Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/080304

Approximate Bayes Estimators of The Inverted Kumarswamy Distribution Parameters Based on Progressive Type-II Censoring Scheme

A. Rashad[1], *M. Yusuf*[1,*] *and Sara Moheb*[2]

[1] Department of Mathematics, Faculty of Science, Helwan University, Ain Helwan, Cairo, Egypt.
[2] Department of Basic Science, Canadian International College, Cairo, Egypt.

Received: 15 Aug. 2018, Revised: 13 Jul. 2019, Accepted: 23 Sep. 2019
Published online: 1 Nov. 2019

Abstract: The purpose of this study is to estimate unknown parameters, reliability, the hazard rate, and the reversed hazard rate function of the inverted Kumaraswamy distribution based on progressively type-II censoring samples. Maximum likelihood estimators are obtained. The Bayesian estimation is studied under quadratic (squared error) and general entropy loss functions using Lindley's approximation.Non-informative prior distributions are used for the parameters. Based on a Monte Carlo simulation study, the statistical performances of Bayes estimators based on quadratic and general entropy loss functions are compared between various estimates.

Keywords: Inverted Kumaraswamy distribution, progressively type II censoring, General Entropy loss functions, Quadratic loss functions, Lindley's approximation.

1 Introduction

In 1980, Kumaraswamy proposed a continuous probability distribution for double-bounded random variable defined on the interval [0,1]. It is identified as an applicable alternative to beta distribution because they both have same shape properties such as unimodal, uniantimodal, increasing, decreasing, monotone or constant.
The inverted distributions are widely employed in economic, biological, engineering sciences, survey sampling, medical research, finance literature, environmental studies, survival, and reliability theory problems. The inverted Kumaraswamy distribution is considered one of the most important distributions that play an important role in the field of life testing and studying reliability measures where the failure time of component is observed to the nearest hours, days or months, hydrological data like daily rainfall and daily streamflow, atmospheric temperatures and growth models such as epidemiology. For more details [See [1], [4], [5], [6] and [7].

In statistical inference, there are two approaches to estimate the population parameters of several phenomena. The first one is the non-Bayesian approach (maximum likelihood estimation) where the population parameters under study are fixed but unknown and the second one is the Bayesian approach in which the population parameters under study are viewed to be a random variable with a prior probability distribution. This prior distribution supplies an additional information about unknown parameter θ that was available before the data were obtained, and there are two types of priors: informative and non-informative.
The Bayesian approach merge the information in the study sample with the available prior information about population parameters, and the result of the merger is the posterior distribution of the population parameters, see [9], [8] and [10].
In the Bayesian approach, in order to specify a single value as expressing our "best" estimators of the unknown parameter, a loss function must be defined. There are various types of loss functions such as the square error loss function (SE) which is a quadratic symmetric function suitable in the cases of highest estimate and lowest estimates, and the general entropy loss function which is asymmetric about the point of origin.

* Corresponding author e-mail: mohammed.yousof@yahoo.com

There are numerous phenomena in life-testing, in reliability experiments, and survival analysis in which the experimenter usually doesn't have complete control of failure times from the experiments in hand. For example, patients may enroll a clinic for treatment from a certain disease at random points of time and leave before completion of treatment or die from a cause different from the one under consideration. In an industrial experiment, units may break accidentally. In several situations, however, the removal of units prior to failure is prepared in advance that provides savings in terms of time and cost associated with testing. Hence, the recorded survival times involve randomly censored data. A progressive type-II censoring scheme can be described as follows: Suppose n units are placed on a life test and the experimenter decides beforehand the quantity m and the number of failures to be observed. Now at the time of the first failure, R_1 of the remaining $n-1$ surviving units are randomly removed from the experiment. At the time of the second failure, R_2 of the remaining $n-R_1-2$ units are randomly removed from the experiment. Finally, at the time of the m^{th} failure, all the remaining surviving units $R_m = n-m-R_1-\ldots-R_{m-1}$ are removed from the experiment. Therefore, a progressive type-II censoring scheme consists of m, and $R_1,\ldots,R_{m-1}$, such that $R_1+\ldots+R_m = n-m$ The failure times obtained from a progressive type-II censoring scheme is denoted by $x_1,\ldots,x_m$
Assuming X is a random variable which has the Inverted Kumaraswamy distribution $(IKum)$ with shape parameters, α and $\beta > 0$, denoted by $X \sim IKum(\alpha,\beta)$.
The probability density function (pdf) and cumulative distribution function (cdf) [See, [11]] are given, respectively, by

$$f(x;\alpha,\beta) = \alpha\beta(1+x)^{-(\alpha+1)}\left[1-(1+x)^{-\alpha}\right]^{\beta-1}, x>0, \alpha,\beta>0. \tag{1.1}$$

And

$$F(x;\alpha,\beta) = \left[1-(1+x)^{-\alpha}\right]^{\beta}, x>0, \alpha,\beta>0. \tag{1.2}$$

The Reliability Function is

$$R(x) = P(X>x) = 1-\left(1-(1+x)^{-\alpha}\right)^{\beta}, x>0. \tag{1.3}$$

The hazard (hrf) denoted by $h_1(x)$ and reversed hazard rate functions $(rhrf)$ denoted by $h_2(x)$ are given by

$$h_1(x) = \frac{f(x)}{R(x)} = \frac{\alpha\beta(1+x)^{-(\alpha+1)}\left[1-(1+x)^{-\alpha}\right]^{\beta-1}}{1-\left(1-(1+x)^{-\alpha}\right)^{\beta}}, x>0, \alpha,\beta>0. \tag{1.4}$$

$$h_2(x) = \frac{f(x)}{F(x)} = \alpha\beta(1+x)^{-(\alpha+1)}[1-(1+x)^{-\alpha}]^{-1}, x>0, \alpha,\beta>0. \tag{1.5}$$

This paper is organized as follows: maximum likelihood estimation for the parameters, reliability function, hazard rate function, and reversed hazard rate function of the $IKum$ distribution based on progressive type-II censored samples are obtained in section 2, Bayesian estimation of the parameters, reliability, hazard rate function, and reversed hazard rate function of the $IKum$ distribution based on progressive type-II censored samples under the square error loss function, and general entropy loss function are derived in section 3. A simulation study applying Monte Carlo method is presented in section 4 to illustrate results developed in this paper.

2 Maximum Likelihood Estimation

This section develops the maximum likelihood estimation (MLE) of the shape parameters, reliability function (rf), hrf, and rhrf based on progressively type-II censored samples. Based on the observed sample $x_1,\ldots,x_m$ from a progressive type-II censoring scheme $(R_1,\ldots,R_m)$, the likelihood function can be written as

$$L(\bar{x};\alpha,\beta) = c\prod_{i=1}^{m} f(x_i)\left[1-F(x_i)\right]^{R_i}, \tag{2.1}$$

where $c = n(n-1-R_1)\ldots(n-R_1-\ldots-R_{m-1}-m-1)$, $f(.)$ and $F(.)$, are given by (1.1) and (1.2) respectively. Then

$$L(\bar{x};\alpha,\beta) = c\alpha^m\beta^m\prod_{i=1}^{m}\left(1+x_{(i)}\right)^{-(\alpha+1)}\prod_{i=1}^{m}\left(1-\left(1+x_{(i)}\right)^{-\alpha}\right)^{\beta-1}\left[1-\left(1-(1+x_i)^{-\alpha}\right)^{\beta}\right]^{R_i}$$

The log-likelihood function can be written as

$$
\begin{aligned}
Log(L) = \ell = \operatorname{Log}[c] + m\operatorname{Log}[\alpha] + m\operatorname{Log}[\beta] - (\alpha+1)\sum_{i=1}^{m}\left(\operatorname{Log}[1+x_i]\right) \\
+(\beta-1)\sum_{i=1}^{m}\left(\operatorname{Log}\left[1-(1+x_i)^{-\alpha}\right]\right) + \sum_{i=1}^{m}\left(R_i\operatorname{Log}\left[1-\left(1-(1+x_i)^{-\alpha}\right)^{\beta}\right]\right)
\end{aligned}. \tag{2.2}
$$

The MLEs of the unknown parameters can be obtained by differentiating the log-likelihood function (2.2) with respect to the unknown parameters and equating to zero, we get

$$
\left.
\begin{aligned}
\frac{m}{\alpha} - \sum_{i=1}^{m}\operatorname{Log}[1+x_i] + (-1+\beta)\sum_{i=1}^{m}\frac{\operatorname{Log}[1+x_i](1+x_i)^{-\alpha}}{1-(1+x_i)^{-\alpha}} \\
+\sum_{i=1}^{m} - \frac{\beta\operatorname{Log}[1+x_i]R_i(1+x_i)^{-\alpha}\left(1-(1+x_i)^{-\alpha}\right)^{-1+\beta}}{1-\left(1-(1+x_i)^{-\alpha}\right)^{\beta}} = 0, \\
\frac{m}{\beta} + \sum_{i=1}^{m}\operatorname{Log}\left[1-(1+x_i)^{-\alpha}\right] + \sum_{i=1}^{m} - \frac{\operatorname{Log}\left[1-(1+x_i)^{-\alpha}\right]R_i\left(1-(1+x_i)^{-\alpha}\right)^{\beta}}{1-\left(1-(1+x_i)^{-\alpha}\right)^{\beta}} = 0
\end{aligned}
\right\}. \tag{2.3}
$$

The solution of the non-linear equations numerically (2.3) is $\hat{\alpha}, \hat{\beta}$.

The MLEs of the reliability function, the hazard rate function, and reversed hazard rate functions are given as

$$
\hat{R}(t) = 1 - \left[1-(1+t)^{-\hat{\alpha}}\right]^{\hat{\beta}}, t > 0 \tag{2.4}
$$

$$
\hat{h}_1(t) = \frac{\hat{\alpha}\hat{\beta}(1+t)^{-(\hat{\alpha}+1)}\left[1-(1+t)^{-\hat{\alpha}}\right]^{\hat{\beta}-1}}{1-\left[1-(1+t)^{-\hat{\alpha}}\right]^{\hat{\beta}}}, t > 0 \tag{2.5}
$$

$$
\hat{h}_2(t) = \hat{\alpha}\hat{\beta}(1+t)^{-(\hat{\alpha}+1)}\left[1-(1+t)^{-\hat{\alpha}}\right]^{-1}, t > 0. \tag{2.6}
$$

3 Bayesian Estimates for the Unkown Parameters α and β

In this section, Bayesian estimation of the parameters of the inverted Kumaraswamy distribution along with reliability function, hazard rate, and reversed hazard rate functions, using progressive type-II censoring samples, based on the square error loss function, and general entropy loss function, assuming non-informative prior, are obtained.
Assuming that α, β are independent random variables, and no information about α and β is available, considering a non-informative prior distribution for α in the form

$$
\pi(\alpha) \propto \frac{1}{\alpha}; 0 < \alpha < \infty
$$

and a non-informative prior distribution for β in the form

$$
\pi(\beta) \propto \frac{1}{\beta}; 0 < \beta < \infty.
$$

The joint non-informative prior for α and β is given by

$$
\pi(\alpha,\beta) \propto \frac{1}{\alpha\beta}; 0 < \alpha < \infty, 0 < \beta < \infty \tag{3.1}
$$

by using equations (2.1, 3.1) we get the joint posterior distribution for α and β as follows

$$
\begin{aligned}
\pi(\alpha,\beta|\bar{x}) &= \frac{\pi(\alpha,\beta)L(\bar{x};\alpha,\beta)}{\int_0^\infty\int_0^\infty \pi(\alpha,\beta)L(\bar{x};\alpha,\beta)d\alpha d\beta} \\
&= \frac{c\alpha^{m-1}\beta^{m-1}\prod_{i=1}^{m}\left(1+x_{(i)}\right)^{-(\alpha+1)}\prod_{i=1}^{m}\left(1-\left(1+x_{(i)}\right)^{-\alpha}\right)^{\beta-1}\left[1-\left(1-(1+x_i)^{-\alpha}\right)^{\beta}\right]^{R_i}}{\int_0^\infty\int_0^\infty c\alpha^{m-1}\beta^{m-1}\prod_{i=1}^{m}\left(1+x_{(i)}\right)^{-(\alpha+1)}\prod_{i=1}^{m}\left(1-\left(1+x_{(i)}\right)^{-\alpha}\right)^{\beta-1}\left[1-\left(1-(1+x_i)^{-\alpha}\right)^{\beta}\right]^{R_i}d\alpha d\beta}
\end{aligned} \tag{3.2}
$$

Integration in equation (3.2) cannot be obtained in a closed form, so we solve it numerically. In the following sections we derive Bayesian estimators for shape parameters, the reliability function, the hazard rate function and reversed hazard rate under two different loss functions.

3.1 Bayesian Estimators Under Square Error Loss Function

1. Bayesian estimator for shape parameter α

$$\hat{\alpha}_{sq} = E(\alpha) = \int_0^\infty\int_0^\infty \alpha \frac{c\alpha^{m-1}\beta^{m-1}\prod_{i=1}^{m}\left(1+x_{(i)}\right)^{-(\alpha+1)}\prod_{i=1}^{m}\left(1-\left(1+x_{(i)}\right)^{-\alpha}\right)^{\beta-1}\left[1-\left(1-(1+x_i)^{-\alpha}\right)^{\beta}\right]^{R_i}}{\int_0^\infty\int_0^\infty c\alpha^{m-1}\beta^{m-1}\prod_{i=1}^{m}\left(1+x_{(i)}\right)^{-(\alpha+1)}\prod_{i=1}^{m}\left(1-\left(1+x_{(i)}\right)^{-\alpha}\right)^{\beta-1}\left[1-\left(1-(1+x_i)^{-\alpha}\right)^{\beta}\right]^{R_i} d\alpha d\beta} \tag{3.3}$$

Provided that $E(\alpha)$ exists and is finite. This integration cannot be solved analytically, so we use Lindley's Bayes approximation, [2]. Let $u(\alpha,\beta)$ be a function of α and β, and we want to find Bayes estimator for it, based on $\pi(\alpha,\beta)$ as a prior distribution. The log-likelihood function for the inverted Kumaraswamy distribution based on progressive type-II censored samples is given by (2.2), Bayes estimate using Lindley approximation is obtained as follows:

$$E(u(\alpha,\beta)|\bar{x}) = \frac{\int_0^\infty\int_0^\infty u(\alpha,\beta)\pi(\alpha,\beta)L(\bar{x};\alpha,\beta)d\alpha d\beta}{\int_0^\infty\int_0^\infty \pi(\alpha,\beta)L(\bar{x};\alpha,\beta)d\alpha d\beta}$$

Let $Q(\alpha,\beta) = \log[\pi(\alpha,\beta)]$

$$E(u(\alpha,\beta)|\bar{x}) = \left(u(\alpha,\beta) + \frac{1}{2}\left[\sum_i\sum_j (u_{ij}+2u_iQ_j)\tau_{ij} + \sum_i\sum_j\sum_k\sum_w L_{ijk}u_w\tau_{ij}\tau_{kw}\right]\right)_{(\alpha,\beta)_{ML}} \tag{3.4}$$

$$\forall i,j,k,w = 1,2, Q_1 = \frac{\partial Q(\alpha,\beta)}{\partial\alpha}, Q_2 = \frac{\partial Q(\alpha,\beta)}{\partial\beta}, u_1 = \frac{\partial u(\alpha,\beta)}{\partial\alpha}, u_2 = \frac{\partial u(\alpha,\beta)}{\partial\beta},$$
$$u_{11} = \frac{\partial^2 u(\alpha,\beta)}{\partial\alpha^2}, u_{22} = \frac{\partial^2 u(\alpha,\beta)}{\partial\beta^2}, u_{12} = \frac{\partial^2 u(\alpha,\beta)}{\partial\alpha\partial\beta}, L_{11} = \frac{\partial^2\ell}{\partial\alpha^2}, L_{12} = \frac{\partial^2\ell}{\partial\alpha\partial\beta},$$
$$L_{22} = \frac{\partial^2\ell}{\partial\beta^2}, L_{111} = \frac{\partial^3\ell}{\partial\alpha^3}, L_{112} = \frac{\partial^3\ell}{\partial\alpha^2\partial\beta}, L_{122} = \frac{\partial^3\ell}{\partial\alpha\partial\beta^2}, L_{222} = \frac{\partial^3\ell}{\partial\beta^3}$$

Calculate the elements of matrix $\{-L_{ij}\}$

$$\sum = \begin{bmatrix} -\frac{\partial^2\ell}{\partial\alpha^2} & -\frac{\partial^2\ell}{\partial\alpha\partial\beta} \\ -\frac{\partial^2\ell}{\partial\alpha\partial\beta} & -\frac{\partial^2\ell}{\partial\beta^2} \end{bmatrix}^{-1} = \begin{pmatrix} \tau_{11} & \tau_{12} \\ \tau_{21} & \tau_{22} \end{pmatrix},$$

by using Mathematica, the values of ,i,j = 1,2, can be obtained where τ_{ij} are the elements of the information matrix Σ.

Substituting in equation (3.4), $Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = \alpha$, the Bayesian estimator for shape parameter α is given as

$$\hat{\alpha}_{sq} \approx \alpha - \frac{\tau_{11}}{\alpha} - \frac{\tau_{12}}{\beta} + \frac{1}{2}(L_{111}\tau_{11}{}^2 + 3L_{112}\tau_{11}\tau_{12} + L_{122}(\tau_{22}\tau_{11} + 2\tau_{12}{}^2) + L_{222}\tau_{12}\tau_{22})$$

2. Bayesian estimator for shape parameter β

Substituting in equation (3.4),

$$Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = \beta,$$

the Bayesian estimator for shape parameter β is given as

$$\hat{\beta}_{sq} \approx \beta - \frac{\tau_{12}}{\alpha} - \frac{\tau_{22}}{\beta} + \frac{1}{2}\left(L_{111}\tau_{11}\tau_{12} + L_{112}\left(\tau_{22}\tau_{11} + 2\tau_{12}{}^2\right) + 3L_{122}\tau_{12}\tau_{22} + L_{222}\tau_{22}{}^2\right)$$

3. Bayesian estimator for reliability function $R(t)$

Substituting in equation (3.4), $Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = R(t)$, the Bayesian estimator for reliability function $R(t)$ is given as

$$\hat{R}_{sq} \approx R(t) - \frac{1}{\alpha}(u_1\tau_{11} + u_2\tau_{12}) - \frac{1}{\beta}(u_1\tau_{12} + u_2\tau_{22}) + \frac{1}{2}(u_{11}\tau_{11} + 2u_{12}\tau_{12} + u_{22}\tau_{22})$$
$$+\frac{1}{2}\begin{bmatrix} L_{111}\left(u_1\tau_{11}{}^2 + u_2\tau_{11}\tau_{12}\right) + L_{112}\left(3u_1\tau_{11}\tau_{12} + u_2\left(\tau_{11}\tau_{22} + 2\tau_{12}{}^2\right)\right) + \\ L_{122}\left(u_1\left(2\tau_{12}{}^2 + \tau_{11}\tau_{12}\right) + 3u_2\tau_{12}\tau_{22}\right) + L_{222}\left(u_1\tau_{12}\tau_{22} + u_2\tau_{22}{}^2\right) \end{bmatrix}$$

4. Bayesian estimator for Hazard rate function $h_1(t)$

Substituting in equation (3.4), $Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = \mathrm{h_1(t)}$, the Bayesian estimator for Hazard rate function $h_1(t)$ is given as

$$\hat{h}_{1sq} \approx \mathrm{h}_1(t) - \frac{1}{\alpha}(u_1\tau_{11} + u_2\tau_{12}) - \frac{1}{\beta}(u_1\tau_{12} + u_2\tau_{22}) + \frac{1}{2}(u_{11}\tau_{11} + 2u_{12}\tau_{12} + u_{22}\tau_{22}) + \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau_{11}{}^2 + u_2\tau_{11}\tau_{12}) + L_{112}(3u_1\tau_{11}\tau_{12} + u_2(\tau_{11}\tau_{22} + 2\tau_{12}{}^2)) \\ +L_{122}(u_1(2\tau_{12}{}^2 + \tau_{11}\tau_{12}) + 3u_2\tau_{12}\tau_{22}) + L_{222}(u_1\tau_{12}\tau_{22} + u_2\tau_{22}{}^2) \end{bmatrix}$$

5. Bayesian estimator for Reversed Hazard rate function $h_2(t)$

Substituting in equation (3.4), $Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = \mathrm{h}_2(\mathrm{t})$, the Bayesian estimator for reversed hazard rate function $h_2(t)$ is given as

$$\hat{h}_{2sq} \approx \mathrm{h}_2(t) - \frac{1}{\alpha}(u_1\tau_{11} + u_2\tau_{12}) - \frac{1}{\beta}(u_1\tau_{12} + u_2\tau_{22}) + \frac{1}{2}(u_{11}\tau_{11} + 2u_{12}\tau_{12} + u_{22}\tau_{22}) + \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau_{11}{}^2 + u_2\tau_{11}\tau_{12}) + L_{112}(3u_1\tau_{11}\tau_{12} + u_2(\tau_{11}\tau_{22} + 2\tau_{12}{}^2)) \\ +L_{122}(u_1(2\tau_{12}{}^2 + \tau_{11}\tau_{12}) + 3u_2\tau_{12}\tau_{22}) + L_{222}(u_1\tau_{12}\tau_{22} + u_2\tau_{22}{}^2) \end{bmatrix}$$

3.2 Bayesian Estimator Under General Entropy Loss Function

1. Estimator for shape parameter α

$$\hat{\alpha}_{Gentropy} = \left[E\left(\alpha^{-q}\right)\right]^{-\frac{1}{q}}$$

Provided that $E\left(\alpha^{-q}\right)$ exists and is finite. Substituting in equation (3.4), $Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = \alpha^{-q}$, the Bayesian estimator for shape parameter α is given as

$$\hat{\alpha}_{Gentropy} \approx \begin{bmatrix} \alpha^{-q} + \frac{q\alpha^{-q-1}\tau_{11}}{\alpha} + \frac{q\alpha^{-q-1}\tau_{12}}{\beta} + \frac{q(q+1)\alpha^{-q-2}\tau_{11}}{2} \\ -\frac{q\alpha^{-q-1}}{2}(\ell_{111}\tau_{11}{}^2 + 3\ell_{112}\tau_{11}\tau_{12} + \ell_{122}(\tau_{11}\tau_{22} + 2\tau_{12}{}^2) + \ell_{222}\tau_{12}\tau_{22}) \end{bmatrix}^{\frac{-1}{q}}$$

2. Bayesian estimator for shape parameter β

Substituting in equation (3.4), $Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = \beta^{-q}$, the Bayesian estimator for shape parameter β is given as

$$\hat{\beta}_{Gentropy} \approx \begin{pmatrix} \beta^{-q} + \frac{q\beta^{-q-1}\tau_{12}}{\alpha} + \frac{q(q+1)\beta^{-q-2}\tau_{22}}{2} + \frac{q\beta^{-q-1}\tau_{22}}{\beta} \\ +\frac{1}{2}q\beta^{-q-1}(L_{111}\tau_{11}\tau_{12} + 3L_{112}(\tau_{22}\tau_{11} + 2\tau_{12}{}^2) + 3L_{122}\tau_{12}\tau_{22} + L_{222}\tau_{22}{}^2) \end{pmatrix}^{\frac{-1}{q}}$$

3. Bayesian estimator for reliability function $R(t)$

Substituting in equation (3.4), $Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = (R(t))^{-q}$, the Bayesian estimator for reliability function $R(t)$ is given as

$$\hat{R}_{Gentropy} \approx (R(t))^{-q} - \frac{1}{\alpha}(u_1\tau_{11} + u_2\tau_{12}) - \frac{1}{\beta}(u_1\tau_{12} + u_2\tau_{22}) + \frac{1}{2}(u_{11}\tau_{11} + 2u_{12}\tau_{12} + u_{22}\tau_{22}) + \frac{1}{2}\begin{bmatrix} L_{111}\left(u_1\tau_{11}{}^2 + u_2\tau_{11}\tau_{12}\right) \\ +L_{112}\left(3u_1\tau_{11}\tau_{12} + u_2\left(\tau_{11}\tau_{22} + 2\tau_{12}{}^2\right)\right) \\ +L_{122}\begin{pmatrix} u_1\left(2\tau_{12}{}^2 + \tau_{11}\tau_{12}\right) \\ +3u_2\tau_{12}\tau_{22} \end{pmatrix} \\ +L_{222}\left(u_1\tau_{12}\tau_{22} + u_2\tau_{22}{}^2\right) \end{bmatrix}$$

4. Bayesian estimator for Hazard rate function $h_1(t)$

Substituting in equation (3.4), $Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = (h_1(t))^{-q}$, the Bayesian estimator for hazard rate function $h_1(t)$ is given as

$$\hat{h}_{1Gentropy} \approx (h_1(t))^{-q} - \frac{1}{\alpha}(u_1\tau_{11} + u_2\tau_{12}) - \frac{1}{\beta}(u_1\tau_{12} + u_2\tau_{22}) + \frac{1}{2}(u_{11}\tau_{11} + 2u_{12}\tau_{12} + u_{22}\tau_{22}) + \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau_{11}{}^2 + u_2\tau_{11}\tau_{12}) + L_{112}(3u_1\tau_{11}\tau_{12} \\ +u_2(\tau_{11}\tau_{22} + 2\tau_{12}{}^2)) + L_{122}(u_1(2\tau_{12}{}^2 + \tau_{11}\tau_{12}) \\ +3u_2\tau_{12}\tau_{22}) + L_{222}(u_1\tau_{12}\tau_{22} + u_2\tau_{22}{}^2) \end{bmatrix}$$

5. Bayesian estimator for Reversed Hazard rate function $h_2(t)$

Substituting in equation (3.4), $Q_1 = \frac{-1}{\alpha}, Q_2 = \frac{-1}{\beta}, u = (h_2(t))^{-q}$, the Bayesian estimator for reversed hazard rate function $h_2(t)$ is given as

$$\hat{h}_{2Gentropy} \approx (h_2(t))^{-q} - \frac{1}{\alpha}(u_1\tau_{11} + u_2\tau_{12}) - \frac{1}{\beta}(u_1\tau_{12} + u_2\tau_{22}) + \frac{1}{2}(u_{11}\tau_{11} + 2u_{12}\tau_{12} + u_{22}\tau_{22}) + \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau_{11}{}^2 + u_2\tau_{11}\tau_{12}) + \\ L_{112}(3u_1\tau_{11}\tau_{12} + u_2(\tau_{11}\tau_{22} + 2\tau_{12}{}^2)) \\ +L_{122}(u_1(2\tau_{12}{}^2 + \tau_{11}\tau_{12}) + 3u_2\tau_{12}\tau_{22}) \\ +L_{222}(u_1\tau_{12}\tau_{22} + u_2\tau_{22}{}^2) \end{bmatrix}$$

It is worth noting that when the value $q = -1$, the general entropy loss function is the same as the squared error loss function.

4 Simulation Studies

To prove the importance of the results obtained in the previous sections, simulation studies are performed. For this study, by using the Monte Carlo method, with fixed sample size n (the total items put in a life test), with constant censoring scheme, where $R_1 = R_2 = ... = R_m$, where m is the sample size of progressively censored from the sample of size n.

The following algorithm is used to generate sample based on progressive type-II censoring scheme, based on any continuous df F, see [3].

1. Generate m independent uniform $(0,1)$ observations $W_1, ..., W_m$.
2. Set $V_i = W_i^{\frac{1}{\gamma_i}}, \gamma_i = \left(i + \sum_{j=m-i+1}^{m} R_j\right)$ $, i = 1, 2, ..., m.$
3. Set $U_i = 1 - V_m V_{m-1} \ldots V_{m-i+1}, i = 1, 2, \ldots, m.$
4. Set $X_i = F^{-1}(U_i)$, ,then X_i for $i = 1, 2, \ldots, m$ is the progressive type-II censoring scheme based on the df F.
5. We have repeated steps $1, 2, 3 and 4$ (10000) times, for different values of n and m.

Estimation average $= \frac{\sum_{i=1}^{10000} \widehat{\theta}_i}{10000}$, mean square error $= \frac{\sum_{i=1}^{10000} (\hat{\theta}_i - \theta)^2}{10000}$, where, θ is the parameter and $\widehat{\theta}$ is the estimator.
Extensive computations are performed using Mathematica 11.
Since the non-linear equations (2.3) are not solvable analytically, numerical methods can be used, as Newton Raphson method with initial values closed to real values of the parameters.

Throughout this section we use the following abbreviations:

1. MLE : means that the estimate by using the maximum likelihood method,
2. Est.: estimator,
3. E_{sq} : means that the estimate under squared error loss function,
4. $E_{Ge,q=0.3}$: means that the estimate under general entropy loss function at $q = 0.3$.

From the simulation studies we noted that:

1. In general, the Bayesian estimators have mean square error less than that of the MLE.
2. In general, the Bayesian estimators under general entropy loss function is better than the Bayesian estimators under squared error loss function
3. Increasing the sample size leads to decrease mean square error and increase the accuracy of estimators.
4. For the parameter α, the estimate under general entropy loss function is the best especially at the value $q = 1$, and it followed by the squared error loss function.
5. Furthermore, by decreasing the value of the parameter β, the accuracy of estimator α increases and mean square error decreases.
6. For the parameter β, the estimate under general entropy loss function is the best especially at the value $q = 2$, and it is followed by the squared error loss function.
7. Furthermore, by increasing the value of the parameter α, the accuracy of estimator β increases and mean square error decreases.
8. The estimate of the reliability function $R(t)$under squared error loss function is the best, and it is followed by the general entropy loss function.
9. The estimate of the hazard rate function $h_1(t)$ under general entropy loss function is the best especially at the value $q = 1$, and it is followed by the squared error loss function.
10. The estimate of the reversed hazard rate function $h_2(t)$ under general entropy loss function is the best.
11. By decreasing the time t, the accuracy of estimate $R(t)$, $h_1(t)$ and $h_2(t)$ increases and mean square error decreases

Table 1. The average, mean square error, $(\alpha = 0.8, \beta = 1.5, t = 2)$.

n	m	Est.	Real	MLE	E_{sq}	$E_{GE,q=0.3}$	$E_{GE,q=0.7}$	$E_{GE,q=1}$	$E_{GE,q=2}$	$E_{GE,q=5}$
200	100	$\hat{\alpha}$	0.8	0.819 (0.0118)	0.8147 (0.0009)	0.8046 (0.0002)	0.8035 (0.0001)	0.7992 (1.5×10^{-6})	0.7957 (0.0002)	0.7795 (0.0004)
		$\hat{\beta}$	1.5	1.5371 (0.0381)	1.5324 (0.0014)	1.5181 (0.0003)	1.5155 (0.0002)	1.507 (0.0004)	1.5026 (0.0001)	1.473 (0.0007)
		$\hat{R}(t)$	0.5528	0.5505 (0.0015)	0.5520 (7.3×10^{-7})	0.5512 (2.8×10^{-6})	0.5504 (5.9×10^{-6})	0.5501 (7.6×10^{-6})	0.5492 (0.0001)	0.5461 (0.0004)
		$\hat{h_1}(t)$	0.2297	0.2343 (0.0008)	0.2331 (0.0001)	0.2306 (7.6×10^{-7})	0.2304 (4.1×10^{-7})	0.2295 (8.1×10^{-8})	0.2285 (1.5×10^{-6})	0.2246 (0.0002)
		$\hat{h_2}(t)$	0.2840	0.2861 (0.0001)	0.2866 (6.8×10^{-6})	0.2852 (1.4×10^{-6})	0.285 (8.7×10^{-7})	0.2841 (2.2×10^{-8})	0.2839 (2.9×10^{-8})	(0.2812) (8.1×10^{-6})
100	50	$\hat{\alpha}$	0.8	0.8360 (0.0261)	0.8263 (0.0006)	0.812 (0.0001)	0.8047 (0.0002)	0.8045 (0.0002)	0.7902 (0.0001)	0.7651 (0.0012)
		$\hat{\beta}$	1.5	1.5754 (0.0897)	1.5666 (0.0044)	1.5376 (0.0014)	1.5294 (0.0086)	1.5258 (0.0007)	1.5011 (0.0003)	1.455 (0.0021)
		$\hat{R}(t)$	0.5528	0.5491 (0.0023)	0.5523 (3.5×10^{-7})	0.5484 (0.0002)	0.5489 (0.0001)	0.5472 (0.0003)	0.5458 (0.0005)	0.5408 (0.0001)
		$\hat{h_1}(t)$	0.2297	0.2382 (0.0017)	0.2354 (0.0003)	0.2323 (6.6×10^{-6})	0.2303 (3.7×10^{-7})	0.2304 (5.1×10^{-7})	0.2268 (8.5×10^{-6})	0.2202 (0.0009)
		$\hat{h_2}(t)$	0.2840	0.2885 (0.0001)	0.2888 (0.0002)	0.2862 (4.7×10^{-6})	0.286 (4.1×10^{-6})	0.2854 (2×10^{-6})	0.2835 (4.4×10^{-7})	0.2792 (0.0002)

Table 2. The average, mean square error, $(\alpha = 0.8, \beta = 1, t = 2)$.

n	m	Est.	Real	MLE	E_{sq}	$E_{GE,q=0.3}$	$E_{GE,q=0.7}$	$E_{GE,q=1}$	$E_{GE,q=2}$	$E_{GE,q=5}$
200	100	$\hat{\alpha}$	0.8	0.8225 (0.0161)	0.817 (0.0002)	0.8079 (0.0006)	0.802 (4.3×10^{-6})	0.8007 (8.8×10^{-7})	0.7941 (0.0003)	0.7751 (0.0006)
		$\hat{\beta}$	1	1.0208 (0.0143)	1.0168 (0.0002)	1.0109 (0.0001)	1.006 (0.0003)	1.007 (0.0005)	0.9993 (1.1×10^{-6})	0.9859 (0.0002)
		$\hat{R}(t)$	0.4152	0.4119 (0.0013)	0.4132 (4.2×10^{-6})	0.4106 (0.0002)	0.4104 (0.0002)	0.4103 (0.0002)	0.4074 (0.0006)	0.4036 (0.0013)
		$\hat{h_1}(t)$	0.2666	0.2729 (0.0013)	0.2714 (0.0002)	0.2692 (6.4×10^{-6})	0.2677 (1.1×10^{-6})	0.2672 (3.3×10^{-7})	0.2658 (7.2×10^{-7})	0.2605 (0.0003)
		$\hat{h_2}(t)$	0.1893	0.1897 (0.0001)	0.1897 (1.1×10^{-7})	0.1891 (4.7×10^{-8})	0.1888 (3.08×10^{-7})	0.1891 (7.6×10^{-8})	0.188 (1.8×10^{-6})	0.187 (5.4×10^{-6})
100	50	$\hat{\alpha}$	0.8	0.8472 (0.0362)	0.836 (0.0013)	0.8175 (0.0003)	0.8065 (0.0004)	0.8062 (0.0004)	0.791 (0.0008)	0.7621 (0.0014)
		$\hat{\beta}$	1	1.0434 (0.0315)	1.0353 (0.0012)	1.0232 (0.0005)	1.0137 (0.0018)	1.0146 (0.0002)	1.003 (0.0001)	0.9764 (0.0005)
		$\hat{R}(t)$	0.4152	0.4084 (0.0029)	0.4107 (0.0002)	0.4057 (0.0009)	0.4050 (0.0001)	0.4034 (0.0004)	0.4006 (0.0001)	0.3926 (0.0005)
		$\hat{h_1}(t)$	0.2666	0.2799 (0.003)	0.2767 (0.0001)	0.2721 (0.0003)	0.2693 (7.21×10^{-6})	0.2692 (6.83×10^{-6})	0.2651 (2.63×10^{-6})	0.2566 (0.0001)
		$\hat{h_2}(t)$	0.1893	0.1901 (0.0001)	0.1900 (4.3×10^{-7})	0.1889 (1.6×10^{-7})	0.1882 (1.24×10^{-6})	0.1882 (1.23×10^{-6})	0.1874 (3.7×10^{-6})	0.1849 (0.0001)

Table 3. The average, mean square error, $(\alpha = 1, \beta = 1.5, t = 2)$.

n	m	Est.	Real	MLE	E_{sq}	$E_{GE,q=0.3}$	$E_{GE,q=0.7}$	$E_{GE,q=1}$	$E_{GE,q=2}$	$E_{GE,q=5}$
200	100	$\hat{\alpha}$	1	1.0251 (0.0188)	1.0189 (0.0003)	1.0084 (0.0006)	1.0047 (0.0002)	1.0024 (6.7×10^{-6})	0.9935 (0.0004)	0.9748 (0.0006)
		$\hat{\beta}$	1.5	1.5389 (0.0391)	1.534 (0.0116)	1.5194 (0.0037)	1.5149 (0.0022)	1.5123 (0.0014)	1.5017 (6.8×10^{-6})	1.4721 (0.0008)
		$\hat{R}(t)$	0.4557	0.4524 (0.0013)	0.4541 (2.5×10^{-6})	0.452 (0.0001)	0.4516 (0.0001)	0.4513 (0.0001)	0.4504 (0.0002)	0.4455 (0.0001)
		$\hat{h}_1(t)$	0.2986	0.3055 (0.0014)	0.3037 (0.0002)	0.3009 (5.2×10^{-6})	0.2999 (1.6×10^{-6})	0.2992 (4.2×10^{-7})	0.2968 (3.6×10^{-6})	0.2918 (0.0004)
		$\hat{h}_2(t)$	0.25	0.2509 (0.0001)	0.2512 (1.4×10^{-6})	0.2502 (4.7×10^{-8})	0.25 (6.7×10^{-9})	0.2498 (2.3×10^{-8})	0.2494 (3.6×10^{-7})	0.247 (8.9×10^{-6})
100	50	$\hat{\alpha}$	1	1.051 (0.0423)	1.0388 (0.0015)	1.012 (0.0001)	1.01 (0.0001)	1.0061 (0.0004)	0.9904 (0.0001)	0.9596 (0.0016)
		$\hat{\beta}$	1.5	1.5779 (0.0896)	1.5691 (0.0047)	1.5345 (0.0011)	1.293 (0.0008)	1.525 (0.0006)	1.5037 (0.0004)	1.4568 (0.0019)
		$\hat{R}(t)$	0.4557	0.4486 (0.0028)	0.452 (0.0001)	0.4495 (0.0003)	0.447 (0.0007)	0.4465 (0.0008)	0.4442 (0.0013)	0.4367 (0.0003)
		$\hat{h}_1(t)$	0.2986	0.3129 (0.0034)	0.3091 (0.0001)	0.3018 (0.0001)	0.3015 (8.3×10^{-6})	0.3002 (2.7×10^{-6})	0.2958 (8.7×10^{-6})	0.2869 (0.0003)
		$\hat{h}_2(t)$	0.25	0.2514 (0.0001)	0.2521 (4.5×10^{-6})	0.2504 (2.3×10^{-7})	0.2497 (1.2×10^{-7})	0.2496 (1.7×10^{-7})	0.2485 (2.3×10^{-6})	0.2449 (0.0002)

Table 4. The average, mean square error, $(\alpha = 0.8, \beta = 1.5, t = 0.6)$.

n	m	Est.	Real	MLE	E_{sq}	$E_{GE,q=0.3}$	$E_{GE,q=0.7}$	$E_{GE,q=1}$	$E_{GE,q=2}$	$E_{GE,q=5}$
200	100	$\hat{\alpha}$	0.8	0.8205 (0.0117)	0.8154 (0.0002)	0.8068 (0.0004)	0.804 (0.0001)	0.8021 (4.9×10^{-6})	0.7958 (0.0001)	0.7787 (0.0004)
		$\hat{\beta}$	1.5	1.542 (0.039)	1.5346 (0.0019)	1.5224 (0.0004)	1.5153 (0.0002)	1.5121 (0.0001)	1.5029 (0.0001)	1.4713 (0.0008)
		$\hat{R}(t)$	0.8245	0.8249 (0.0005)	0.8253 (5.8×10^{-7})	0.8254 (7×10^{-7})	0.8248 (5.7×10^{-8})	0.8248 (3×10^{-8})	0.8248 (3.2×10^{-8})	0.8234 (1.3×10^{-6})
		$\hat{h}_1(t)$	0.3496	0.3524 (0.0013)	0.3508 (1.3×10^{-6})	0.3477 (3.5×10^{-6})	0.3476 (4.1×10^{-6})	0.347 (6.9×10^{-6})	0.3448 (0.0002)	0.3401 (0.0008)
		$\hat{h}_2(t)$	1.6431	1.6735 (0.0001)	1.6707 (0.0007)	1.6621 (0.0003)	1.6559 (0.0001)	1.6535 (0.0001)	1.6468 (0.0001)	1.6206 (0.0005)
100	50	$\hat{\alpha}$	0.8	0.8248 (0.0265)	0.8299 (0.0008)	0.8142 (0.0002)	0.8086 (0.0007)	0.8027 (0.0001)	0.7937 (0.0005)	0.7686 (0.0010)
		$\hat{\beta}$	1.5	1.5765 (0.0869)	1.5677 (0.0045)	1.5442 (0.0019)	1.5322 (0.0010)	1.5234 (0.0055)	1.5052 (0.0005)	1.4563 (0.0020)
		$\hat{R}(t)$	0.8245	0.8249 (0.0010)	0.8258 (1.5×10^{-6})	0.8260 (1.8×10^{-6})	0.8253 (5.2×10^{-7})	0.8253 (4.7×10^{-7})	0.8245 (1.2×10^{-8})	0.8223 (5.2×10^{-6})
		$\hat{h}_1(t)$	0.3496	0.3555 (0.0031)	0.3520 (5.8×10^{-6})	0.3463 (0.0001)	0.3452 (0.0001)	0.3435 (0.0003)	0.3408 (0.0007)	0.3328 (0.0002)
		$\hat{h}_2(t)$	1.6431	1.7014 (0.0001)	1.6962 (0.0028)	1.6800 (0.0013)	1.6703 (0.0073)	1.6637 (0.0042)	1.6492 (0.0005)	1.6078 (0.0013)

Table 5. The average, mean square error, $(\alpha = 0.8, \beta = 1, t = 0.6)$.

n	m	Est.	Real	MLE	E_{sq}	$E_{GE,q=0.3}$	$E_{GE,q=0.7}$	$E_{GE,q=1}$	$E_{GE,q=2}$	$E_{GE,q=5}$
200	100	$\hat{\alpha}$	0.8	0.8214 (0.016)	0.8159 (0.0002)	0.8043 (0.0001)	0.8034 (0.0001)	0.8017 (3.3×10^{-6})	0.7925 (0.0005)	0.7746 (0.0006)
		$\hat{\beta}$	1	1.02 (0.0139)	1.0166 (0.0002)	1.0084 (0.0007)	1.0081 (0.0006)	1.0074 (0.0005)	0.9998 (6×10^{-7})	0.9855 (0.0002)
		$\hat{R}(t)$	0.6866	0.6861 (0.0008)	0.6865 (2.5×10^{-8})	0.6857 (8.4×10^{-7})	0.6854 (1.3×10^{-6})	0.6854 (1.4×10^{-6})	0.6846 (3.9×10^{-6})	06831 (0.0001)
		$\hat{h_1}(t)$	0.5	0.5067 (0.0031)	0.5046 (0.0002)	0.501 (9.3×10^{-7})	0.5007 (4.5×10^{-7})	0.5 (1.1×10^{-8})	0.4971 (8.4×10^{-6})	04902 (0.0009)
		$\hat{h_2}(t)$	1.0954	1.1096 (0.0001)	1.1068 (0.0001)	1.1008 (0.0002)	1.1006 (0.0002)	1.1003 (0.0002)	1.0945 (1.2×10^{-6})	1.0838 (0.0013)
100	50	$\hat{\alpha}$	0.8	0.8466 (0.0358)	0.8354 (0.0012)	0.8124 (0.0001)	0.8096 (0.0009)	0.8064 (0.0004)	0.787 (0.0001)	0.7606 (0.0015)
		$\hat{\beta}$	1	1.0445 (0.0320)	1.0364 (0.0013)	1.0197 (0.0001)	1.0165 (0.0002)	1.0157 (0.0002)	0.9992 (4.9×10^{-6})	0.9738 (0.0007)
		$\hat{R}(t)$	0.6866	0.6854 (0.0016)	0.6861 (2.2×10^{-7})	0.6846 (3.9×10^{-6})	0.6836 (9×10^{-6})	0.6838 (7.7×10^{-6})	0.6825 (0.0001)	0.679 (0.0005)
		$\hat{h_1}(t)$	0.5	0.5149 (0.0067)	0.5105 (0.0001)	0.5032 (0.0001)	0.5027 (7.3×10^{-6})	0.50106 (1.1×10^{-6})	0.4945 (0.0003)	0.4831 (0.0028)
		$\hat{h_2}(t)$	1.0954	1.1254 (0.0001)	1.1202 (0.0006)	1.1082 (0.0001)	1.1056 (0.0001)	1.1057 (0.0001)	1.0931 (7.5×10^{-6})	1.0729 (0.0005)

Table 6. The average, mean square error, $(\alpha = 1, \beta = 1.5, t = 0.6)$.

n	m	Est.	Real	MLE	E_{sq}	$E_{GE,q=0.3}$	$E_{GE,q=0.7}$	$E_{GE,q=1}$	$E_{GE,q=2}$	$E_{GE,q=5}$
200	100	$\hat{\alpha}$	1	1.0241 (0.0189)	1.0179 (0.0003)	1.0069 (0.0004)	1.0052 (0.0002)	1.0014 (3×10^{-6})	0.9941 (0.0003)	0.9744 (0.0006)
		$\hat{\beta}$	1.5	1.5379 (0.0392)	1.5332 (0.0010)	1.5184 (0.0003)	1.5166 (0.0002)	1.5112 (0.0001)	1.5004 (4.3×10^{-6})	1.4744 (0.0006)
		$\hat{R}(t)$	0.7704	0.7704 (0.0006)	0.7710 (3.8×10^{-7})	0.7705 (8.6×10^{-9})	0.7705 (1.1×10^{-8})	0.7703 (1.1×10^{-8})	0.7697 (4.5×10^{-7})	0.7689 (2.1×10^{-6})
		$\hat{h_1}(t)$	0.4658	0.4709 (0.0025)	0.4685 (7.5×10^{-6})	0.4653 (2.5×10^{-7})	0.4645 (1.7×10^{-6})	0.4635 (5.2×10^{-6})	04614 (0.0001)	0.4544 (0.0001)
		$\hat{h_2}(t)$	1.5625	1.5875 (0.0001)	1.5853 (0.0005)	1.5755 (0.0001)	1.5747 (0.0001)	1.5709 (0.0006)	1.5634 (2.3×10^{-6})	1.5456 (0.0002)
100	50	$\hat{\alpha}$	1	1.0494 (0.042)	1.0402 (0.0016)	1.0198 (0.0003)	1.012 (0.0001)	1.0042 (0.0002)	0.9880 (0.0001)	0.9608 (0.0015)
		$\hat{\beta}$	1.5	1.5761 (0.0905)	1.5745 (0.0055)	1.5451 (0.0020)	1.5345 (0.0011)	1.5209 (0.0004)	1.5001 (0.0003)	1.4589 (0.0018)
		$\hat{R}(t)$	0.7703	0.7702 (0.0013)	0.7719 (2.3×10^{-6})	0.7708 (1.6×10^{-7})	0.7705 (2.5×10^{-8})	0.7696 (6.1×10^{-7})	0.7693 (1.2×10^{-6})	0.7672 (0.0001)
		$\hat{h_1}(t)$	0.4658	0.4771 (0.0053)	0.472 (0.0003)	0.4659 (1.1×10^{-7})	0.4635 (5×10^{-6})	0.4622 (0.0001)	0.4565 (0.0008)	0.4461 (0.0003)
		$\hat{h_2}(t)$	1.5624	1.6171 (0.0001)	1.6138 (0.0026)	1.5946 (0.0010)	1.5877 (0.0006)	1.5774 (0.0002)	1.564 (0.0001)	1.5338 (0.0008)

5 Conclusions

In this study, Bayesian and non-bayesian maximum likelihood estimators of the two-shape parameters, reliability, hazard rate, and reversed hazard rate functions for the inverted Kumaraswamy distribution based on progressively type-II censoring samples under the non-informative prior distribution by using Lindley's approximation are accomplished. By using a Mathematica program, we have generated the samples to compute the estimated parameters. based on a Monte Carlo simulation study, it has been seen that the Bayes estimator performs better than the MLE's and the Bayes estimator under general entropy loss function has the smallest MSE's as compared with Bayes estimator under squared error loss function. The accuracy of estimators increases and mean square error decreases when increasing the sample sizes.The simulation also stresses the importance of general Entropy loss functions which is applicable in the case studied.

Acknowledgement

The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

References

[1] Kumaraswamy, P. (1980). A Generalized Probability Density Function for Double-Bounded Random Processes. Journal of Hydrology, 46, 79-88.
[2] Lindley, D. V. (1980). Approximate Bayesian Methods, Trabajos deEstadistica Y de Investigacion Operativa,31:1, 223-245.
[3] Balakrishnan, N. and Aggarwala, R. (2000). Progressive Censoring: Theory, Methods and Applications, Birkhauser, Boston.
[4] Jones, M.C. (2009) Kumaraswamy's Distribution: A Beta-Type Distribution withSome Tractability Advantages. Statistical Methodology , 6, 70-81.
[5] Golizadeh, A., Sherazi, M.A. and Moslamanzadeh, S. (2011). Classical and Bayesian estimation on Kumaraswamy distribution using grouped and ungrouped data under difference of loss functions. Journal of Applied Sciences, 11(12), 2154-2162.
[6] Sindhu, T.N., Feroze, N. and Aslam, M. (2013). Bayesian analysis of the Kumaraswamy distribution under failure censoring sampling scheme. International Journal of Advanced Science and Technology, 51, 39-58.
[7] Sharaf, E.L., Deen, M. M. , AL-Dayian, G. R. and EL-Helbawy, A.A. (2014) Statistical Inference for Kumaraswamy Distribution Based on Generalized Order Statistics with Applications. British Journal of Mathematics of Computer Science , 4, 1710- 1743.
[8] Rashad, A., Mahmoud, M. and Yusuf, M. (2016). Bayes estimation of the logistic distribution parameters based on progressive sampling. Applied Mathematics & Information Sciences. 10 (6), pp. 2293-2301
[9] Mahmoud, M., Rashad, A. and Yusuf, M. (2016). Approximate Bayes estimators of the logistic distribution parameters based on progressive type-II censoring scheme. Pakistan Journal of Statistics and Operation Research. 12 (3), pp. 519-531.
[10] Rashad, A., Mahmoud, M. and Yusuf, M. (2017). Bayes estimation of the scale parameter of the logistic distribution based on progressive first-failure censored scheme. Journal of Statistics Applications & Probability. 6 (1), pp. 49-63.
[11] A. M. Abd AL-Fattah, A.A. EL-Helbawy and G.R. AL-Dayian (2017), Inverted Kumaraswamy Distribution: Properties And Estimation. Pakistan Journal of Statistics and Operation Research. 33(1), 37-61