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Non Parametric Test for Testing Exponentiality Against Exponential Better Than Used in Laplace Transform Order

Mohamed A. W. Mahmoud[1] *and Mahmoud M. M. Mansour*[2,*]

[1] Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt
[2] Department of Basic Science, Faculty of Engineering, The British University in Egypt, El Sherouk City, Cairo, Egypt

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Abstract: In this paper, the test statistic for testing exponentiality against exponential better than used in Laplace transform order (EBUL) based on the Laplace transform technique is proposed. Pitman's asymptotic efficiency of our test is calculated and compared with other tests. The percentiles of this test are tabulated. The powers of the test are estimated for famously used distributions in aging problems. In the case of censored data, our test is applied and the percentiles are also calculated and tabulated. Finally, real examples in different areas are utilized as practical applications for the proposed test.

Keywords: Aging, Testing exponentiality, U- statistic, Censored data, Monte Carlo simulation, EBUL.

1 Introduction

Concepts of ageing describe how a component or system improves or deteriorates with age. Many classes of life distributions are defined in the literature according to their ageing properties, see for example Barlow and Proschan [6], Abouammoh and Ahmad [1] and El-Batal [8]. Life testing can be defined as the process done under specified conditions to determine how and when the system will fail under the same conditions. The purpose of these tests is improvement the reliability of components or systems and knowing the weakest and strongest points of the system. One of the most life testing is testing exponentiality versus classes of life distributions. Testing exponentiality determines whether the system·s life distribution exponential. On other hand, from these tests, we ask about if the system has a constant failure rate.

For testing against increasing failure rate (IFR), see Proschan and Pyke [23], Barlow [5] and Ahmad [2], among others. For testing against new better than used (NBU), see Hollander and Proschan [9], Koul [15], Kumazawa [16] and Ahmad [3]. For testing against new better than used in Laplace transform order (NBUL) and exponential better than used (EBU), see Diab et al. [7] and El-Batal [8], respectively. Testing exponentiality based on goodness of fit approach against many classes of life distributions was studied by some authors such as Kayid et al. [13], Ismail and Abu- Youssef [10] and Mahmoud et al. [21]. For testing exponentiality using Laplace transform technique see, Atallah et al. [4]. Here, some basic definitions are mentioned to explain how our test is constructed :

Definition 1. *A life distribution F, with F (0) = 0, survival function $\bar{F}$ and finite mean μ is said to be EBU if*

$$\bar{F}(x+t) \le \bar{F}(t)e^{-\frac{x}{\mu}}\ , x,t > 0, \tag{1}$$

or

$$\bar{F}_t \le e^{-\frac{x}{\mu}}, x,t > 0,$$

where $\bar{F}_t = \frac{\bar{F}(x+t)}{\bar{F}(t)}$ represents the survival function related to the random residual life time X_t.

* Corresponding author e-mail: mahmoud.mansour@bue.edu.eg

Note that, the above definition which is introduced by El-Batal [8], is motivated by comparing the life length X_t of a component of age t with another new component of life length X which is exponential with the same mean μ.

Recently a new class of life distribution named exponential better than used in Laplace transform order (EBUL) is introduced by Mahmoud et al. [21] which expand the EBU class :

Definition 2. *X is said to be EBUL if*

$$\int_0^\infty e^{-sx}\bar{F}(x+t)dx \leq \frac{\mu}{(\mu s+1)}\bar{F}(t),\ x,t>0, s\geq 0. \tag{2}$$

In the current work, Laplace transform technique is used to testing exponentiality versus (EBUL). In Section 2 based on U- statistic our test is developed and its asymptotic properties are studied. In that section, Monte Carlo null distribution critical points are simulated for samples size $n = 5, 10, 11, 15(5)25, 29, 30(5)50$ and the power estimates are also calculated and tabulated. In Section 3, we dealing with right-censored data and selected critical values are tabulated. Finally, in Section 4 we discuss some applications to clarify the utility of the proposed test in reliability analysis.

2 Testing Against EBUL Alternatives

In this part a test statistic is constructed to test exponentiality versus (EBUL) based on Laplace transform technique. The following lemma is required :

Lemma 1. *If X a random variable with distribution function F and F belongs to EBUL class, then*

$$\beta(s\mu+1)(1-\phi(s)) \leq s(\beta\mu+1)(1-\phi(\beta)), \quad \beta, s\geq 0, \beta\neq s. \tag{3}$$

where $\phi(s) = Ee^{-sX} = \int_0^\infty e^{-sx}dF(x)$.

Proof.

Since F is EBUL then,

$$\int_0^\infty e^{-sx}\bar{F}(x+t)dx \leq \frac{\mu}{(\mu s+1)}\bar{F}(t),\ x,t>0\ .$$

By using Laplace transform and integrating both sides over $(0,\infty)$ with respect to t, we get

$$\int_0^\infty\int_0^\infty e^{-\beta t}e^{-sx}\bar{F}(x+t)dxdt \leq \frac{\mu}{(\mu s+1)}\int_0^\infty e^{-\beta t}\bar{F}(t)dt\ . \tag{4}$$

Setting

$$\begin{aligned} I_1 &= \int_0^\infty e^{-\beta t}\bar{F}(t)dt = E\int_0^\infty e^{-\beta t}I(X>t)dt \\ &= E\int_0^X e^{-\beta t}dt = \frac{1}{\beta}(1-Ee^{-\beta X}), \end{aligned}$$

it is easy to show that

$$I_1 = \frac{1}{\beta}(1-\phi(\beta)). \tag{5}$$

Setting

$$I_2 = \int_0^\infty\int_0^\infty e^{-\beta t}e^{-sx}\bar{F}(x+t)dxdt.$$

So I_2 can be put in the following form

$$I_2 = \int_0^\infty \int_v^\infty e^{-\beta v} e^{-s(u-v)} \bar{F}(u) du dv$$
$$= \int_0^\infty \int_0^v e^{-u(\beta-s)} e^{-sv} \bar{F}(v) du dv$$
$$= \frac{1}{\beta - s} \left[\int_0^\infty e^{-sv} \bar{F}(v) dv - \int_0^\infty e^{-\beta v} \bar{F}(v) dv \right],$$

therefore

$$I_2 = \frac{1}{\beta - s} \left[\frac{1}{s} (1 - \phi(s)) - \frac{1}{\beta} (1 - \phi(\beta)) \right]. \tag{6}$$

Substituting (5) and (6) into (4), we get

$$\beta (s\mu + 1)(1 - \phi(s)) \le s(\beta\mu + 1)(1 - \phi(\beta)).$$

This completes the proof.

Let $X_1, ..., X_n$ denote a random sample from a distribution F, we desire to test $H_\circ$: F is exponential against H_1 :F is EBUL and not exponential. By using the following as a measure of departure from $H_\circ$ in favor of H_1

$$\zeta(s) = s(\beta\mu + 1)(1 - \phi(\beta)) - \beta(s\mu + 1)(1 - \phi(s)),$$

note that, under $H_\circ$, $\zeta(s) = 0$, while it is positive under H_1. Define the test statistic $\zeta_n(s)$ as follows

$$\zeta_n(s) = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n [s(1 + \beta X_i)\left(1 - e^{-\beta X_j}\right) - \beta(1 + sX_i)\left(1 - e^{-sX_j}\right)].$$

To make the test invariant, let

$$\Delta_n(s) = \frac{\zeta_n(s)}{\bar{X}^2},$$

where $\bar{X} = \sum_{i=1}^n \frac{X_i}{n}$ is the sample mean.

Then

$$\Delta_n(s) = \frac{1}{n^2 \bar{X}^2} \sum_i \sum_J \phi(X_i, X_j), \tag{7}$$

where

$$\phi(X_i, X_j) = s(1 + \beta X_i)\left(1 - e^{-\beta X_j}\right) - \beta(1 + sX_i)\left(1 - e^{-sX_j}\right). \tag{8}$$

The following theorem summarizes the asymptotic properties of the test.

Theorem 1. *As $n \to \infty$, $(\Delta_n(s) - \zeta(s))$ is asymptotically normal with mean 0 and variance $\sigma^2(s)/n$, where $\sigma^2(s)$ is given in (9). Under $H_\circ$, the variance reduces to (10).*

Proof.

Using standard U-statistic theory,cf. Lee [18], one can see that:

$$\sigma^2 = V\{E[\phi(X_1, X_2) \mid X_1] + E[\phi(X_1, X_2) \mid X_2]\}.$$

Recall the definition of $\phi(X_i, X_j)$ in (8), thus it is easy to show that

$$E[\phi(X_1, X_2) \mid X_1] = s - \beta - s\int_0^\infty e^{-\beta x} dF(x) + \beta \int_0^\infty e^{-sx} dF(x) + s\beta X \int_0^\infty e^{-sx} dF(x) - s\beta X \int_0^\infty e^{-\beta x} dF(x),$$

Similarly

$$E[\phi(X_1,X_2)\mid X_2] = s-\beta-se^{-\beta X}+\beta e^{-sX}+s\beta e^{-sX}\int_0^\infty xdF(x)-s\beta e^{-\beta X}\int_0^\infty xdF(x).$$

Hence

$$\sigma^2(s,\beta)=Var\{2s-2\beta-s\int_0^\infty e^{-\beta x}dF(x)+\beta\int_0^\infty e^{-sx}dF(x)+s\beta X\int_0^\infty e^{-sx}dF(x)-s\beta X\int_0^\infty e^{-\beta x}dF(x)$$
$$-se^{-\beta X}+\beta e^{-sX}+s\beta e^{-sX}\int_0^\infty xdF(x)-s\beta e^{-\beta X}\int_0^\infty xdF(x)\}. \tag{9}$$

Under $H_\circ$

$$\sigma_\circ^2(s)=\frac{s^2\beta^2(s-\beta)^2\left(1+s+\beta+2s^2\beta^2\right)}{(2s+1)(2\beta+1)(s+\beta+1)(s+1)^2(\beta+1)^2}. \tag{10}$$

2.1 Asymptotic efficiency

To decide the quality of this procedure, its Pitman asymptotic efficiency (PAE) is compared with some other tests in Table 1 for the following alternative distributions.

(i)The Weibull distribution: $\bar{F}_1(x)=e^{-x^\theta}, x\geq 0, \theta\geq 1$.
(ii)The linear failure rate distribution (LFR): $\bar{F}_2(x)=e^{-x-\frac{\theta}{2}x^2}, x\geq 0, \theta\geq 0$.
(iii)The Makeham distribution: $\bar{F}_3(x)=e^{-x-\theta\left(x+e^{-x}-1\right)}, x\geq 0, \theta\geq 0$.

Note that when $\theta=1$and 0, the(i), (ii) and (iii) distributions reduce to the exponential distribution. The PAE is defined by:

$$PAE\ (\Delta_n(\beta,s))=\frac{1}{\sigma_\circ(\beta,s)}\left|\frac{d}{d\theta}\zeta(\beta,s)\right|_{\theta\to\theta_\circ}.$$

at $\beta=0.5$ and $s=0.2 \implies \sigma_\circ=0.0100187$.
This leads to:

$$PAE\ [\Delta_n(0.5,0.2), Weibull]=39.385,\quad PAE\ [\Delta_n(0.5,0.2), LFR]=27.7261 \quad \text{and}$$
$$PAE\ [\Delta_n(0.5,0.2), Makeham]=9.57809.$$

Table 1: Comparison Between The PAE of Our Test and Some PAEs of Other Tests

Test	Weibull	LFR	Makeham
Kango [11]	0.132	0.433	0.144
Mugdadi and Ahmad [22]	0.170	0.408	0.039
Mahmoud and Abdul Alim [19]	0.405	0.433	0.289
Mahmoud et al. [21]	7.34682	4.19399	1.71572
Our test $\Delta_n(0.5,0.2)$	39.385	27.7261	9.57809

It is clear that our test has the greatest efficiency in all cases. It is noticed that the proposed test $\Delta_n(0.5,0.2)$ enjoys with efficiency better than other tests for all alternatives and especially at $\beta=0.5$ and $s=0.2$,where our test was computed at different values of β and s to obtain the best efficiency for all alternatives.

2.2 Monte Carlo null distribution critical values

In this subsection Monte Carlo null distribution critical points for our test $\Delta_n(0.5,0.2)$ are simulated based on 10000 generated samples from the standard exponential distribution using Mathematica 8 program. Table 2 gives the upper percentile points of $\Delta_n(0.5,0.2)$, for $n=5,10,11,15(5)25,29,30(5)50$.

Table 2: The Upper Percentile Points of $\Delta_n(0.5, 0.2)$ with 10000 Replications

n	90%	95%	99%
5	0.00711445	0.00825135	0.0102042
10	0.00485419	0.00580865	0.00753558
11	0.00455357	0.00547267	0.00699703
15	0.00387027	0.00464434	0.00605413
20	0.00331803	0.00404287	0.00527937
25	0.0028943	0.00351706	0.00467283
29	0.00266871	0.00327249	0.00442804
30	0.00261159	0.00319335	0.00443350
35	0.00240493	0.00296787	0.00390736
40	0.00221051	0.00276719	0.00366207
45	0.00207163	0.00256106	0.00351028
50	0.00200803	0.00248306	0.00336085

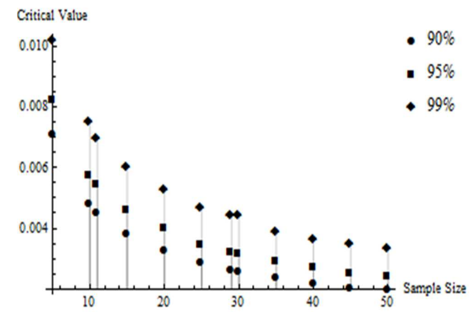

Fig. 1: The Relation Between Sample Size and Critical Values

From Table 2 and Figure 1, it is obvious that the critical values are decreasing as the samples size increasing and they are increasing as the confidence levels increasing.

2.3 The power of test

In this subsection the power of our test $\Delta_n(0.5, 0.2)$ will be estimated at significance level $\alpha = 0.05$ with respect to two alternatives Weibull and linear failure rate (LFR) distributions based on 10000 samples. Table 3 gives the power estimates with parameter $\theta = 2, 3$ and 4 at $n = 10, 20$ and 30.

Table 3: The Power Estimates of The Statistic $\Delta_n\,(0.5, 0.2)$

n	θ	Weibull	LFR
10	2	0.7593	0.4418
	3	0.9964	0.5501
	4	0.9999	0.6335
20	2	0.9875	0.702
	3	1.0000	0.8095
	4	1.0000	0.8652
30	2	0.9998	0.8487
	3	1.0000	0.9196
	4	1.0000	0.9567

It is obviously that the power estimates increase as the sample size increases for each value of the parameter θ.

3 Testing for Censored Data

Here, a test statistic is proposed to test $\mathrm{H}_\circ$ versus H_1 with randomly right -censored data. Such a censored data is usually the only available information in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can be modeled as follows. Suppose n objects are put on test, and $X_1, X_2, ..., X_n$ denote their true life times. Let $X_1, X_2, ..., X_n$ be independent and identically distributed (i.i.d) according to a continuous life distribution F. Let $Y_1, Y_2, ..., Y_n$ be (i.i.d) according to a continuous life distribution G. Also we assume that X's and Y's are independent. In the randomly right -censored model, the pairs $(Z_j\ , \delta_j)$, $j = 1, ..., n$,are observed, where $\mathrm{Z}_j = \min(X_j, Y_j)$ and

$$\delta_j = \begin{cases} 1, \text{ if } & \mathrm{Z}_j = \mathrm{X}_j \ \text{(j-th observation is uncensored)} \\ 0, \text{ if } & \mathrm{Z}_j = \mathrm{Y}_j \ \text{(j-th observation is censored)} \end{cases}$$

Let $\mathrm{Z}_{(0)} = 0\ < \mathrm{Z}_{(1)}\ < \mathrm{Z}_{(2)}\ < ... < \mathrm{Z}_{(n)}$ denote the orderd Z's and $\delta_{(j)}$ is δ_j corresponding to $\mathrm{Z}_{(j)}$. Using the censored data $(\mathrm{Z}_j\ , \delta_j)$, $j = 1, ..., n$. Kaplan and Meier [12] proposed the product limit estimator, as follows

$$\bar{F}_n(X) = \prod_{[j:\, Z(j) \le X]} \{(n-j)/(n-j+1)\}^{\delta_{(j)}}, X\ \in [0, Z_{(n)}]$$

Now, for testing $\mathrm{H}_\circ : \zeta_c\,(s) = 0$ against $\mathrm{H}_1 : \zeta_c\,(s) > 0$, using the randomly right censored data, the following test statistic is proposed

$$\hat{\zeta}_c\ = s\,(\beta\mu + 1)\,(1 - \phi\,(\beta)) - \beta\,(s\mu + 1)\,(1 - \phi\,(s))\,.$$

For computational purpose, $\hat{\zeta}_c$ may be rewritten as

$$\hat{\zeta}_c\ = s\,(\beta\Omega + 1)\,(1 - \tau) - \beta\,(s\Omega + 1)\,(1 - \eta)\,, \text{where}$$

$$\eta = \sum_{j=1}^{n} e^{-s\,Z_{(j)}}[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)}], \Omega = \sum_{k=1}^{n}[\prod_{m=1}^{k-1} C_m^{\delta(m)}\left(Z_{(k)} - Z_{(k-1)}\right)],$$

$$\tau = \sum_{j=1}^{n} e^{-\beta\,Z_{(j)}}[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)}]$$

and

$$dF_n(Z_j) = \bar{F}_n(Z_{j-1}) - \bar{F}_n(Z_j), \ \ c_k = [n-k]\,[n-k+1]^{-1}\,.$$

To make the test invariant, let

$$\Delta_c\,(\beta, s) = \frac{\zeta_c\,(\beta, s)}{\bar{Z}^2}, \text{where } \bar{Z} = \sum_{i=1}^{n} \frac{Z_{(i)}}{n}. \tag{11}$$

3.1 Monte Carlo null distribution critical values in censored case

In this subsection the monte Carlo null distribution critical values of Δ_c at $\beta = 0.5$ and $s = 0.2$ for samples sizes $n = 20, 25, 30, 40, 50, 51, 60, 70, 81$ with 10000 replications are simulated from the standard exponential distribution by using Mathematica 8 program. Table 4 gives the upper percentile points of the statistic $\Delta_c\,(0.5, 0.2)$.

Table 4: The Upper Percentile Points of $\Delta_c\,(0.5, 0.2)$.

n	90%	95%	99%
20	0.0519223	31.478	53343.0
30	0.932315	1430.06	5.04584×10^{8}
40	4.58761	104973.0	4.55874×10^{11}
50	23.2245	3.05692×10^{6}	3.31635×10^{15}
51	31.4132	3.78736×10^{6}	5.66593×10^{15}
60	113.921	7.31707×10^{7}	3.27441×10^{18}
70	760.501	2.24853×10^{10}	3.24708×10^{22}
81	5365.96	9.15652×10^{11}	2.08289×10^{26}

Table 4 shows that the critical values increase as the sample size and the confidence level increase.

4 Some Applications

In this section, our test is applied to some real data-sets at 95% confidence level.

4.1 Case of non censored data

In this section two examples are presented considering $\beta = 0.5$ and $s = 0.2$.

*Example 1.*Let us consider the following data, which represent failure times in hours, for a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress (Lawless [17]):

0.205	0.363	0.407	0.770	0.720	0.782
1.178	1.255	1.592	1.635	2.310	

It was found that $\Delta_n\,(0.5, 0.2) = 0.00500741$ and this value less than the critical value in Table 2. Then we conclude that this data set have the exponential property.

Example 2. We consider a classical real data in Keating et al. [14] set on the times, in operating days, between successive failures of air conditioning equipment in an aircraft. Theses data are recorded.

3.750	0.417	2.500	7.750	2.542	2.042	0.583
1.000	2.333	0.833	3.292	3.500	1.833	2.458
1.208	4.917	1.042	6.500	12.917	3.167	1.083
1.833	0.958	2.583	5.417	8.667	2.917	4.208
8.667						

Since $\Delta_n\,(0.5, 0.2) = 0.0086296$ and this value is greater than the critical value in Table 2. Then we conclude that this data set have EBUL property.

4.2 Case of censored data

In this section two example are presented considering $\beta = 0.5$ and $s = 0.2$.

*Example 3.*Consider the data in Susarla and Vanryzin [24]. These data represent 81 survival times of patients of melanoma. Of them 46 represent whole life times(non-censored data) and the observed values are:

13	14	19	19	20	21	23	23	25	26	26	27
27	31	32	34	34	37	38	38	40	46	50	53
54	57	58	59	60	65	65	66	70	85	90	98
102	103	110	118	124	130	136	138	141	234		

The ordered censored observations are:

16	21	44	50	55	67	73	76	80	81	86	93
100	108	114	120	124	125	129	130	132	134	140	147
148	151	152	152	158	181	190	193	194	213	215	

We get $\Delta_c\,(0.5, 0.2) = 2.44476 \times 10^{94}$ which is greater than the critical value of Table 4. Then we accept H_1 which states that the data set have EBUL property.

*Example 4.*Consider the data in Mahmoud et al. [20] which represent 51 liver cancers patients taken from Elminia cancer center Ministry of Health – Egypt, which entered in (1999). Out of these 39 represents non-censored data, and the others represents censored data.The ordered life times (in days)

– Non-censored data

10	14	14	14	14	14	15	17	18	20
20	20	20	20	23	23	24	26	30	30
31	40	49	51	52	60	61	67	71	74
75	87	96	105	107	107	107	116	150	

– Censored data

30	30	30	30	30	60	150	150	150	150	150	185

The value of $\Delta_c\,(0.5, 0.2)$ is computed to be 4.07023×10^{69} which is greater than the critical value of Table 4. There is enough to accept H_1which states that the data set have EBUL property.

5 Conclusion

Laplace transform technique is used to testing exponentiality versus (EBUL). The Pitman asymptotic efficiency of this test is studied. The upper percentiles and the power of the proposed test are calculated and tabulated. In case of censored data the critical values of this test are calculated and tabulated. Our test is applied to some real data. Finally, the proposed test in the two cases seems to be simple. It is noted that our test is more efficient than Kango [11], Mugdadi and Ahmad [22], Mahmoud and Abdul Alim [19] and Mahmoud et al. [21], based on the approach of PAE. One can also note that it has good estimated power and this increases as n and θ increase. The proposed test can be applied to assess the efficacy for all treatment methods in all different fields in medical research regardless of knowing the nature of the used treatment method. But it is not recommended applying this test in case of comparing two treatment methods.

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Mohamed A. W. Mahmoudis presently employed as a pro- fessor of Mathematical statistics in Department of Mathematics and Dean of Faculty of Science, Al-Azhar University, Cairo, Egypt. He received his PhD in Mathematical statistics in 1984 from Assiut University, Egypt. His research in- terests include: Theory of reliability, ordered data, characterization, statistical inference, distribution theory, discriminant analysisand classes of life distribu- tions. He published and Co-authored more than 100 papers in reputed interna- tional journals. He supervised more than 62 M. Sc. thesis and more than 75 Ph. D. thesis.

Mahmoud M. M. Mansour is a lecturer of Mathematical Statis- tics at Basic Science Department, Faculty of Engineering, The Britsh University in Egypt, Cairo, Egypt. He received Ph.D. from Faculty of Science AL -Azhar University Egypt in 2017. His research interests include: Theory of reliability, Censored data, Life testing, distribution theory.