Applied Mathematics & Information Sciences
An International Journal

Resource Constrained Time-cost Trade-off Problem and its Genetic Algorithm Solution

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Received: 8 Oct. 2012; Revised 12 Dec. 2012; Accepted 25 Dec. 2012
Published online: 1 Mar. 2013

Abstract: As the fact that renewable resources were used in majority projects in modern enterprises, the classic time-cost trade-off problem was extended and a new resource-constrained time-cost trade-off problem based on random chance constrained programming was proposed, where the renewable resources were emphasized. The project cost was calculated based on renewable resources. Each activity could be implemented with compressed manner in which the renewable resources were devoted to shorten the activity duration. According to the characteristics of the model, an genetic algorithm based on random simulation technique was presented to solve this model. Finally, the practical example verified the validity of the model and effectiveness of the proposed algorithm.

Keywords: Product management, chance constrained programming, random simulation, genetic algorithm

1. Introduction

There are many uncertain factors of technique, finance, environment and so on in product development project. So there is often unreasonable project planning resulting in project delay and over-budget cost. For the success of the project, we must take rigid project management [1,2]. The determination of project scheme must take the uncertainty into consideration. The objective of this paper is to find the appropriate execution option and resource utilization for each activity, so as to minimize project duration and cost, that is, the time-cost trade-off problem.

The time-cost trade-off problem has extensively been studied for many years and has been recognized as a particularly difficult combinatorial problem [3–5]. Perera [6] suggested a linear programming model to minimize the total project cost for a specified completion time using the concept of chain-bar charts. The formulation, however, assumes time-cost linearity and the critical path remains the same during project crashing. Morgan et al. [7] developed a theory of efficiency and performance tradeoffs for new product development projects, they employed a sequential data envelopment analysis (DEA) methodology to incorporate multiple factors including product development cost, product cost, and project lead time. The limitation of their researchlies in that the examination of the market success of the new products was limited to managers perception of overall financial performance, other dimensions of market success were not addressed. Shtub et al. [8] introduced the integer programming model which integrates indirect costs in the objective function and considers discrete time-cost curves towards a more realistic representation of actual problems. Robinson [9] presented a dynamic programming algorithm that considers arbitrary time-cost functions and decomposes the objective function into sequences of one-dimensional optimization problems.

The literature on the time-cost trade-off problem is rich and this indicates the scientific interest on this subject. This paper aims to construct the time-cost trade-off problem with random information by chance constrained programming technique, which is solved using a random simulation based genetic algorithm. The vague, uncertain and imprecise information such as time and cost associated with project activities are considered as random numbers with a predefined probability distribution. Where, the critical path method [10] is applied to represent project duration, the project cost is calculated based on use of renewable resources. In section 5, as a case study, this methodology is applied to the project planning of injection molding machine product in order to demonstrate its efficacy for a real-life project.

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2. Chance constrained programming with random decisions

Project planning is an objective optimization problem in an uncertain environment. Liu and Iwamura [11] provided a spectrum of chance constrained programming as well as chance constrained multi-objective programming and chance constrained goal programming with random coefficients but crisp decisions, and designed a random simulation based genetic algorithm for solving these models. Random chance constrained programming is mainly for planning problems with constraints containing random variables, assuming the possibility of constraint satisfaction is a certain predetermined value, which is also called a confidence level. So, the chance here means the possibility of constraints to be satisfied. A random chance constrained programming may be written as follows:

$$\begin{cases} \min \overline{f} \\ \text{s.t.} \Pr\{f(x,\zeta) \le \overline{f}\} \ge \beta, \\ \quad \Pr\{g_j(x,\zeta) \le 0, j = 1,2,\ldots,p\} \ge \alpha \end{cases} \tag{1}$$

where x is a n-dimension random decision vector. ζ is a vector of fuzzy parameters, $f(x,\zeta)$ is the return function, $g_j(x,\zeta)$ are constraint functions, $j = 1,2,\ldots,p$, α and β are predetermined confidence levels for constraints and objectives, respectively, and $Pr\cdot$ denotes that the possibility measure of the event $\{g_j(x,\zeta) \le 0, \quad j = 1,2,\ldots,p\}$ is at least α. For each fixed solution x, the objective value $\overline{f}$ should be the minimum achieved by the objective function $f(x,\zeta)$ with at least possibility β, as follows:

$$\overline{f} = \min\{f|\Pr|\{f(x,\zeta) \le \overline{f}\} \ge \beta\} \tag{2}$$

3. Optimization model for time-cost trade-off problem

The whole process of product development project can be decomposed into a limited number of activities, among which there are minimum-interval constraints. Intuitively, we use an AoN (Activity on Node) graph $G = (V,E)$ to illustrate that process, where, V is node set in AoN, $i \in V$ is the development activity which constitute the project, E is the set of preposing relationships, $e_{ij} \in E$ is the directed arc from node i to node j, the connection sequences between nodes are indicated by matrix C:

$$C = (e_{ij})_{n\times n}, \quad e_{ij} = \begin{cases} 1, \{i\} \prec \{j\}; \\ 0,\text{other}. \end{cases}$$

where, $\{i\} \prec \{j\}$ denotes adjacency relationship between activity i and j, furthermore, i starts before j.

There is renewable resource set K, for each resource $k \in K$, having available resource amount R_k in the whole development project. We have execution mode set M_i for each development activity i, when the activity is implemented with a particular mode, the corresponding completion time of the activity is a variable d_{im}, and the set of resources needed is K_i, for each resource $k \in K_i$, consumption amount is r_{imk}. In addition, only one activity mode will be selected to perform, and no interruption during the working process.

3.1. Time optimization model based on critical path method

Provided a product development project is composed of N activities, a set of routes from starting activity $\{1\}$ to ending activity $\{\text{n}\}$ is denoted as

$$L = \{e_{ij} = 1 | e_{ij} \in E, i = 1,2,\ldots,n-1; j = 2,3,\ldots,n\}$$

so the overall time of the project will be

$$T_L(d_{im}) = \sum_{e_{ij}\in L, m\in M_i} e_{ij} d_{im}$$

The critical path is the longest route from starting activity to ending activity, therefore, its overall time is

$$F_t(d_{im}) = \max T_L(d_{im}) = \max \sum_{e_{ij}\in L, m\in M_i} e_{ij} d_{im}$$

Take s_i as the start time for activity i, f_i as the ending time, s_1 and f_n denote the earliest start time and the earliest finish time, respectively. Considering project completion time and resources constraints, the chance constrained programming model for critical path based time optimization will be:

$$\begin{cases} \min F_t(d_{im}) = \min(\max \sum_{e_{ij}\in L, m\in M_i} e_{ij} d_{im}) \\ \text{s.t.} \sum_{m\in M_i} x_{im} = 1, i \in V \\ \quad s_j - s_i \ge \sum_{m\in M_i} x_{im} d_{im} \\ \quad f_n - s_1 \le T_E \\ \quad \sum_{i\in V} \sum_{m\in M_i} x_{im} r_{imk} \le R_k \\ \quad \Pr\{\max \sum_{e_{ij}\in L, m\in M_i} e_{ij} d_{im} \le T_E\} \ge \beta_T \\ \quad \Pr\{d_i \le d_{im} \le D_i\} \ge \alpha_T \\ \quad i = 1,2,\ldots,n-1; j = 2,3,\ldots,n; k = 1,2,\ldots,K \\ \quad x_{im} \in \{0,1\}, \forall e_{ij} \in E \end{cases} \tag{3}$$

where, T_E is the expected project time, d_i and D_i are the minimum duration and normal duration of activity i, respectively. α_T, β_T are the predetermined confidence levels for the time constraint and the objective function, respectively. The first constraint set in Eq.(3) requires that each activity should be executed with one mode only, the second constraint set requires all activities must satisfy the sequence constraints.

3.2. Project cost computation

Product development project is a knowledge-intensive activity, whose necessary resources are mainly the renewable human resources, so the project cost generates mainly based on the usage of human resources. The cost consists of two types, static cost and dynamic cost. Static cost is paid to project members in the form of weekly or monthly salaries. Furthermore, it presents a linear increasing relationship between static costs and the project time, the longer the time, the more static costs are. Dynamic cost is the expenditure dominated by the project manager. In many cases, for the reason of shorten the project completion time, we need to apply the way of overtime working to reach our goal, that is in the price of dynamic cost.

The renewable resource set K denotes the categories of project members, such as senior engineer, engineer and technician. For any category $k \in K$, whose static cost during the project period will be $sc_k = f_n c_k$, where c_k is the unit time cost paid in the form of salaries, f_n is the project time. The dynamic cost for human resource k is $dc_k = \sum_{i\in V} \triangle d_{im} c'_k$, where c'_k is unit overtime cost of resource k, fuzzy variable $\triangle d_{im}$ is the compression time obtained by overtime working. For every activity i, select a mode $m \in M_i$, then the overall project cost can be expressed as

$$F_c(c_k, c'_k, \triangle d_{im}) = \sum_{k\in K} R_k c_k f'_n + \sum_{i\in V}\sum_{m\in M_i} x_{im} y_i \sum_{k\in K_{im}} (r_{imk} \triangle d_{im} c'_k)$$

whose first item is the static cost, the second is the dynamic cost. Project time-cost chance constraint programming model with renewable resource constraints and time constraint is:

$$\begin{cases} \min F_c(c_k, c'_k, \triangle d_{im}) \\ \text{s.t.} \\ \quad 0 \le \triangle d_{im} \le D_i - d_i \\ \quad \sum_{m\in M_i} x_{im} = 1, i \in V \\ \quad \{\sum_{i\in V}\sum_{m\in M_i} x_{im} r_{imk} \le R_k\} \ge \alpha_R \\ \quad \Pr\{(\sum_{k\in K} R_k c_k f_n + \sum_{i\in V}\sum_{m\in M_i} x_{im} y_i \sum_{k\in K_{im}} \\ \qquad (r_{imk}\triangle d_{im} c'_k)) \le C_E\} \ge \beta_C \\ \quad \Pr\{\max \sum_{e_{ij}\in L, m\in M_i} e_{ij}(d_{im} - y_i \triangle d_{im}) \le T_E\} \ge \alpha'_T \\ \quad i = 1,2,\ldots,n-1; j = 2,3,\ldots,n; k = 1,2,\ldots,K \\ \quad e_{ij}, x_{im}, y_i \in \{0,1\}, \forall e_{ij} \in E \end{cases} \tag{4}$$

where, k_{im} represents the renewable resource set needed by activity i with mode m; r_{imk} is the demand amount of resource k when activity i implemented with mode m; α'_T is the confidence level for time constraint of critical path. β_C is the predetermined confidence level for the project cost function.

3.3. Optimization model for time-cost trade-off problem

In product development projects, both the project time and cost objectives should be took into account to make project planning, and decision makers hope to get a compromising project scheme. Generally, it is difficult to find a solution to make each single objective be its optimal, as the two objectives restrict mutually. This paper suggests a compromising model, which is a single objective function composed of two weighted dimensionless objectives:

$$F(x_{im}, d_i, \triangle d_i) = \omega_t \frac{F_T - F_{Tmin}}{F_{Tmax} - F_{Tmin}} + \omega_c \frac{F_C - F_{Cmin}}{F_{Cmax} - F_{Cmin}}, \tag{5}$$

where, F_{Tmax}, F_{Cmax} are the maximums of project time and cost, respectively. F_{Tmin}, F_{Cmin} are the minimums of project time and cost, respectively. ω_t, ω_c are the weights of project time and cost, respectively, having $\omega_t + \omega_c = 1$.

Construct compromising optimization model based on chance constrained programming with time, cost and resource constraints:

$$\begin{cases} \min F(x_{im}, d_i, \triangle d_i) \\ \text{s.t.} \sum_{m\in M_i} x_{im} = 1, i \in V \\ \quad 0 \le \triangle d_i \le D_i - d_i \\ \quad s_j - s_i \ge \sum_{m\in M_i} x_{im} \widetilde{d_{im}} \\ \quad f'_n - s_1 \le T_E \\ \quad \sum_{i\in V}\sum_{m\in M_i} x_{im} r_{imk} \le R_k \\ \quad \Pr\{\max \sum_{e_{ij}\in L, m\in M_i} e_{ij} d_{im} \le T_E\} \ge \beta_T \\ \quad \Pr\{(\sum_{k\in K} R_k c_k f_n + \sum_{i\in V}\sum_{m\in M_i} x_{im} y_i \sum_{k\in K_{im}} \\ \qquad (r_{imk}\triangle d_{im} c'_k)) \le C_E\} \ge \beta_C \\ \quad \Pr\{d_i \le d_{im} \le D_i\} \ge \alpha_T \\ \quad \Pr\{\max \sum_{e_{ij}\in L, m\in M_i} e_{ij}(d_{im} - y_i \triangle \widetilde{d_i}) \le T_E\} \ge \alpha'_T \\ \quad i = 1,2,\ldots,n-1; j = 2,3,\ldots,n; k = 1,\ldots,K \\ \quad e_{ij}, x_{im}, y_i \in \{0,1\}, \forall e_{ij} \in E \end{cases} \tag{6}$$

4. Model solving

We apply the genetic algorithm based on random simulation technique to solve the optimization model. Restrict to the space, the algorithm is simply described following.

Representative structure: use a vector $x = (x_1, x_2, \ldots x_n, x_{n+1}, \ldots x_{2n})$, the former n dimension represent the activity mode and the latter n dimension represent the activity working manner.

Handling the constraints: use the technique of random simulation to check whether the chromosomes generated by genetic operators are feasible.

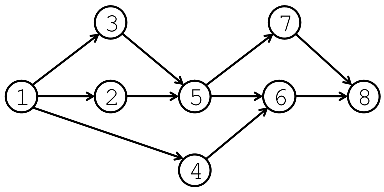

Figure 1 AON of product implementation planning for injection molding machine

Initializing process: generate a random vector x from its region until a feasible one is accepted as a chromosome. Repeat the above process Npop-size times, then we have Npop-size initial feasible chromosomes.

Evaluation function: the fitness of each chromosome is calculated by Eq.(5).

The algorithm flow is presented following:

Step 1: Input the paremeters, $N_{pop-size}$, P_c, P_m.

Step 2: Initialize $N_{pop-size}$ chromosomes, the feasibility of which may be checked by random simulation.

Step 3: update the chromosomes by crossover and mutation operations and random simulation is used to check the feasibility of offspring.

Step 4: compute the fitness of each chromosome.

Step 5: select the chromosomes by spinning the roulette wheel.

Step 6: repeat the second to fourth steps for a given number of cycles.

Step 7: report the best chromosome as the optimal solution.

Fig. 1. shows the AON diagram which illustrates the activity sequence. Table 1 defines the time parameters of each activity by each mode. There are three types of resources, namely senior engineer, engineer and technician, whose number are 1, 3, 2, unit cost are 450, 250, 200 yuan, overtime cost are 1100, 650, 550 yuan. The predilection degree of the time-cost-quality model of this project is: $w_t = 0.35$, $w_c = 0.65$. The random simulation based genetic algorithm was realized by the VC programming, and the specific parameters are: population size Npop-size = 30, maximum evolution generation MAXGEN = 1000. Then these parameters are input into the algorithm program, the given confidence levels are: $\alpha_T = 0.95$, $\beta = 0.90$, $\beta_T = 0.95$, $\beta_C = 0.95$. It is determined after computation that the critical path of the AoN network is $1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 8$.

5. A case study

To describe the proposed methodology of this paper clearly, a case study of product development project for injection molding machine was conducted in order to demonstrate the efficacy of this method for time-cost equilibrium optimization of product development project.

Table 2 shows the optimization values of the decision variable. According to which, when considered the time constraints only, the critical path based time optimization model obtained the shortest project completion time; considered the time and cost constraints, the time-cost optimization model obtained lowest project cost; considered all of the time, cost and resource constraints, the compromising optimization model obtained time-cost tradeoff solution. Take the form of [activity number, activity mode, working mode] to indicate the optimal results for activities, so the activity planning of the project is [1,1,0], [2,2,1], [3,2,0], [4,2,0], [5,1,1], [6,1,0], [7,2,0] and [8,2,1].

Table 1 Time and quality parameters of each activity by each mode

ACT	Activity Name	Predecessor	Resource 1	2	3	d_{im}	$\triangle d_{im}$
1	Injection system	-	1	0	0	N(30,1)	N(8,1)
			0	1	0	N(34,1)	N(9,1)
			0	0	1	N(38,1)	N(12,1)
2	Mold combination device	1	1	0	0	N(32,1)	N(10,1)
			0	1	0	N(38,1)	N(11,1)
			0	0	1	N(46,1)	N(13,1)
3	Mold adjustion device	1	0	1	0	u(20,40)	u(4,8)
			0	0	1	u(25,50)	u(7,15)
4	Feeding system	1	0	1	0	N(16,1)	N(6,1)
			0	0	1	N(21,1)	N(4,1)
5	Push-out device	3	0	1	0	N(18,1)	N(5,1)
			0	0	1	N(22,1)	N(6,1)
6	Cooling system	3	0	1	0	u(22,45)	u(6,12)
			0	0	1	u(27,50)	u(7,15)
7	Hydraulic system	6	1	0	0	N(30,1)	N(6,1)
			0	1	0	N(34,1)	N(8,1)
8	Control system	2,6,7	0	1	0	N(24,1)	N(7,1)
			0	0	1	N(28,1)	N(9,1)

Table 2 Time and quality parameters of each activity by each mode

Optimization model	Time constraint	Cost constraint	Resource constraint	Time	Cost
Time optimization	130	-	R1:2 R2:6 R3:4	93	-
Time cost optimization	130	75000	R1:2 R2:6 R3:4	108	59500
Compromise optimization	130	75000	R1:2 R2:6 R3:4	116	64300

6. Conclusion

This paper proposed a time-cost tradeoff model for product development project, established three random chance constrained programming models including a project completion time optimization model based on critical path, a project time-cost optimization model based on employment of renewable resources and a time-cost compromising optimization model. These models were solved with genetic algorithm based on random simulation technique, the practical example of product development project for injection molding machine verified this methodology can balance time and cost objectives of projects, providing the quantitative and authentic decision-making basis for planning of product development projects.

Acknowledgement

This work was supported by the National 973 Program, China (No. 2011CB706500), the National Natural Science Foundation, China (No. 51175456). Sincere appreciation is extended to the reviewers of this paper for their helpful comments.

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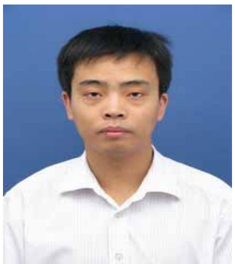

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