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Generalized Ghost Dark Energy Model in the Framework of Quantum Cosmology for Different Scale Factor Choices

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Abstract: We intend to analyze the behavior of generalized ghost dark energy model (GGDE) in the framework $\hbar$ - corrected Friedmann Equations for three different choices of cosmological scale factor namely emergent, intermediate and logimidate scenarios. The behavior of some cosmological parameters are studied showing some consistency with the observations. For the stability analysis, we discuss the role of the squared speed of dark energy fluid v_D^2 we observe a type of fluctuation between stability and unstability behaviors in our model Scenarios.

Keywords: Dark energy, $\hbar$ corrected gravity

1 Introduction

Recent cosmological observations show that our universe is in accelerating mode [1,2,3,4,5]. This cosmic acceleration can be explained by assuming a fluid with large negative pressure denoted as dark energy (DE) or the energy of vacuum. The simplest candidate for dark energy is the cosmological constant Λ [6,7]. Since mid-nineties, the behavior of dark energy is studied using models like quintessence [8], tachyon [9], phantom [10], dilaton [11] and quintom field [12], Holographic DE [13] and cold dark matter ΛCDM [14]. A new model of dark energy called Veneziano ghost dark energy (GDE) is proposed to find a solution for the $U(1)$ Quantum chromodynamics QCD problem [7]. Within this theory ghosts field may be considered as a candidate for the dark energy where no new degree of freedom or new parameter is added to modify gravity and that what gives this theory its power compared to the others modified gravity models. Also, its vacuum energy can be used to explain the time-varying cosmological constant in a space-time, since the ghost field has no contribution to the vacuum energy in the Minkowskian spacetime. The energy density of the vacuum ghost field is proportional to $\Lambda_{QCD}^3 H$, where Λ_{QCD} is the QCD mass scale and H is the Hubble parameter [16,17]. A correct choice of Λ_{QCD} can eliminate the fine tuning problem proposed by standard cosmology. In [16,18] the GDE model is considered to have the energy density in the form:

$$\rho = \alpha H, \tag{1}$$

where, α is some constant parameter, $H = \dot{a}(t)/a(t)$ represents the Hubble parameter, $a(t)$ is the scale factor and $\dot{a}$ is its first derivative with respect to cosmic time t.

The vacuum energy of the Veneziano ghost field in QCD takes the form $H + O(H^2)$. That way Eq.(1) can be modified to a more general form which is known as generalized ghost dark energy GGDE [19,20,21]:

$$\rho = \alpha H + \delta H^2, \tag{2}$$

where, δ is a constant parameter. The addition of the second high order term shows a good agreement with observational data more than the normal GDE model.

Putting the GGDE together with quantum corrected Friedmann equations on the same frame is the aim of this work to investigate the behavior of the equation of state parameter ω, the deceleration parameter q, square speed of DE fluid V_D^2 and state finder operators s and r.
This work is arranged as follows. In next section the model is considered. In section 3, we study the evolution of the cosmological parameters. Namely, equation of state

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parameter ω, deceleration parameter q and geometrical state-finder parameters r and s for the emergent, intermediate and logimidate scenarios and finally the perspective is presented in section 4.

2 The Model

Effective field theory is considered to study the quantum structure of general relativity at scales below the Planck mass [19,20].

Effective field theory plays an important rule to distinguish between the quantum effects of the low energy particles from the physics at high energy. The latter effects are represented by the most general series of effective Lagrangians which are consistent with the symmetry of general relativity [21]. In the effective field theory of gravity, quantum loop calculations lead to well defined results in the low energy limit, which for one loop correction is proportional to $\hbar$. By using the Fourier transform, one can write the quantum mechanical correction to the potential in the form [23,22]:

$$\Phi(r) = -\frac{GM_1M_2}{r}[1+\lambda\frac{G(M_1+M_2)}{rc^2} - \tilde{\gamma}\frac{G\hbar}{r^2c^3}+...], \quad (3)$$

where, λ and $\tilde{\gamma}$ are two parameters that can take different values depending on the authors and r is the distance between two objects, G is the gravity constant and $\hbar$ is the planks constant, we redefine $r \longrightarrow r\prime = r(1+aGM/r)$). Many of the different values for γ_q which vary in sign are found in the literature [17,23,22]. By considering the gravity as an effective theory, one can write the corrected potential as a $\hbar$ - corrections to the Newtonian potential [23]:

$$\Phi(r) = -\frac{GM_1M_2}{r}[1-\gamma_q\frac{G\hbar}{r^2c^3}+....]. \quad (4)$$

By assuming the quantum correction as mentioned before, the total energy due to the quantum correction is given by [23]:

$$E = \frac{1}{2}m(\frac{da}{dt})^2 - \frac{GMm}{a} + \gamma_q\frac{G^2\hbar Mm}{a^3c^3}. \quad (5)$$

By using the energy density ρ and plank length l_p one can write [23]:

$$\frac{2E}{ma^2} = \frac{1}{a^2}(\frac{da}{dt})^2 - \frac{8}{3}\pi G\rho(1-\frac{l_p^2\gamma_q}{a^2}). \quad (6)$$

Since $H = a(\dot{t})/a(t)$, the first Friedmann equation with an $\hbar$-correction can be written as [23]:

$$H^2 + \frac{k}{a^2} = \frac{8}{3}\pi G\rho(1-\gamma_q\frac{l_p^2\gamma_q}{a^2}). \quad (7)$$

In this work we consider the system of units in which G and l_p equal to 1. By using the equation of continuity, which represents the conservation of energy, which has the form:

$$\dot{\rho} + 3H\rho(1+\omega) = 0. \quad (8)$$

One can write the second Friedman equation with an $\hbar$-correction as:

$$\ddot{a} = \frac{4\pi G}{3}a\rho(1+\omega) - 4\pi G\rho\gamma_q l_p^2\frac{\rho(1+\omega)}{a}. \quad (9)$$

Eqs. (7) and (9) are considered as the quantum corrected Friedmann equations derived within the context of Newtonian mechanics. The fractional energy densities for DE, DM and curvature parameter are given by the following quantities[4]:

$$\Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{8\pi G(t)\rho_D}{6H^2}, \quad (10)$$

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{8\pi G(t)\rho_m}{6H^2}, \quad (11)$$

$$\Omega_k = \frac{k}{a^2H^2}, \quad (12)$$

where, ρ_{cr} indicates the critical energy density and G is the gravitational constant.

3 GGDE in the $\hbar$ Correction Frame for Different Scale Factor Models

In this section, we are going to analyze our model through studying the evolution of some cosmological parameters for the three different choices of the cosmological scale factor. One can write Eq.(8) for dark energy dominate universe as:

$$\dot{\rho_D} + 3H\rho_D(1+\omega_D) = 0. \quad (13)$$

For interacting case, one finds:

$$\dot{\rho_D} + 3H\rho_D(1+\omega_D) = Q, \quad (14)$$

where, Q represents the interaction term which can be considered as an arbitrary function of cosmological parameters, like the Hubble parameter H and energy densities of the model. We assume the form $Q = b\dot{\rho_D}$ [24] for dark energy dominate universe, where, b is the coupling parameter between dark matter (DM) and dark energy (DE). Due to the unknown nature of DM and DE, different Lagrangians have been proposed to generate this interaction term. Actually, a suitable form of Q can be reconstructed using the theory of quantum gravity or through an observation scheme using the SNIa data [25, 26].

By differentiating Eq.(7) with respect to the cosmic time t and using Eq.(13), then after some algebraic steps, one finds:

$$\dot{H} - \frac{k}{a^2} = \frac{8\pi}{3}[2(1+\omega_D) + \frac{\alpha t^{-2\alpha-1}\gamma_q}{a_o^2H} - \frac{2\gamma_q(1+\omega_D)}{a_o^2t^{2\alpha}} \quad (15)$$

Adding Eq.(7) to Eq.(15) and dividing by H^2 one finds:

$$1+\frac{\dot{H}}{H^2}=1+\frac{8\pi G\left(\dot{\rho_{\text{GDE}}}-\frac{L_p^2\gamma_q\left(\dot{\rho_{\text{GDE}}}-\frac{2\dot{a}\rho_{\text{GDE}}}{a}\right)}{a^2}\right)}{\gamma H^3}. \tag{16}$$

Now, we derive the reconstructed cosmological quantities that we need for our model by assuming three different scenarios for the cosmological scale factor.

3.1 Emergent Scenario of the Cosmological Scale Factor

The emergent scenario assumes that the universe is isotropic and homogeneous at large scales. Also the universe is accelerating as suggested by recent cosmological measurements. The emergent scale factor could be written as [27]:

$$a=a_0\left(\lambda+e^{\mu t}\right)^{\beta} \tag{17}$$

where, a_0,λ,μ and β are four positive parameters of the model. In this case one can find an expression of Hubble parameter as:

$$H=\frac{\dot{a}}{a}=\frac{\beta\mu e^{\mu t}}{\lambda+e^{\mu t}} \tag{18}$$

Instead, the general expression of ρ_{EGDE} could be written, using Eq.(2), as:

$$\rho_{EGDE}=\frac{\alpha\beta\mu e^{\mu t}}{\lambda+e^{\mu t}}+\frac{\beta^2\delta\mu^2e^{2\mu t}}{\left(\lambda+e^{\mu t}\right)^2} \tag{19}$$

we assume parameters values $a_o=0.12,\gamma_q=\frac{-41}{10},\lambda=0.3,\mu=0.3,\beta=0.3,0.5$ and 0.7 for emergent case. In Fig. (1-a), the growth of emergent energy density against cosmic time is considered, showing an increasing behavior over the given time range.

Using Eq.(10) and Eq.(8), it is possible to obtain the derivative of Ω_D with respect to the cosmic time t. Since $\Omega_D'=\frac{d\Omega_D}{dx}=\frac{\dot{\Omega}_D}{H}$, we can write the mathematical expression for the fractional DE density which is used to study the evolution of dark energy, as:

$$\Omega_D'=-\frac{4\pi\alpha Ge^{-3\mu t}\left(\lambda+e^{\mu t}\right)^3\left(\frac{\beta\mu^2e^{\mu t}}{\lambda+e^{\mu t}}-\frac{\beta\mu^2e^{2\mu t}}{\left(\lambda+e^{\mu t}\right)^2}\right)}{3\beta^3\mu^3}, \tag{20}$$

where, we have used the fact that $H'=\frac{a'}{a}=1$. The primes indicate derivatives with respect to $x=\ln a$. By using the present values for the parameters, Fig.(1-b) shows that $\Omega_D'<0$.

Combining Eq.(13) and Eq.(14) together with the first derivative of ρ in Eq.(2), we can study the evolution of EoS parameters ω_D for both non-interacting and interacting models against cosmic time as shown in Figs.(1-c and 1-d). We observe that $\omega<-1$. This indicates phantom regime is started, showing that the universe grows without bound over time.

Now, we study the behavior of the deceleration parameter q that gives an idea about the rate at which the expansion of the universe is slowing down due to what is called self gravitation. It is given by:

$$q=-1-\frac{\dot{H}}{H^2}. \tag{21}$$

In Fig.(1-e), the deceleration parameter q is plotted as a function of cosmic time t . We notice that $q<0$ indicates accelerated expansion behavior of the universe which supports recent observations. To Study the stability of our model, we analyze the squared speed of DE fluid which is the ratio of dark energy pressure to the energy density of dark energy [23]. For GGDE assuming $\hbar$ -quantum corrected universe, the squared speed of dark fluid liquid takes the form [17, 18]:

$$V_D^2=\frac{\dot{P_D}}{\dot{\rho_D}}, \tag{22}$$

where,

$$\dot{P_D}=\dot{\omega_D}\rho_D+\dot{\rho_D}\omega_D. \tag{23}$$

Combining Eq.(22) with Eq.(23), yields:

$$V_D^2=\omega_D(t)-\frac{t\dot{\omega_D}(t)}{4\alpha\left(\omega_D(t)+1\right)}. \tag{24}$$

The sign of V_D^2 is very important to determine the stability of the evolution of our model [26]. A positive value indicates that the model is stable whereas instability of a given perturbation corresponds to the negative value of V_D^2. In Figs.(1-f and 1-g), the evolution of V_D^2 against the cosmic time is studied for both non-interacting and interacting cases, It can be observed that our model for both cases, shows a type of stability $V_D^2>0$ at low values of β while instability is observed for high value of β over the considered time range, since $V_D^2<0$. For completeness, we make some analyses for the geometrical dimensionless parameters called state finder parameters r and s. These parameters are given by [3]:

$$r=2q^2+q-\frac{\dot{q}}{H}, \tag{25}$$

$$s=\frac{r-1}{3\left(q-\frac{1}{2}\right)}. \tag{26}$$

We can easily find a single parametric relation between r and s in $s-r$ plane as shown in Fig.(1-h). The trajectories in $r-s$ plane lead to different cosmological models demonstrating different behavior over the considered phantom regime. Actually, these types of parameters are used to investigate the universe expansion scenarios.

3.2 Intermediate Scenario of the Scale Factor

In the intermediate scenario, the scale factor $a(t)$ is given by [27]:

$$a = e^{Bt^m}, \tag{27}$$

where, B and m are two positive constants which should satisfy the following conditions: $Bm > 0$, $B > 0$ and $0 < m < 1$ [27]. For the intermediate scenario, the expansion of universe is faster than Power-Law form, where the scale factor takes the form, $a(t) = t^n$; $n > 1$. Moreover, the expansion of the universe [25,26] is slower for standard de-sitter scenario where $m = 1$.

The Hubble parameter could be written as:

$$H = Bmt^{m-1}. \tag{28}$$

The energy density for the intermadate case ρ_{EGDE} could be written, using Eq.(2) as:

$$\rho_{IGDE} = m^2B^2\delta t^{2m-2} + \alpha mBt^{m-1} \tag{29}$$

The parameters values are $B = 2, \gamma_q = \frac{-41}{10}, m = 0.3, 0.5$ and 0.7 are used for intermediate case. Figure(2-a), shows the growth of intermediate energy density against cosmic time, showing a decreasing behavior over the given time range. To study the evolution of GDE, we derive an expression for Ω'_D as:

$$\Omega'_D = -\frac{4\pi\alpha G(m-1)t^{1-2m}}{3B^2m^2}. \tag{30}$$

Inserting the present values for the parameters, Fig(1-b) shows that $\Omega'_D > 0$, indicating that DE now is greater than the past.

Using Eq.(13) and Eq.(14) together with the first derivative of ρ in Eq.(2) respectively, the evolution of EoS parameters ω_D for both non-interacting and interacting model against cosmic time is considered in Figs.(2-c and 2-d). We observe that $-1 < \omega < -\frac{1}{3}$, indicating quintessence like behavior.

The intermediate deceleration parameter for non-interacting case using Eq.(21), is given by:

$$q = -1 - ((-1+m)t^{-m})/(Bm). \tag{31}$$

In Fig.(2-e), the deceleration parameter q is plotted as a function of cosmic time. We notice that q starts with a positive value showing a decreasing behavior with time staying at a negative level indicating a type of contraction followed by accelerated expansion of the universe.

In Figs.(2-f and 2-g), we observe a type of instability for both scenarios; since $V_D^2 < 0$.

Using Eqs. (25) and (26) together with Eqs.(28) and (31), the trajectories of $r-s$ plane are plotted in Fig.(2-h) showing a decreasing behavior over the considered time range. It is well known that in $s-r$ plane, a quintessence-like behavior of dark energy corresponds to $s > 0$ while a phantom-like model of dark energy corresponds to $s < 0$. The transition from phantom to quintessence or the opposite is represented by crossing the point $r = 1$ and $s = 0$ which denotes a ΛCDM behavior.

3.3 the Logamediate Scenario of the Scale Factor

We now consider the logamediate scenario of the scale factor, which is defined as [27]:

$$a = a_o e^{A\log^{\zeta}(t)} \tag{32}$$

where, A and ζ are two constant parameters which satisfying the conditions $A\zeta > 0$ and $\zeta > 1$. The Hubble parameter is given by:

$$H = \frac{A\zeta\log^{\zeta-1}(t)}{t}, \tag{33}$$

and the logamediated energy density is given by:

$$\rho_{LGDE} = \frac{A^2\delta\zeta^2\log^{2\zeta-2}(t)}{t^2} + \frac{\alpha A\zeta\log^{\zeta-1}(t)}{t}. \tag{34}$$

We assume the parameters values $a_o = 0.12, \gamma_q = \frac{-41}{10}, \lambda = 0.3, \mu = 0.3, \zeta = 3, 5$ and 7, for logamediate case. In Fig. (1-a), the growth of logamediate energy density against cosmic time is considered, showing an increasing behavior over the given time range. Following the same method as before, one finds the fractional energy density as:

$$\Omega'_D = -\frac{4\pi\alpha Gt^3\log^{3-3\zeta}(t)\left(\frac{A(\zeta-1)\zeta\log^{\zeta-2}(t)}{t^2} - \frac{A\zeta\log^{\zeta-1}(t)}{t^2}\right)}{3A^3\zeta^3} \tag{35}$$

By using the present values for the parameters, Fig. (1-b) shows that $\Omega'_D < 0$.

Using Eq.(13) and Eq.(14) together with the first derivative of ρ in Eq.(2), the evolution of EoS parameters ω_D for both non-interacting and interacting model against cosmic time is considered in Figs.(3-c and, 3-d), we notice that $\omega < -1$.

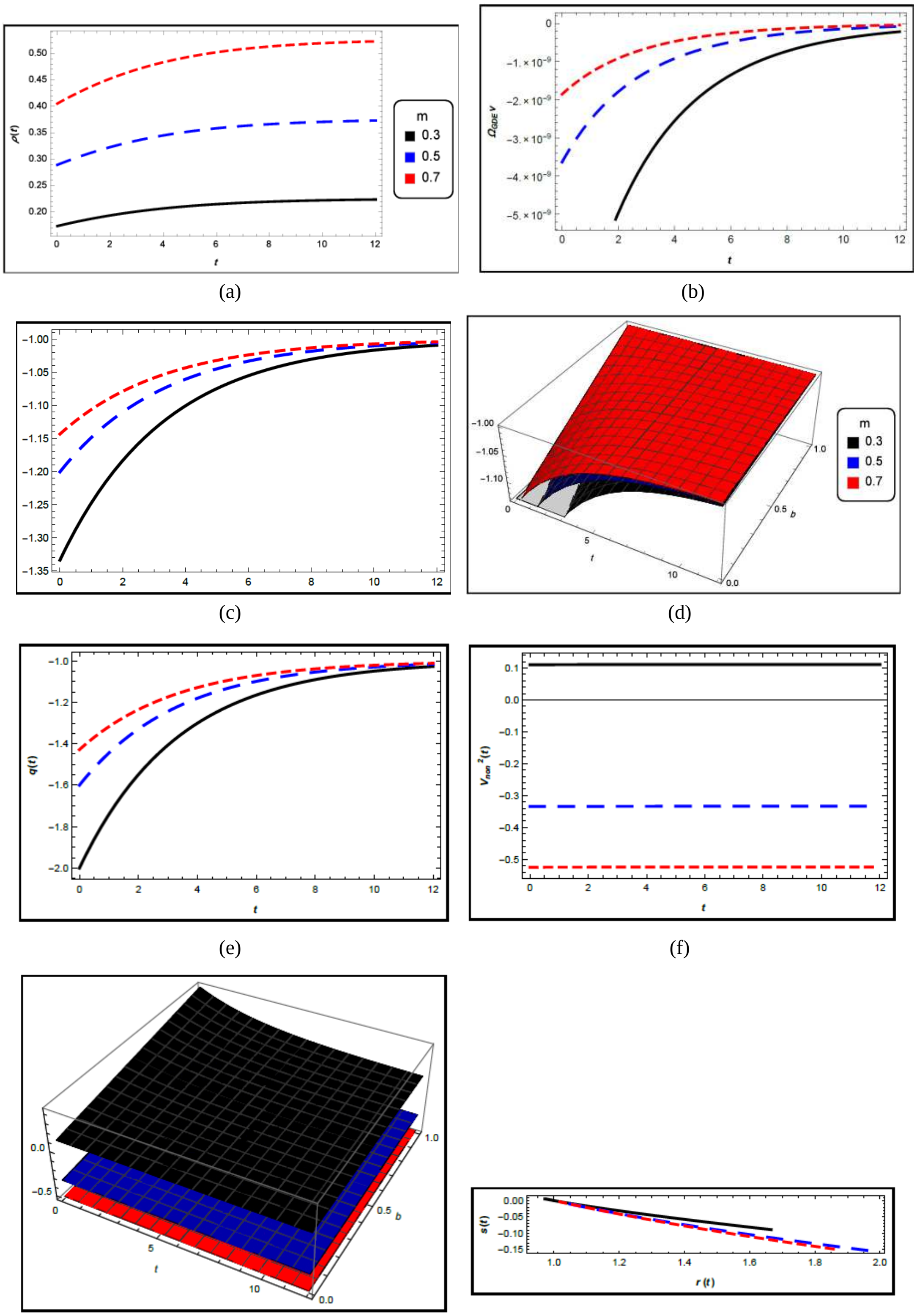
The deceleration parameter, using Eq.(21) for non-interacting case is given by:

$$q = -\frac{t^2\log^{2-2\zeta}(t)\left(\frac{A(\zeta-1)\zeta\log^{\zeta-2}(t)}{t^2} - \frac{A\zeta\log^{\zeta-1}(t)}{t^2}\right)}{A^2\zeta^2} - 1 \tag{36}$$

In Fig.(3-e), the deceleration parameter q is plotted as a function of cosmic time t. We notice that q stays at a negative level indicating an accelerated expansion of the universe. In Figs. (3-f and 3-g), the evolution of V_D^2 against the cosmic time is considered for both non-interacting and interacting cases. We notice that both cases show a type of stability over the considered time range, since $V_D^2 > 0$.

4 Perspective

The generalized ghost dark energy GGDE in framework of $\hbar$-quantum corrected universe is considered for



(a) (b)

(c) (d)

(e) (f)

(g) (h)

Fig. 1: Emergent Scenario of the cosmological Scale Factor

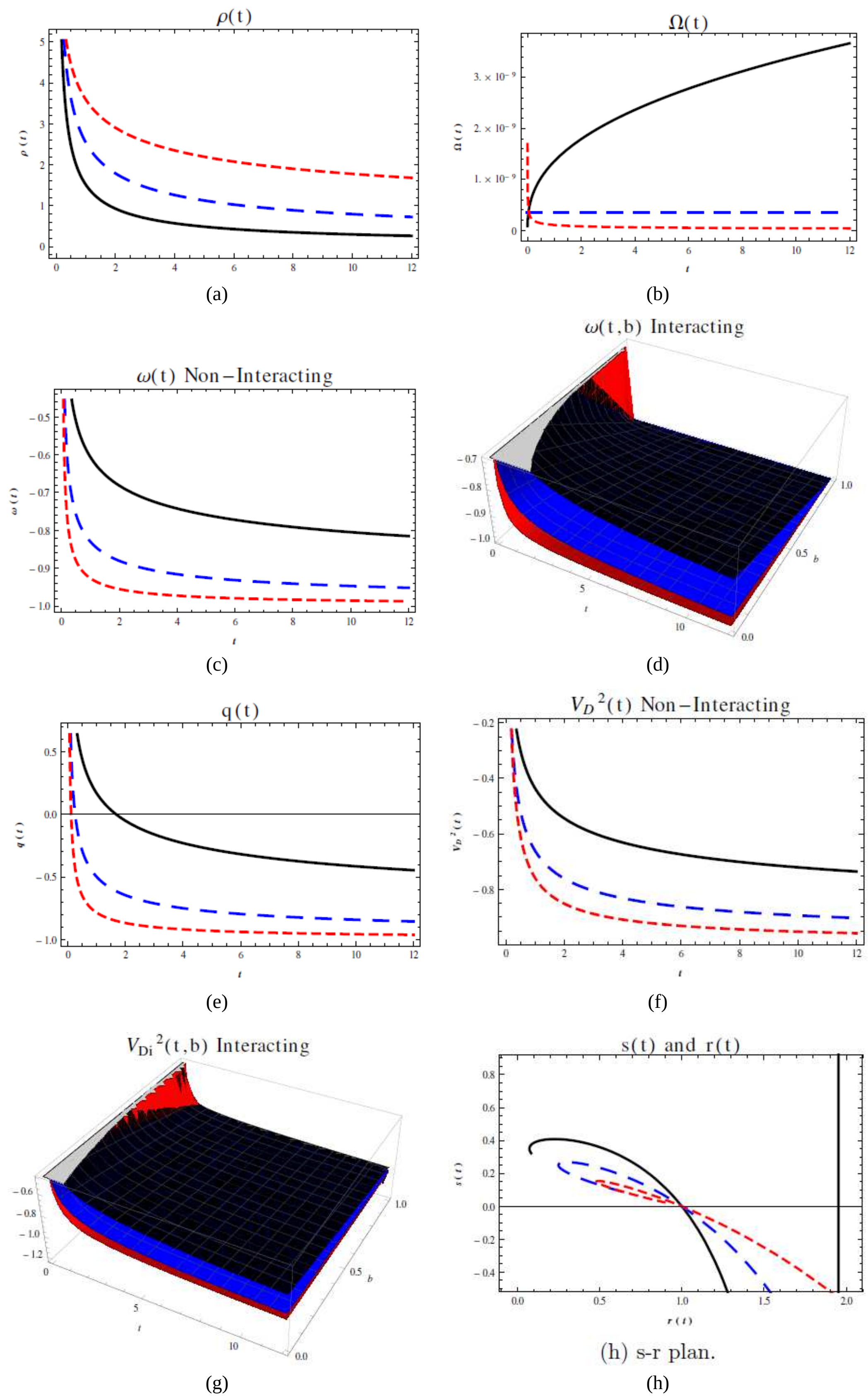


Fig. 2: Intermediate Scenario of the cosmological Scale Factor.

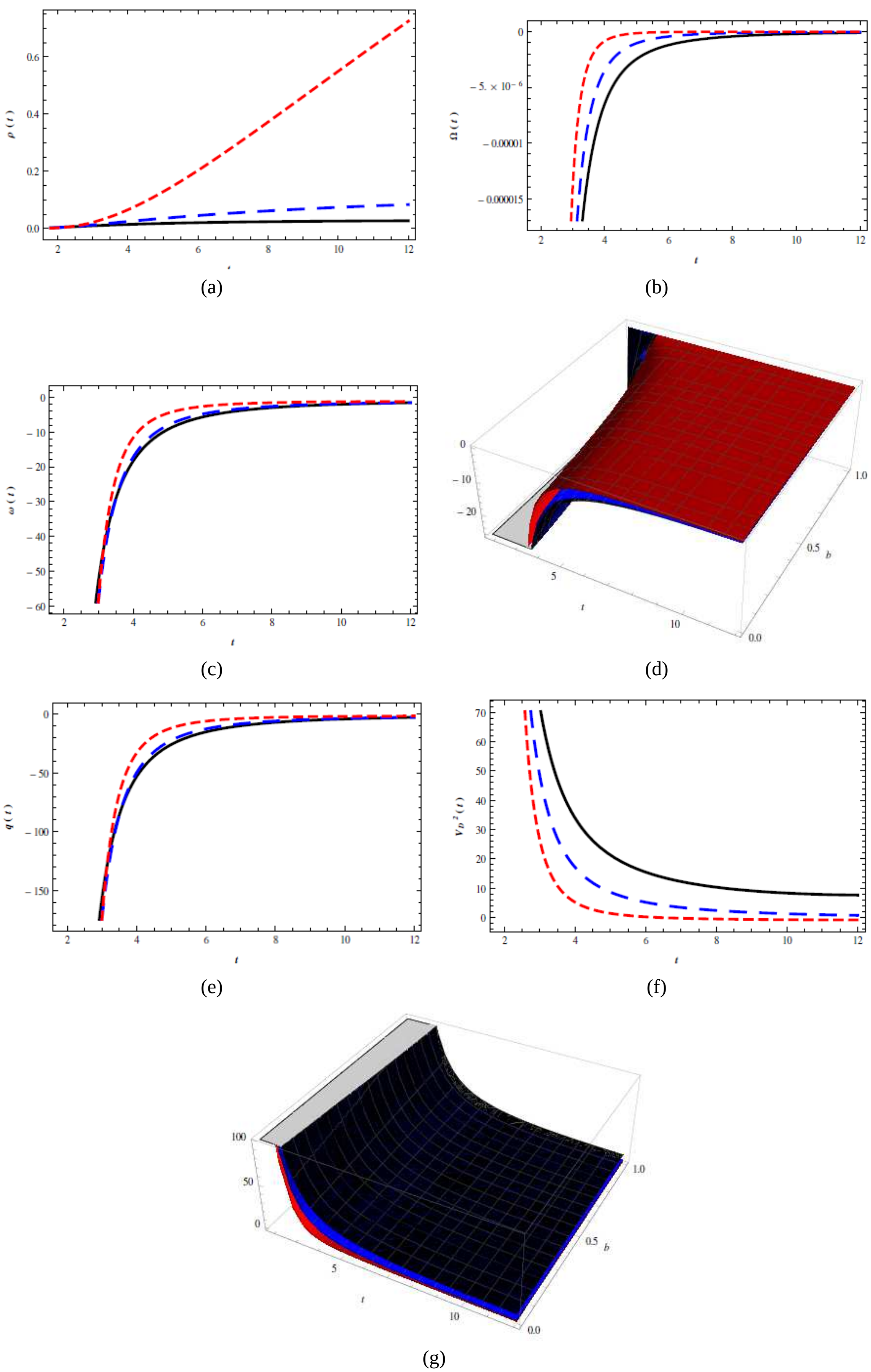


Fig. 3: Logamediate Scenario of the Scale Factor.

different choices of the cosmological scale factor namely emergent, intermediate and logimedate scenarios. The behavior of the reconstructed equation state parameter (EoS) for both interacting and non-interacting dark energy models is studied. We observe EoS stays generally in the negative level, for intermediate cases $\omega < \frac{-1}{3}$ shows a type of quintessence like behavior while for emergent and logimediate case a phantom behavior is noticed $\omega < -1$ implies a type of negative kinetic energy representing the necessary condition for universe expansion and forming what is known as big rip. On studying the evolution of the deceleration parameter q with cosmic time for the three scenarios, we notice that the deceleration parameter passes from decelerated to accelerated phases in some cases but generally we found that $q < 0$, leading to the fact that the universe is in accelerated expansion mode. Through the study of the behavior of the square of speed of DE fluid, it is noticed that $v_D^2 < 0$ leading to a type of instability for both emergent and intermediate cases while a type of stability is observed for logimidate case since $v_D^2 > 0$ over the considered time range.
Finally, an investigation for the behavior of state finder parameters s and r is assumed for emergent and intermediate cases. One notices that a decreasing behavior with the quintessence evolution of our model, showing that our model can verify the ΛCDM phase of the universe which implies the interaction between dust and ΛCDM phase of the universe. By considering the most recent negative γ_q [23], one finds that the considered cosmological parameters are assumed to be strongly dependent on time in a nonlinear manner. The assumed model shows some good consistency with the recent observed data [1]. Actually, the evidence for accelerating expansion of the universe which is more significant now, makes a progressive step to understand the nature of DE [1].

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