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An Exponential Estimator over Regression Estimator Using Two Auxiliary variables

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Abstract: In the present study, we propose an estimator for population mean when the information is available for two variables under simple random sampling. Expressions for the MSE's of the proposed estimator are derived up to the first degree of approximation. The theoretical conditions have also been verified by a numerical example. It has been shown that the proposed estimator is more efficient than usual regression estimator for two variables.
Keywords: Simple random sampling, auxiliary variable, mean square error, efficiency.

1 Introduction

The use of auxiliary information can increase the precision of an estimator when study variable y is highly correlated with auxiliary variable x. The regression estimator for estimating the unknown population mean

$$t_1=\bar{y}+b\left(\bar{X}-\bar{x}\right) \tag{1}$$

The MSE expression of regression estimator is

$$MSE\left(t_1\right)=f_1\bar{Y}^2C_y^2\left(1-\rho^2\right) \tag{2}$$

Where, $f_1=\dfrac{N-n}{Nn}$, n is the sample size, N is the population size, $\left(C_y,C_x\right)$ are the coefficients of variation of the variates (y, x) respectively.

When there are two auxiliary variables X and Z, the regression estimator of $\bar{Y}$ is

$$t_2=\bar{y}+b_1\left(\bar{X}-\bar{x}\right)+b_2\left(\bar{Z}-\bar{z}\right) \tag{3}$$

Where $b_1=\dfrac{s_{yx}}{s_x^2}$ and $b_2=\dfrac{s_{yz}}{s_z^2}$. Here s_x^2 and s_z^2 are the sample variances respectively, s_{yx} and s_{yz} are the sample covariance respectively. The MSE expression of this estimator is

$$MSE\left(t_2\right)=f_1\bar{Y}^2C_y^2\left(1-\rho_{yx}^2-\rho_{yz}^2+2\rho_{yx}\rho_{yz}\rho_{xz}\right) \tag{4}$$

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2 The proposed Estimator

Following Malik and Singh [1], Bahl and Tuteja [2] , we propose the multivariate ratio estimator over regression estimator using information of two auxiliary variables as follows

$$t_p = \bar{y}\exp\left(\alpha_1 \frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\exp\left(\alpha_2 \frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right) + b_1\left(\bar{X}-\bar{x}\right) + b_2\left(\bar{Z}-\bar{z}\right) \tag{5}$$

To obtain the MSE of t_p to the first degree of approximation, we define

$$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}},\ e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}},\ \ e_2 = \frac{\bar{z}-\bar{Z}}{\bar{Z}}$$

Such that, $E(e_i) = 0;\ \ i = 0, 1, 2.$

$$E(e_0^2)=f_1C_y^2,\ E(e_1^2)=f_1C_x^2,\ E(e_2^2)=f_1C_z^2,$$

$$E(e_0e_1)=f_1K_{yx}C_x^2,\ E(e_0e_2)=f_1K_{yz}C_z^2,\quad E(e_1e_2)=f_1K_{xz}C_z^2,$$

$$K_{yx}=\rho_{yx}\frac{C_y}{C_x},\ K_{yz}=\rho_{yz}\frac{C_y}{C_z},\ K_{xz}=\rho_{xz}\frac{C_x}{C_z}.$$

Expressing equation (5) in terms of e's, we have

$$t_p = \bar{Y}\left(1+e_0\right)\left(\exp\left(\alpha_1\frac{-e_1}{2+e_1}\right)\exp\left(\alpha_2\frac{-e_2}{2+e_2}\right)\right)-b_1\bar{X}e_1-b_2\bar{Z}e_2$$

$$=\bar{Y}\left[1+e_0-\frac{\alpha_1e_1}{2}+\frac{\alpha_1^2e_1^2}{8}-\frac{\alpha_2e_2}{2}-\frac{\alpha_1\alpha_2e_1e_2}{4}+\frac{\alpha_2^2e_2^2}{8}-\frac{\alpha_2e_0e_2}{2}-\frac{\alpha_1e_0e_1}{2}\right]-b_1\bar{X}e_1-b_2\bar{Z}e_2 \tag{6}$$

Retaining the term's up to single power of e's in (6) we have

$$t_p-\bar{Y}=\left\{\bar{Y}\left[e_0-\frac{\alpha_1e_1}{2}-\frac{\alpha_2e_2}{2}\right]-b_1\bar{X}e_1-b_2\bar{Z}e_2\right\} \tag{7}$$

Squaring both sides of (7) and then taking expectations, we get the MSE of the estimator t_p up to the first order of approximation, as

$$MSE(t_p)=f_1\left\{\bar{Y}^2\left[C_y^2+\frac{\alpha_1^2C_x^2}{4}+\frac{\alpha_2^2C_z^2}{4}+\frac{\alpha_1\alpha_2k_{xz}C_z^2}{2}-\alpha_1k_{yx}C_x^2-\alpha_2k_{yz}C_z^2\right]\right.$$

$$+B_1^2\bar{X}^2C_x^2+B_2^2\bar{Z}^2C_z^2+2B_1B_2\bar{X}\bar{Z}k_{xz}C_z^2-2\bar{Y}\left[B_1\bar{X}k_{yx}C_{p_1}^2+B_2\bar{Z}k_{yz}C_z^2-\frac{\alpha_1B_1\bar{X}C_x^2}{2}\right.$$

$$-\frac{\alpha_2 B_2 \bar{Z} C_z^2}{2}-\frac{\alpha_1 B_2 \bar{Z} k_{xz} C_z^2}{2}-\frac{\alpha_2 B_1 \bar{X} k_{xz} C_z^2}{2}\Bigg]\Bigg\} \quad (8)$$

$$MSE(t_p)=A_1+\alpha_1^2 A_2+\alpha_2^2 A_3+\alpha_1\alpha_2 A_4-\alpha_1 A_5-\alpha_2 A_6 \quad (9)$$

$$A_1=f_1[\bar{Y}^2 C_y^2+B_1^2\bar{X}^2 C_x^2+B_2^2\bar{Z}^2 C_z^2+2B_1B_2\bar{X}\bar{Z}K_{xz}C_z^2-2\bar{Y}\bar{X}B_1K_{yx}C_x^2-2\bar{Y}\bar{Z}B_2K_{xz}C_z^2]$$

$$A_2=\frac{f_1\bar{Y}^2 C_x^2}{4}, A_3=\frac{f_1\bar{Y}^2 C_z^2}{4}, A_4=\frac{f_1\bar{Y}^2 K_{xz}C_z^2}{2}$$

$$A_5=f_1\left[K_{yx}C_x^2-B_1\bar{Y}\bar{X}C_x^2-\bar{Y}\bar{Z}B_2K_{xz}C_z^2\right]$$

$$A_6=f_1\left[K_{xz}C_z^2-B_1\bar{Y}\bar{X}K_{xz}C_z^2-\bar{Y}\bar{Z}B_2C_z^2\right]$$

Where, $B_1=\frac{S_{yx}}{S_x^2}$ and $B_2=\frac{S_{yz}}{S_z^2}$.

Minimising equation (9) with respect to α_1 and α_2 we get the optimum values as

$$\alpha_1=\frac{A_4A_6-2A_3A_5}{A_4^2-4A_2A_3} \text{ and } \alpha_2=\frac{A_4A_5-2A_2A_6}{A_4^2-4A_2A_3}$$

3 Numerical Illustrations

We have applied the traditional and proposed estimator on the data of apple production amount in 1999 (as interest of variate) and number of apple trees in 1999 (as first auxiliary variate), apple production amount in 1998 (as second auxiliary variate) of 204 villages in Black Sea Region of Turkey (Source: Institute of Statistics, Republic of Turkey).For this data, we have

$N=204$, $n=50$, $\bar{Y}=966$, $\bar{X}=26441$, $\bar{Z}=1014$, $S_y=2389.76$, $S_x=45402.78$, $S_z=2521.40$,

$S_{xz}=94636084$, $S_{yx}=77372777$, $S_{yz}=5684276$, $\rho_{xz}=0.83$, $\rho_{yx}=0.71$, $\rho_{yz}=0.94$

Table 3.1: MSE values of estimators.

Estimators	$\bar{y}$	t_2 (Regression)	t_p (Proposed)
MSE Values	86224.19	62097.28	2853.37

4 Conclusions

In this paper, we have proposed an exponential estimator over regression estimator for estimating unknown population mean of study variable using information on two auxiliary variables. From Table 3.1, we observe that the proposed estimator t_p is best followed by the regression estimator for two auxiliary variables.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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