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A Direct Approach to Transient Queue-Size Distribution in a Finite-Buffer Queue with AQM

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Abstract: A finite-buffer $M/G/1$-type queueing model is considered in which the level of saturation of the buffer is controlled by a dropping function. A direct analytical method to the study of the transient queue-size distribution is proposed. Applying the embedded Markov chain paradigm and the formula of total probability, a specific-type system of integral equations for the transient queue-size distributions, conditioned by the number of packets present in the system at the opening, is derived. The corresponding system of linear equations built for the Laplace transforms is written in a matrix form and solved directly. The $M/M/1/2-$type system is analyzed as a special case separately. Numerical utility of the approach is illustrated as well.

Keywords: Dropping function, finite buffer, formula of total probability, integral equation, queue size, transient state.

1 Preliminary

Applications of finite-buffer queueing systems in modelling of the evolution of telecommunication networks, especially in the TCP/IP packet networks are well-known. One of hot topics in present scientific research devoted to teletraffic problems is the analysis of the phenomena of buffer congestions and losses of packets being transmitted, which are typical ones in Internet routers. To meet these challenges the idea of the Active Queue Management (AQM) was proposed, in which a dropping function for controlling the process of enqueueing of the arriving packets was introduced. A dropping function rejects the incoming packets with a probability depending on the instantaneous or average queue size at the pre-arrival epoch, even when the buffer is not saturated yet. In [5] the first AQM algorithm, called RED one, with a linear dropping function was analyzed. Other types of dropping functions were investigated in [1] and [14] (an exponential one - REM algorithm), [6] and [15] (a “gentle” RED algorithm (GRED) i.e. a doubly linear dropping function) and [21] (a quadratic dropping function). More information about AQM schemes and their applications one can find e.g. in [7], [16], [17] and [20].

Although AQM algorithms assume a much greater “flexibility” of the system for the process of buffer saturating as compared to the classical TD (Tail Drop) scheme, where the arriving packets are lost only during the buffer overflow period, they have not been widely developed in the Internet. One of the reasons seems to be the fact that the impact of these algorithms on the key stochastic and performance characteristics of the system, such as the queue-size distribution or time between successive losses, especially in the transient state, have not yet been sufficiently investigated analytically. The compact representation for the stationary queue-size distribution at an arbitrary epoch in the $M/G/1/N$-type system with packet dropping was obtained in [3]. In [8] the $M/M/1/N$ system with single and batch arrivals, controlled by a general-type dropping function, was analyzed. The formulae for three important stationary-state stochastic characteristics of the system were found there: the queue-size distribution, the number of packets (batches of packets) lost consecutively, and the time between two successive losses. The results from [8] were partially generalized in [18] and [19] where the representations for the stationary queue-size distribution in the case of generally distributed packet volumes and bounded total system capacity were derived for the $M/M/1$ and $M/G/1$ queues respectively.

As one can note, almost all of the results related to AQM models concern systems in equilibrium. However, a comprehensive analysis of the system operation should be based, if possible, on transient characteristics. Indeed,

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rapidly changing parameters of the packet traffic, or changing network specifications, at simultaneous heavy traffic load, cause that the system has no chance to stabilize in practice and the stationary analysis of stochastic system characteristics does not fit with reality.

In the paper we study the transient queue-size distribution in the $M/G/1/N-$type queue in which an arriving packet is dropped with a probability d_n depending on the actual number n of packets present in the system at the pre-arrival epoch. To find the representation for the Laplace transform of the queue-size distribution at a fixed moment t we apply the approach being a mixture of different techniques. Firstly, we find an explicit formula for the conditional probability that exactly k packets join the queue "physically" in the system with dropping. Next, using this result and the paradigm of embedded Markov chain, we construct a specific-type system of integral equations for the transient queue-size conditional distributions, where the condition is a number of packets present in the system initially. The technique of integral equations, combined with the potential approach, was sucessfully used in [9–13] for finite-buffer queues without restrictions in the enqueueing process. From the original system we obtain the corresponding one written for the Laplace transforms. Transforming it to the matrix form we obtain the general solution.

So, the remaining part of the article is organized as follows. In the next Section 2 we give a mathematical description of the system, introduce necessary notations and prove a theorem that gives the explicit formula for the probability $a_{n,k}(y)$ of k "real" arrivals in the period $(0, y]$ on condition that the system contains n packets at $t = 0$. In Section 3, applying the idea of embedded Markov chain and results from Section 2, we build the system of integral equations for the conditional queue-size distributions. In Section 4 we rewrite in a matrix form the corresponding system built for the Laplace transforms and find its general solution. In Section 5 we present explicit results for the special case of the $M/M/1-$type system with packet dropping and a one-place buffer. Section 6 contains exemplary numerical computations, and the last Section 7 is devoted to concluding remarks.

2 Queueing model and auxiliary results

Let us consider the one-server $M/G/1/N$-type queueing system with single Poisson arrivals with intensity λ, and generally distributed service times with a distribution function $F(\cdot)$. The maximum system capacity equals N i.e. we have $(N-1)$-place buffer queue and one place for service. The input stream is being "filtered" by a dropping function d_n, expressing the probability that the arriving packet will be dropped if the system "state" at the pre-arrival epoch equals n (we have n packets present in the system at this time). So, of course, $0 \le d_n \le 1$ for any $0 \le n \le N$, and $d_N = 1$.

Introduce te following set of notations which will be used in the paper:

- ⋄ $X(t)$ – for the number of packets present in the system at time t;
- ⋄ $\overline{F}(t) = 1 - F(t)$ – for the tail of the distribution function $F(\cdot)$ at point t;
- ⋄ $\delta_{i,j}$ – for the Kronecker delta function.

Denote by $h_{n,k}(y)$ the probability that exactly $0 \le k \le n$ packets join "physically" the system (are accepted for service without dropping) during $(0, y]$, on condition that the number of packets present at the opening equals $0 \le n \le N$, and the time y precedes the first service completion epoch occurring after $t = 0$. In [3] the formula for the Laplace transform of $h_{n,k}(y)$ was found. In the present paper we need and find the explicit formula for "pure" $h_{n,k}(y)$.

Theorem 2.1. In the $M/G/1/N$-type queue with dropping function the probability $h_{n,k}(y)$ that before the first service completion epoch y after the opening, exactly k packets "physically" enter the system, on the condition that the number of packets present in the system initially equals $0 \le n \le N$, is given by the following formulae:

$$h_{n,0}(y) = e^{-\lambda(1-d_n)y}, \tag{1}$$

$$h_{n,k}(y) = \sum_{i=k}^{\infty} \frac{(\lambda y)^i}{i!} e^{-\lambda y} \times \prod_{j=0}^{k-1}(1-d_{n+j}) \sum_{\substack{j_0,\dots,j_k \ge 0,\\ j_0+\dots+j_k = i-k}} d_n^{j_0} d_{n+1}^{j_1} \dots d_{n+k}^{j_k},$$

$$\text{where } 1 \le k \le N-n, \text{ and} \tag{2}$$

$$h_{n,k}(y) = 0, \quad k > N-n. \tag{3}$$

Proof. The probability that the Poisson arrival process "jumps" i times before the first service completion epoch y and all the packets are dropped equals $\frac{(\lambda y)^i}{i!} e^{-\lambda y} d_n^i$. Summing over all possibilities i.e. i from 0 to ∞ gives (1).

In the case of k "physical" inputs, the number of $i \ge k$ jumps of the Poisson arrival process must occur before y, that explains the first factor on the right side of (2). Moreover, successive packets from k ones are being accepted for service independently with probabilities $1 - d_n$, $1 - d_{n+1}$, ... $1 - d_{n+k-1}$, that comments the second factor on the right side of (2). Finally, the last factor presents all possibilities of dropping the remaining $i - k$ packets which are lost. For example, in the formula for $h_{n,1}(y)$ the last factor is a sum of the following summands:

$$d_n^0 d_{n+1}^{i-1} + d_n^1 d_{n+1}^{i-2} + \dots + d_n^{i-1} d_{n+1}^0. \tag{4}$$

Successive components in (4) indicate which one of the arriving packets (from i packets occurring), in turn, is accepted for service. For instance, the first summand $d_n^0 d_{n+1}^{i-1}$ relates to the situation in which the first arriving packet is accepted and the next $i-1$ are dropped. Similarly, $d_n^2 d_{n+1}^{i-3}$ describes the situation in which the accepted packet is the third one.

If the system contains exactly n packets at the opening then, of course, the number k of packets which "physically" join the system before the first service completion epoch y must be a number between 0 and $N-n$. So, the formula (3) is trivial. □

In the next section we apply the probabilities $a_{n,k}(y)$ from Theorem 2.1. to write a specific-type system of integral equations for conditional transient queue-size distribution.

3 Integral equations for conditional transient queue-size distributions

In this section we derive a system of integral equations for non-stationary queue-size distributions in the $M/G/1/N$-type queue, conditioned by the numbers of packets present in the system at time $t=0$.

Let us introduce the following notation:

$$Q_n(t,m) = \mathbf{P}\{X(t) = m \,|\, X(0) = n\}, \tag{5}$$

where $0 \le m, n \le N$.

Assume, firstly, that the system is empty at the opening. Let us note that the following integral equation holds then true:

$$Q_0(t,m) = \lambda \int_0^t \Big[d_0 Q_0(t-x,m) + (1-d_0) Q_1(t-x,m) \Big] e^{-\lambda x} dx + e^{-\lambda t}\delta_{m,0}. \tag{6}$$

Indeed, if the first arrival occurs at time $x < t$, then the evolution of the system is being continued, beginning at this time, with no packet present (with probability d_0) or with exactly one packet present (with probability $1-d_0$). In the case of no arrivals before time t the random event $\{X(t) = m\}$ is equivalent to $\{m = 0\}$.

Consider now the situation in which the system contains at least one packet at time $t = 0$. As it is well known, in the $M/G/1$-type system successive service completion epochs are Markov (renewal) moments (see e.g. [4]). Applying the formula of total probability with respect to the first service completion epoch after the opening of the system, we can write the following system of integral equations:

$$Q_n(t,m) = \sum_{k=0}^{N-n} \int_0^t h_{n,k}(y) Q_{n+k-1}(t-y,m) dF(y) + \overline{F}(t) h_{n,m-n}(t), \tag{7}$$

where $1 \le n \le N$, and the representations for $h_{n,k}(t)$ were found in (1)–(3).

Let us comment briefly the last equation. The first summand on the right side of (7) relates to the situation in which the first departure epoch y occurs before t. In the case of k packets "physically" arriving before y, at the Markov moment y the system "renews" its operation with $n+k-1$ packets present. According to the second summand on the right side of (7), if the first service ends after t then $X(t) = m$, $m \ge n$, if and only if exactly $m-n$ packets enter before t.

4 Solution for transforms of conditional queue-size distributions

The main aim of this section is to find the compact formulae for the Laplace transforms of conditional queue-size distributions which, additionally, can be useful in numerical practice.

Let us introduce the following notation:

$$\widehat{Q}_n(s,m) = \int_0^\infty e^{-st} Q_n(t,m) dt, \quad \mathrm{Re}(s) > 0, \tag{8}$$

and define the functions

$$\alpha(s) = \frac{\lambda d_0}{s+\lambda}, \qquad \beta(s) = \frac{\lambda(1-d_0)}{s+\lambda} \tag{9}$$

and

$$\gamma(s,m) = \frac{\delta_{m,0}}{s+\lambda}. \tag{10}$$

Besides, let

$$\widehat{h}_{i,j}(s) = \int_0^\infty e^{-st} h_{i,j}(t) dF(t) \tag{11}$$

and

$$\varDelta_{i,j}(s) = \int_0^\infty e^{-st} h_{i,j}(t) \overline{F}(t) dt. \tag{12}$$

Introducing (8)–(12) into the equations (6)–(7) lead to the following system:

$$\big[1-\alpha(s)\big]\widehat{Q}_0(s,m) - \beta(s)\widehat{Q}_1(s,m) = \gamma(s,m) \tag{13}$$

and

$$\widehat{Q}_n(s,m) = \sum_{k=0}^{N-n} \widehat{h}_{n,k}(s) \widehat{Q}_{n+k-1}(s,m) + \varDelta_{n,m-n}(s), \tag{14}$$

where $1 \le n \le N$.

Let us note that the system of equations (13–(14) can be written in a matrix form.

Indeed, let $\mathbf{A}(s)$ be an $(N+1)\times(N+1)$ matrix, with rows and columns numbered from 0 to N, and with entries defined as follows:

$$A_{0,0}(s)=1-\alpha(s),\quad A_{0,1}(s)=-\beta(s),$$
$$A_{0,k}(s)=0\ \text{ for }\ 2\le k\le N. \tag{15}$$

Besides, for $1\le i\le N$,

$$A_{i,j}(s)=\begin{cases}\widehat{h}_{i,j}(s) & \text{for } 1\le i\le N-2,\ i<j\le N-1,\\ \widehat{h}_{i,0}(s) & \text{for } 1\le i\le N,\ j=i-1,\\ \widehat{h}_{i,i}(s)-1, & \text{for } 1\le i\le N-1,\ j=i,\\ -1 & \text{for } i=j=N,\\ 0 & \text{otherwise.}\end{cases} \tag{16}$$

Similarly, let $\mathbf{B}(s,m)$ and $\mathbf{Q}(s,m)$ be $(N+1)\times 1$ matrices defined in the following way:

$$\mathbf{B}(s,m)=\big[\gamma(s,m),\ -\varDelta_{1,m-1}(s),,\ ...,\ -\varDelta_{N,m-N}(s)\big]^T \tag{17}$$

and

$$\mathbf{Q}(s,m)=\big[\widehat{Q}_0(s,m),\ \widehat{Q}_1(s,m),\ ...,\ \widehat{Q}_N(s,m)\big]^T. \tag{18}$$

Referring to (15)–(18) the system (13)–(14) can be rewritten in the form

$$\mathbf{A}(s)\mathbf{Q}(s,m)=\mathbf{B}(s,m). \tag{19}$$

Of course, just from the definition (5) of $Q_n(t,m)$, follows that the solution of the system is unique, thus it can be derived via Cramer's rule.

Theorem 4.1. In the $M/G/1/N$-type queueing system with a dropping function the vector $\widehat{Q}(s,m)$ of the Laplace transforms of conditional transient queue-size distributions can be found as

$$\mathbf{Q}(s,m)=\mathbf{A}^{-1}(s)\mathbf{B}(s,m), \tag{20}$$

where $\mathbf{A}^{-1}(s)$ is an inverse matrix of $\mathbf{A}(s)$ defined in (15)–(16), and the matrix $\mathbf{B}(s,m)$ is defined in (17).

Due to the finite capacity of the buffer the stationary state of the system exists and then is independent on the number of packets present in the system initially.

Denote

$$Q(m)=\lim_{t\to\infty}Q_n(t,m). \tag{21}$$

From the representation (20) given in Theorem 4.1., for a particular system, we can find the stationary probability using the well-known Tauberian theorem i.e.

$$Q(m)=\lim_{s\downarrow 0}s\widehat{Q}_n(s,m), \tag{22}$$

where n can be chosen arbitrarily from 0 to N.

5 Simplifications for a one-place buffer

In this section we obtain the explicit formulae for conditional distributions $Q_n(t,m)$ in the $M/M/1/2$-type system with one-place buffer (so $N=2$) and exponentially distributed service times with mean μ^{-1}.

Just from the representations (1)–(2) we get

$$h_{1,0}(y)=e^{-\lambda(1-d_1)y}, \tag{23}$$
$$h_{1,1}(y)=1-e^{-\lambda(1-d_1)y}, \tag{24}$$
$$h_{2,0}(y)=1. \tag{25}$$

Moreover, from (11), we obtain

$$\widehat{h}_{1,0}(s)=\frac{\mu}{s+\mu+\lambda(1-d_1)}, \tag{26}$$
$$\widehat{h}_{1,1}(s)=\frac{\lambda\mu(1-d_1)}{(s+\mu)\big(s+\mu+\lambda(1-d_1)\big)}, \tag{27}$$
$$\widehat{h}_{2,0}(s)=\frac{\mu}{s+\mu}. \tag{28}$$

Now, from (15)–(16) we get

$$\mathbf{A}(s)=\begin{bmatrix}1-\alpha(s) & -\beta(s) & 0\\ \widehat{h}_{1,0}(s) & \widehat{h}_{1,1}(s)-1 & 0\\ 0 & \widehat{h}_{2,0}(s) & -1\end{bmatrix}$$
$$=\begin{bmatrix}1-\frac{\lambda d_0}{s+\lambda} & -\frac{\lambda(1-d_0)}{s+\lambda} & 0\\ \frac{\mu}{s+\mu+\lambda(1-d_1)} & \frac{\lambda\mu(1-d_1)}{(s+\mu)\left(s+\mu+\lambda(1-d_1)\right)}-1 & 0\\ 0 & \frac{\mu}{s+\mu} & -1\end{bmatrix}. \tag{29}$$

Similarly, taking into consideration the definition (12), in the considered system we have

$$\varDelta_{i,j}(s)=\frac{\widehat{h}_{i,j}(s)}{\mu}. \tag{30}$$

Referring to (17), the forms of a column matrix $\mathbf{B}(s,m)$ in dependence on m are following:

$$\mathbf{B}(s,0)=\Big[\frac{1}{s+\lambda},\ 0,\ 0\Big]^T, \tag{31}$$
$$\mathbf{B}(s,1)=\Big[0,\ -\frac{1}{s+\mu+\lambda(1-d_1)},\ 0\Big]^T, \tag{32}$$
$$\mathbf{B}(s,2)=\Big[0,\ -\frac{\lambda(1-d_1)}{(s+\mu)\big(s+\mu+\lambda(1-d_1)\big)},\ -\frac{1}{s+\mu}\Big]^T. \tag{33}$$

In the next section we present sample numerical results for the $M/M/1/2-$type system, applying the formulae (29)–(33).

6 Numerical results

Let us consider the $M/M/1/2-$type system with dropping of packets analyzed in the previous section. In this section we investigate the influence of the traffic load, and separately the arrival and service intensities, for the probability that the system is empty at fixed moment of time. Moreover, we analyze the dependence between the shape of a dropping function and the probability of transient buffer saturation. All computations we execute using the *Mathematica* environment.

Example 1. Let us fix $\mu = 4$ and observe the system at the moment $T = 0.5$. Define the dropping function as follows:

$$d_0 = 0.25 \text{ and } d_1 = 0.50. \tag{34}$$

We are interested in probabilities

$$\mathbf{P}\{X(0.5) = 0 \,|\, X(0) = n\}$$

for ten different values of the arrival intensity λ and, in consequence, for different values of the traffic load $\rho = \frac{\lambda}{\mu}$. In Tab. 1 we state results obtained for all possible numbers n ($n = 0, 1, 2$) of packets present in the system at the opening.

No.	λ	ρ	$n=0$	$n=1$	$n=2$
1	0.5	0.125	0.922188	0.795368	0.554044
2	1.0	0.250	0.850577	0.732273	0.517191
3	2.0	0.500	0.724228	0.622449	0.451770
4	3.0	0.750	0.617680	0.531206	0.395930
5	4.0	1.000	0.527995	0.455258	0.348165
6	6.0	1.500	0.389199	0.338926	0.272011
7	8.0	2.000	0.291077	0.257137	0.215478
8	10.0	2.500	0.221432	0.198914	0.173058
9	16.0	4.000	0.109176	0.103040	0.096950
10	24.0	6.000	0.052637	0.052633	0.052631

Table 1 Conditional probabilities $\mathbf{P}\{X(0.5) = 0 \,|\, X(0) = n\}$ in a function of intensity of arrivals λ

The results from Tab. 1 are presented geometrically in Fig. 1, where the solid line relates to the case of $n = 0$ and the dashed and dotted lines - to the cases of $n = 1$ and $n = 2$ respectively.

Let us note that, obviously, as the intensity of arrivals λ increases (so, simultaneously, as the traffic load ρ increases) then the probabilities that the server is idle at $T = 0.5$ decrease. Moreover, let us observe that the reduction is the most visible in the case of $n = 0$, when the system is empty initially. It is intuitively clear. Indeed, in this case the impact of the increasing arrival intensity λ for the probability that the system is empty is the greatest one.

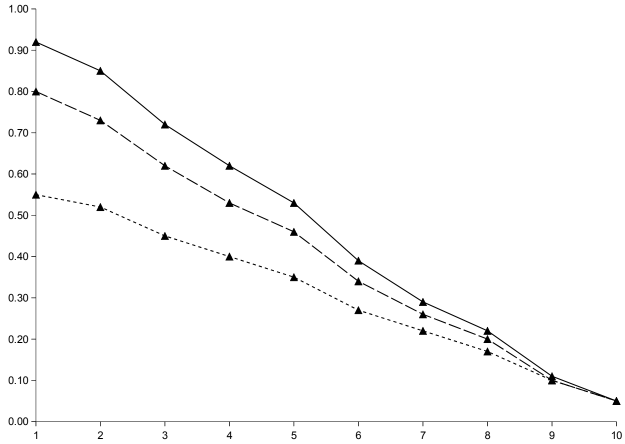

Fig. 1 Conditional probabilities $\mathbf{P}\{X(0.5) = 0 \,|\, X(0) = n\}$ in a function of intensity of arrivals λ

Example 2. Let us investigate the influence of the service intensity μ on the probability that the system is empty at the moment $T = 1.0$. Fix $\lambda = 1$ and take the same dropping function as in Example 1. The values of probabilities

$$\mathbf{P}\{X(1) = 0 \,|\, X(0) = n\}$$

for $n = 0, 1, 2$ and for ten different values of μ are given in Tab. 2 and presented geometrically in Fig. 2. The solid, dashed and dotted lines, as in Example 1, relate to the cases of $n = 0$, $n = 1$ and $n = 2$ respectively.

No.	μ	ρ	$n=0$	$n=1$	$n=2$
1	0.167	6.000	0.503288	0.085170	0.008403
2	0.250	4.000	0.517869	0.124687	0.018012
3	0.400	2.500	0.542721	0.191021	0.042289
4	0.500	2.000	0.558352	0.232020	0.062412
5	0.667	1.500	0.582842	0.295055	0.101003
6	1.000	1.000	0.626520	0.403407	0.189132
7	1.333	0.750	0.664036	0.491721	0.281462
8	2.000	0.500	0.724228	0.622449	0.451770
9	4.000	0.250	0.830293	0.810514	0.756733
10	8.000	0.125	0.909485	0.908848	0.906238

Table 2 Conditional probabilities $\mathbf{P}\{X(1) = 0 \,|\, X(0) = n\}$ in a function of service intensity μ

As one can observe, obviously, as the service intensity μ (and the traffic load ρ) increases then the conditional probabilities $\mathbf{P}\{X(1) = 0\}$ increase too. The increase of probabilities is the greatest one in the case of the system being saturated initially ($n = 2$). Indeed, in this case the influence of the increasing service intensity for the probability of empty the system is the strongest one.

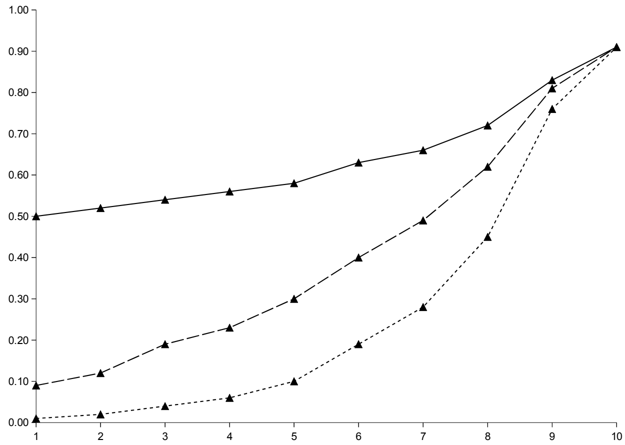

Fig. 2 Conditional probabilities $\mathbf{P}\{X(1) = 0 \,|\, X(0) = n\}$ in a function of service intensity μ

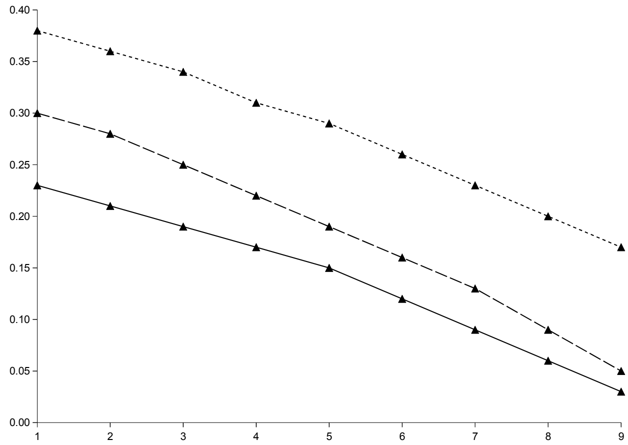

Fig. 3 Conditional probabilities $\mathbf{P}\{X(1) = 2 \,|\, X(0) = n\}$ in a function of d_1

Example 3. Let us now consider the critically loaded system ($\rho = 1$) in which $\lambda = \mu = 2$, and take $d_0 = 0.1$. Investigate the influence of the value of d_1 for the conditional probabilities that the system is saturated at the time $T = 1.0$ i.e. $X(1) = 2$. Results for d_1 changing from 0.1 to 0.9 with step 0.1 are given in Tab. 3 and visualized in Fig. 3, where the solid, dashed and dotted lines denote the cases of $n = 0$, $n = 1$ and $n = 2$ respectively.

No.	d_1	$n=0$	$n=1$	$n=2$
1	0.1	0.232453	0.301621	0.377886
2	0.2	0.212943	0.277333	0.357580
3	0.3	0.192182	0.251294	0.335959
4	0.4	0.170051	0.223314	0.312901
5	0.5	0.146422	0.193176	0.288267
6	0.6	0.121147	0.160631	0.261903
7	0.7	0.094063	0.125394	0.233636
8	0.8	0.064987	0.087138	0.203271
9	0.9	0.033710	0.045486	0.170596

Table 3 Conditional probabilities $\mathbf{P}\{X(1) = 2 \,|\, X(0) = n\}$ in a function of d_1

Of course the values of probabilities of system saturation decrease essentially as the value of d_1 increases. Besides, let us note that in a short period of time the probability of loss of the arriving packet remain relatively high, despite very high values of the dropping probability d_1, in the case of the system that is "full" initially.

Example 4. Let us take now $d_1 = 0.1$ and investigate the influence of increasing values d_0 (from 0.1 to 0.9) for the probability of the saturation of the system at the time $T = 1.0$, if the system is saturated at the opening. Let us consider three different values of the traffic load: the critical load $\rho = 1$ ($\lambda = \mu = 2$), the overload $\rho = 2$ ($\lambda = 4$, $\mu = 2$) and the extreme overload $\rho = 4$ ($\lambda = 8$, $\mu = 2$). The results of the experiment are given in Tab. 4 and presented geometrically in Fig. 4, where the solid, dashed and dotted lines relate to the cases of $\rho = 1$, $\rho = 2$ and $\rho = 4$ respectively.

Let us note that as the traffic load ρ gets larger, the decrease of the corresponding probabilities is less, from about 33 percent in the case of $\rho = 1$ to as little as 25 percent for $\rho = 4$. Indeed, at very high traffic load the probability that the system will become empty before $T = 1.0$ is small, and only in this case the value of d_0 can "act" for reducing the probability of saturation of the system.

No.	d_0	$\rho=1$	$\rho=2$	$\rho=4$
1	0.1	0.301621	0.527825	0.736625
2	0.2	0.293163	0.517356	0.730989
3	0.3	0.283955	0.505096	0.723817
4	0.4	0.273911	0.490635	0.714454
5	0.5	0.262935	0.473447	0.701876
6	0.6	0.250918	0.452861	0.684443
7	0.7	0.237735	0.428012	0.659468
8	0.8	0.223246	0.397780	0.622448
9	0.9	0.207289	0.360711	0.565676

Table 4 Conditional probabilities $\mathbf{P}\{X(1) = 2 \,|\, X(0) = 2\}$ in a function of d_0 for three different values of ρ

7 Conclusion

In the paper we consider the $M/G/1/N$-type queueing system in which the stream of the arriving packets is

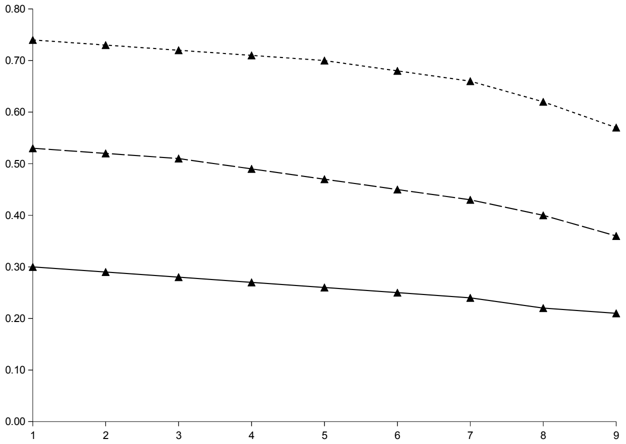

Fig. 4 Conditional probabilities $\mathbf{P}\{X(1) = 2\,|\,X(0) = 2\}$ in a function of d_0 for three different values of ρ

being "screened" by a dropping function. To find the representations for the conditional transient queue-size distributions, where the condition is in the number of packets present in the system initially, we propose a simple direct approach. Firstly, applying the idea of the embedded Markov chain and the formula of total probability, we build a specific-type system of integral equations for the transient queue-size distributions. Using the matrix notation we give the solution of the corresponding system written for the Laplace transforms. Moreover, we consider separately the special case of the $M/M/1/2-$type system, where we obtain all the formulae in the explicit forms. For exemplary queueing models we investigate numerically the influence of the arrival and service intensities, and of the traffic load on the probability that the system is idle at a fixed time. Moreover, we analyze the impact of the shape of a dropping function on the probability of the saturation of the system, in particular for different values of the traffic load.

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