Applied Mathematics & Information Sciences
An International Journal

Projective Lag Synchronization of Chaotic Systems with Parameter Mismatch

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Received: 18 Nov. 2012; Revised 4 Jan. 2012; Accepted 16 Feb. 2012
Published online: 1 Jan. 2013

Abstract: This paper investigates projective synchronization with time delay and parameter mismatch for a lately proposed simple yet complex one-parameter chaotic system, and Rössler system as well. With an active control scheme, it is proved that the zero solution of the dynamical error system is globally asymptotically stable based on Barbalat's lemma. Numerical simulations verify the correctness and effectiveness of the proposed method.

Keywords: Projective lag synchronization, mismatch parameter, one-parameter chaotic system.

1. Introduction

The synchronization of chaotic systems firstly appeared in the article [1] by Pecora and Carroll, thereafter it has attracted considerably increasing attention and has become a hot research topic. It has shown that chaotic synchronization has potential applications in secure communications [2,3], as well as oscillator design. Now there are many candidate chaotic systems, such as, famous Lorenz system [4], Chen system [5], and the unified chaotic system [6, 7], even hyper-chaotic system. In 2012, Wang etc. found a simple yet complex one-parameter chaotic system [8], which has more simple algebraic structure and richer dynamic behaviors than the unified chaotic system. Perhaps it can provide convenience for designing secure communication circuits, and enhances the security in communication.

Currently, there are several types of synchronization in chaotic system: complete synchronization, phase synchronization, generalized synchronization, anti-synchronization, projective synchronization, and lag synchronization [10–17]. Among these, projective synchronization refers to that the corresponding state variables of the drive-response systems evolve in a proportional scale, including complete synchronization and anti-synchronization. That the state variable of the response system is synchronized with the historical state variable of the drive system is called as lag synchronization, which is more reasonable in engineering applications because time delay exists inevitably for finite signal transmission speeds in communication. Therefore, projective lag synchronization, which can suit for more practical cases, has attracted wide attention [18,19]. That they have investigated projective lag synchronization between the drive-response systems is based on the same chaotic system with identical system parameter, that is to say, the drive-response systems are the same and have the identical parameters. However, dynamical systems is easily perturbed by external forces, and it was found that synchronization in two different types of chaotic oscillators can occur in Refs. [20,21], this is very important in engineering applications since no two oscillators can be identical in practical systems. Therefore, it is very meaningful to study synchronization in chaotic systems with parameter mismatch.

In this paper, combining the time delay and parameter mismatch, we extend the case of parameter mismatch in a chaotic system to that of parameter mismatch in a class of chaotic systems, and firstly propose projective lag synchronization in chaotic oscillators with parameter mismatch. The rest of this paper is organized as follows. Projective lag synchronization and a new family of chaotic system are introduced respectively in Section 2 and Section 3 . The theoretical analysis and numerical results of projective lag synchronization in a lately found chaotic system with parameter mismatch, as well as Rössler sys-

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tem [22], are given in Section 4. Finally, some conclusions are drawn in Section 5.

2. Projective lag synchronization with parameter mismatch

Consider a drive system

$$\dot{x} = f(x) \tag{1}$$

and the controlled response system

$$\dot{y} = f(y) + \Delta f(y) + u(t) \tag{2}$$

where, $\Delta f(y)$ contains the mismatch parameter, $x = (x_1, x_2, \cdots, x_n)^T$ and $y = (y_1, y_2, \cdots, y_n)^T$ are the state vectors, and $f : R^n \to R^n$ is a continuous nonlinear vector function, $u(t)$ is the controller to be designed. Define the error vector $e(t) = y(t) - \lambda x(t - \tau) \doteq y - \lambda x_\tau$, where λ is the nonzero scale factor, τ is a constant representing time delay or lag. The dynamical error system is described as follows:

$$\dot{e} = \dot{y} - \lambda \dot{x}_\tau = f(y) + \Delta f(y) + u(t) - \lambda f(x_\tau) \tag{3}$$

Definition:(Projective lag synchronization) For any initial values of systems (1) and (2), if the dynamical error system (3) satisfies $||e(t)|| \to 0$ as $t \to \infty$, the zero solution of the error system is said to be globally stable. The drive-response systems (1) and (2) are said to be in projective lag synchronization. Specifically, for $\lambda = 1, \tau = 0$, it is reduced to the traditionally global complete synchronization. For $\lambda = -1, \tau = 0$, it is the traditionally global anti-synchronization. When $\tau < 0$, it is the globally projective anticipating synchronization.

3. A new one-parameter chaotic system

Recently, Wang et al. [8] found a new family of chaotic systems described by the following system

$$\begin{cases} \dot{x}_1 = -x_1 - x_2 \\ \dot{x}_2 = -x_1 - x_1 x_3 + r x_2 \\ \dot{x}_3 = -0.1 x_3 + x_1 x_2 \end{cases} \tag{4}$$

with only one real parameter r, which can continuously generate a variety of cascading Lorenz-like attractors. Although this new family of chaotic systems has very rich and complex dynamics, it has a very simple algebraic structure with only two quadric terms and all nonzero coefficients in linear part being -1 except -0.1 and r. Surprisingly, although this new system does not belong to the family of Lorenz-type systems in some existing classifications such as the generalized Lorenz canonical form, it can generate not only Lorenz-like attractors but also Chen-like attractors, as shown in Fig 1. This reveals that further study of the system algebraic structure and its effects on the system dynamics remain an important and interesting challenge. In addition, this system with $r \in [0, 0.05)$ is subtle, and contains somewhat complicated dynamical behaviors, but it is still chaotic. When $r \in [0.05, 0.74)$, it has several common features as that of the unified chaotic system [7], which includes Lorenz system [4], Lü system [9] and Chen system [5].

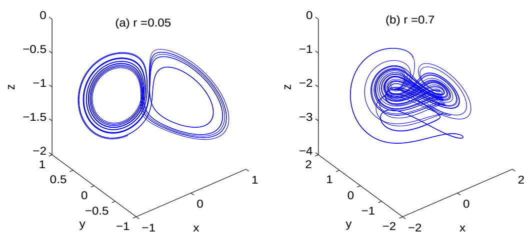

Figure 1 The phase diagram of the new chaotic system.

4. Simulation examples

Choose this new chaotic system (4) as the drive system, where r is any value in $[0, 0.74)$, implying that the system is chaotic and contains some types of chaotic systems. The controlled response system is described as

$$\begin{cases} \dot{x}_1 = -x_1 - x_2 + u_1 \\ \dot{x}_2 = -x_1 - x_1 x_3 + r x_2 + \Delta r x_2 + u_2 \\ \dot{x}_3 = -0.1 x_3 + x_1 x_2 + u_3 \end{cases} \tag{5}$$

Here, Δr is the mismatch in parameter, r and $r + \Delta r \in [0, 0.74)$. By active control technique, one can choose the controllers $u_1 = e_2, u_2 = e_1 - l e_2 + y_1 y_3 - \lambda x_{1\tau} x_{3\tau} - \Delta r y_2, u_3 = -y_1 y_2 + \lambda x_{1\tau} x_{2\tau}$. The error system can be written as

$$\begin{cases} \dot{e}_1 = -e_1 - e_2 + u_1 = -e_1 \\ \dot{e}_2 = -e_1 - y_1 y_3 + \lambda x_{1\tau} x_{3\tau} + r e_2 + \Delta r y_2 + u_2 \\ \quad = -(l - r) e_2 \\ \dot{e}_3 = -0.1 e_3 + y_1 y_2 - \lambda x_{1\tau} x_{2\tau} + u_3 = -0.1 e_3 \end{cases} \tag{6}$$

where, l is a constant and $l > r$. Construct a Lyapunov function as $V(t) = (e_1^2 + e_2^2 + e_3^2)/2$, and its time derivative along the trajectories of the error system (6) is

$$\dot{V}(t) = \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 = -e_1^2 - (l - r) e_2^2 - 0.1 e_3^2 \tag{7}$$

Denote $m = \min\{l - r, 0.1\}$, one has

$$\dot{V}(t) \le -m[e_1^2 + e_2^2 + e_3^2] \le 0 \tag{8}$$

Obviously $0 \leq V(t) \leq V(0)$, so the limit of $V(t)$ exists as $t \to \infty$. From equation (8), one obtains $e_i^2 \leq -\frac{1}{m}\dot{V}(t)(i = 1, 2, 3)$, and

$$\begin{aligned} \lim_{t\to\infty} \int_0^t e_i^2(t)dt &\leq -\frac{1}{m} \lim_{t\to\infty} \int_0^t \dot{V}(t)dt \\ &= \frac{1}{m}[V(0) - \lim_{t\to\infty} V(t)] \end{aligned} \tag{9}$$

According to Barbalat's lemma [23], $e_i(t) \to 0(i = 1, 2, 3)$ as $t \to \infty$. So the error system (6) is global asymptotically stable at the origin, that is to say, the global asymptotical projective lag synchronization occurs between systems (4) and (5) with parameter mismatch. In practice, the error system easily converges to the origin in finite time.

Numerical simulations are shown in Fig. 2 with $\tau = 1, \lambda = 2$ and in Fig. 3 with $\tau = 2, \lambda = -1$. The synchronization errors varying with time are shown in (a) of both figures, implying that synchronization between the drive-response systems can be easily achieved, and it clearly founds that the third components of the new chaotic system spare more time to synchronization than the other two. Specifically, the first components of the drive-response systems evolving with time are plotted in (b), while the two-dimensional and three-dimensional phase diagrams are displayed in (c) and (d) respectively. From all above panels, in which the curves of the drive system are shown in blue, and that of the response system in red, it is well shown that the drive-response systems with parameter mismatch are in lag synchronization in a proportional scale.

The Rössler system is further considered as the drive system,

$$\begin{cases} \dot{x}_1 = -x_2 - x_3 \\ \dot{x}_2 = x_1 + ax_2 \\ \dot{x}_3 = x_3(x_1 - c) + b \end{cases} \tag{10}$$

where, $a = b = 0.2$, and c is any value in the interval $[5, 10]$, implying that the system is chaotic. The controlled system is described as follows:

$$\begin{cases} \dot{y}_1 = -y_2 - y_3 + u_1 \\ \dot{y}_2 = y_1 + ay_2 + \Delta ay_2 + u_2 \\ \dot{y}_3 = y_3(y_1 - c) + b + \Delta cy_3 + \Delta b + u_3 \end{cases} \tag{11}$$

Here, Δa, Δb, and Δc are the mismatches of corresponding parameters. In this paper, we let $\Delta a = \Delta b = 0$, and randomly choose Δc such that $c + \Delta c \in [5, 10]$. Let the controllers $u_1 = e_2 + e_3 - e_1, u_2 = -e_1 - le_2, u_3 = -y_1y_3 + \lambda x_{1\tau}x_{3\tau} - \Delta cy_3 + (\lambda - 1)b$, then the error system can be written as

$$\begin{cases} \dot{e}_1 = -e_2 - e_3 + u_1 = -e_1 \\ \dot{e}_2 = e_1 + ae_2 + u_2 = -(l - a)e_2 \\ \dot{e}_3 = y_1y_3 - \lambda x_{1\tau}x_{3\tau} - ce_3 + \Delta cy_3 + (1 - \lambda)b + u_3 \\ \quad = -ce_3 \end{cases} \tag{12}$$

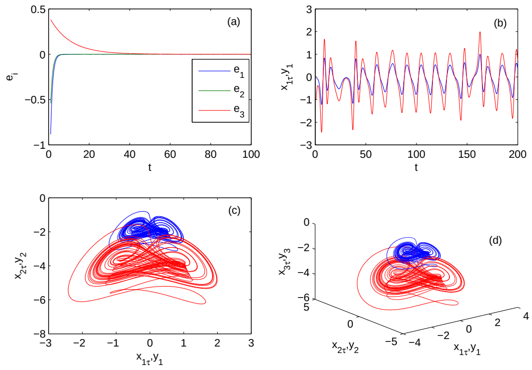

Figure 2 Projective lag synchronization in newest chaotic systems under parameter mismatch ($\tau = 1, \lambda = 2$). (a) The error e_i with time, (b) time series of y_1 and $x_{1\tau}$, (c) the two-dimensional phase diagrams, (d) the three-dimensional phase diagrams.

Construct a Lyapunov function $V(t) = (e_1^2 + e_2^2 + e_3^2)/2$, and its time derivative along the error system is

$$\dot{V}(t) = -e_1^2 - (l - a)e_2^2 - ce_3^2 \tag{13}$$

Choose l such that $l - a > 0$, and let $m = \min\{l - a, c, 1\}$, then

$$\dot{V}(t) \leq -m[e_1^2 + e_2^2 + e_3^2] \leq 0 \tag{14}$$

Similarly, according to Barbalat's lemma, $e_i(t) \to 0(i = 1, 2, 3)$ as $t \to \infty$. Thus the error system (12) is global asymptotically stable at the origin. The drive-response systems (10) and (11) are in projective lag synchronization. Figs. 4 and Fig. 5 respectively show the results with $\tau = 2, \lambda = 0.5$ and with $\tau = 1, \lambda = -1$, which illustrate the effectiveness of the proposed method.

5. Conclusions and discussions

In summary, this paper has studied the projective lag synchronization in chaotic systems with parameter mismatch. To be more specific, the response system follows the driver's by synchronizing with its past state in a proportional way. Based on active control technique and Barbalat's lemma, It has theoretically proven that the drive system and the response system, which have the mismatches in parameter, can be in projective synchronization with delay time. The lately found simple yet complex one-parameter chaotic system and famous Rössler system have been employed to further validate the effectiveness of the proposed method, and numerical results have demonstrated that projective lag synchronization in the drive-response systems is easy to achieved. This method combining time delay and parameter mismatch can be more applicable in practical systems, and perhaps the one-parameter chaotic system can

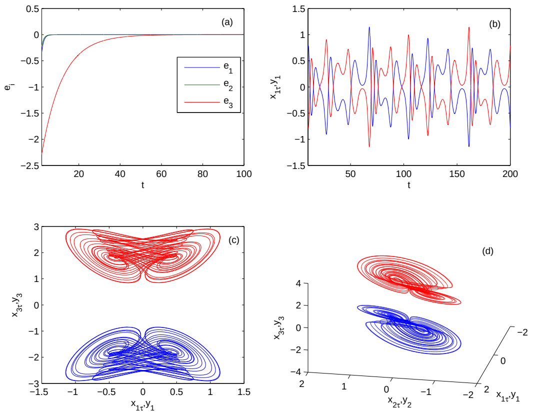

Figure 3 Projective lag synchronization in the new chaotic systems with parameter mismatch($\tau = 2, \lambda = -1$). (a) The error e_i with time, (b) time series of y_1 and $x_{1\tau}$, (c) the two-dimensional phase diagrams, (d) the three-dimensional phase diagrams.

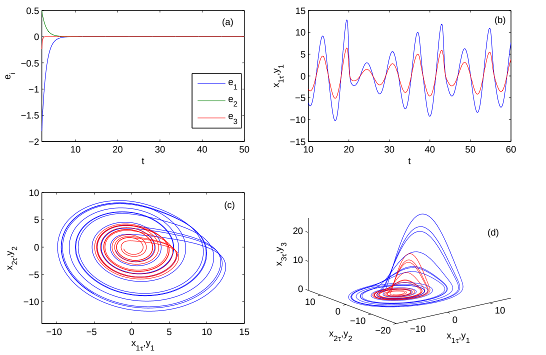

Figure 4 Projective lag synchronization in Rössler systems under parameter mismatch($\tau = 2, \lambda = 0.5$). (a) The error e_i with time, (b) time series of y_1 and $x_{1\tau}$, (c) the two-dimensional phase diagrams, (d) the three-dimensional phase diagrams.

reduce the design in secure communication since it has simple algebra structure and complicated dynamics.

Acknowledgement

The authors acknowledge the financial support of the National Natural Science Foundation of China (Grant No. 11172215) and the Fundamental Research Funds for the Central Universities. The author is grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

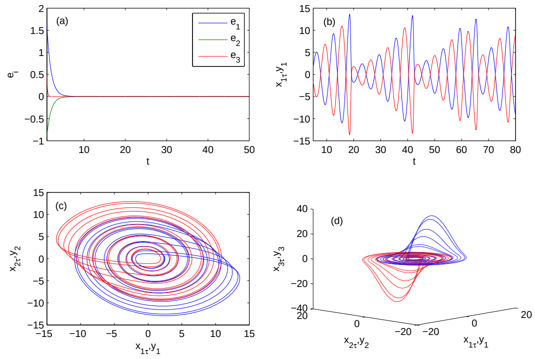

Figure 5 Projective lag synchronization in Rössler systems under parameter mismatch($\tau = 1, \lambda = -1$). (a) The error e_i with time, (b) time series of y_1 and $x_{1\tau}$, (c) the two-dimensional phase diagrams, (d) the three-dimensional phase diagrams.

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