma z_{(j)}}[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)}],$$

$$\Omega = \sum_{j=1}^{n} e^{-z_{(j)}}[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)}],$$

and

$$dx = (Z_{(j)} - Z_{(j-1)}), \quad C_k = [n-k][n-k+1]^{-1}.$$

Table 5. below gives the critical values percentiles of $\hat{\Lambda}_c(\gamma)$ test for sample size $n = 5(5)30(10)70, 81, 86$.

Table 5. The upper percentile of $\hat{\Lambda}_c(\gamma)$ with 5000 replications at $\gamma = 0.2$

n	90%	95%	98%	99%
5	13.5698	15.6251	21.4884	22.853
10	9.39547	11.6982	14.5646	16.1048
15	6.8184	8.77395	11.1652	12.7648
20	5.40007	7.0327	9.33557	10.7836
25	4.3512	5.88305	7.65831	9.22072
30	3.55056	4.97609	6.43102	7.3819
40	2.38012	3.61256	5.13749	6.08144
50	1.81253	2.97982	4.30084	5.24347
60	1.03287	2.07022	3.36469	4.17644
70	0.648289	1.64264	2.81515	3.51092
81	0.357731	1.28685	2.47333	3.18287
86	0.083661	0.95329	1.93505	2.77149

From Table 5.and Fig 2. It can be observed that the critical values are increasing as confidence level increasing and decreasing as the sample size increasing.

4.1 The Power Estimates for $\hat{\Lambda}_c(\gamma)$

The power of the statistic $\hat{\Lambda}_c(\gamma)$ is considered at the significant level $\alpha = 0.05$ with suitable parameters values of θ at $n = 10, 20$ and 30 for three commonly used distributions such as Weibull, linear failure rate and Gamma distributions based on 5000 simulated samples tabulated in Table 6.

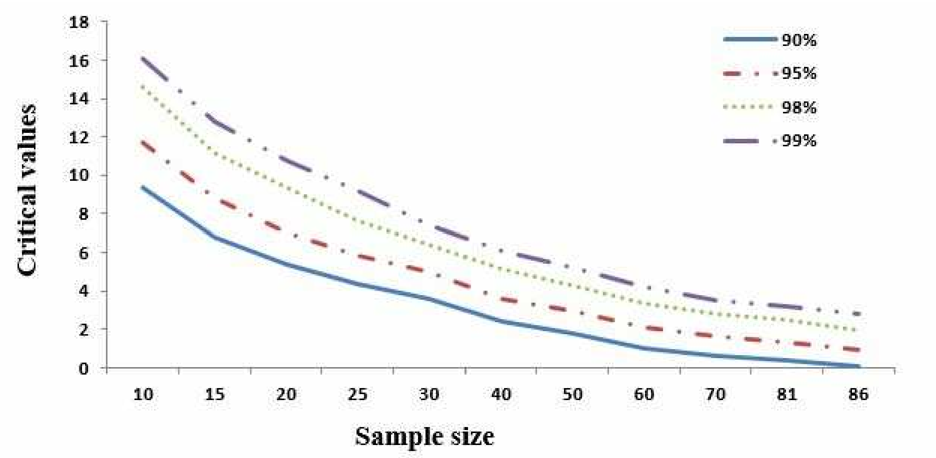

Fig. 2: Relation between critical values,sample size and confidence levels

Table 6. Power Estimates of the Statistic $\hat{\Lambda}_c(\gamma)$

Distribution	**Parameter θ**	**Sample size**		
		n=10	**n=20**	**n=30**
	2	0.9698	0.9928	0.9998
LFR family	**3**	0.9637	0.999	1.0000
	4	0.979	0.999	1.0000
	2	0.9642	0.9822	0.9906
Wiebull family	**3**	0.9736	0.987	0.9952
	4	0.9794	0.9912	0.9958
	2	0.9878	0.9896	0.9892
Gamma family	**3**	0.9996	1.0000	0.9998
	4	1.0000	1.0000	1.0000

From Table 6., we see that our test $\hat{\Lambda}_c(\gamma)$ at $\gamma = 0.2$ has a good power for all alternatives.

4.2 Applications for Censored Data

We present two good real examples to illustrate the use of our test statistics $\hat{\Lambda}_c(\gamma)$ in case of censored data at 95% confidence level.

Data-set #4.

Consider the data-set in [31]. These data represents 81 survival times of patients of melanoma. Out of these 46 represents whole times (non-censored data). We get $\hat{\Lambda}_c(\gamma) = 3.41488 \times 10^{-101}$ which is less than the tabulated value in Table 5. It is evident at the significant level $\alpha = 0.05$. This means that this kind of data doesn't fit with RNBUL property.

Data-set #5.

Consider the data-set given in [28] for lung cancer patients. These data consists of 86 survival times (in month) with 22 right censored. In this case, we get $\hat{\Lambda}_c(\gamma) = 7.08527^{-11}$ which is less than the tabulated value in Table 5. Then, we accept H_0 the null hypotheses which show that the data set has exponential property.

5 conclusion

The RNBUL is defined and a test statistic based on goodness of fit for is presented. The Pitman asymptotic relative efficiences (PARE) are calculated and it is noticed that the PAEs of our new test are better than some old tests for all used alternatives. Monte Carlo null distribution critical points are simulated for sample size $n = 5(5)50$ and the power estimates of this test are also calculated for some common alternatives distribution followed by some numerical examples. The problem in case of right censored data is also handled and selected critical values are tabulated, the power estimates for censor data of this test are tabulated also we discuss some applications to elucate the usefullness of the proposed test in reliability analysis for censored data.

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