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Investigating the Role of Rotation in Factor Analysis, in Regard to the Repeatability of the Extracted Factors: a Simulation Study based on the 2- Parameter Weibull Distribution

Dimitris Panaretos [1], *George Tzavelas* [2], *Malvina Vamvakari* [3] *and Demosthenes Panagiotakos* [1,*]

[1] School of Health Science and Education, Harokopio University, Athens, Greece
[2] Department of Statistics and Insurance Science, University of Piraeus, Piraeus, Greece
[3] Department of Informatics & Telematics, School of Digital Technology, Harokopio University, Athens, Greece

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Abstract: Factor analysis (FA) is the most commonly used pattern recognition methodology in social and health research. A technique that may help to better retrieve true information from FA is the rotation of the information axes. The main goal is to test the reliability of the results derived through FA on skewed distributions and to reveal the best rotation method under various scenarios. Based on the results of the simulations, it was observed that when applying non-orthogonal rotation and particularly the promax method, the results were more repeatable as compared to the orthogonal rotation, or, when no rotation was applied.

Keywords: factor analysis, multivariate analysis, recognition pattern analysis, rotation, repeatability

1 Introduction

The importance of achieving accurate information in both research and clinical practice is a fundamental issue in research methodology in order to make robust conclusions about the research hypothesis. However, all experiments have some degree of random errors that are caused inevitably in an experimental process due to imponderable and uncontrollable factors and by extension influence the accuracy of a research. The past years epidemiologic research has incorporated into the analytical methodologies used pattern recognition analysis. Pattern analysis is a classical multivariate statistical approach that aims to identify patterns in data in order to show certain attributes. In recent years, FA has been widely employed by biomedical researchers and satisfactory progress and results have been achieved in the field of pattern recognition, from genes to human behavior. A technique that may help to better retrieve true information from FA is the rotation of the information axes [1,2]. The rotation can be orthogonal (the factors are uncorrelated) or non-orthogonal (the factors are correlated). The most common methods of orthogonal rotation are Varimax [3] and Quartimax [4,5,6] while the most common methods of non-orthogonal rotation are direct Oblimin [7] and Promax [8].

In a simulation paper [9] it was evaluated whether rotation type, in the presence of random error, influences the repeatability on simulated data derived from symmetric distributions. From the results of the simulation studies performed there it was observed that when applying non-orthogonal rotation, and specifically the Promax method, the results were more robust (i.e., repeatable) as compared to the orthogonal rotation, while when we did not apply any type of rotation, the results were much less repeatable and thus, it was not possible to generalize. The main goal was to test the repeatability of the results derived through a commonly used pattern recognition methodology, i.e., factor analysis, and to reveal the best rotation method under various scenarios in the case of the Uniform and Normal distributions. Therefore, the distribution of initial data was symmetrical, which is not realistic. So, it is of particular interest to study the same research scenario for asymmetric distributions, too, like the Weibull. The probability density function (pdf) of a Weibull is: $f(x) = \frac{\kappa}{\lambda}(\frac{x-\gamma}{\lambda})^{k-1}e^{-(\frac{x-\gamma}{\lambda})^c}$ where $\kappa > 0$ is the shape parameter, also known as the Weibull slope, $\lambda > 0$ is the scale parameter and

* Corresponding author e-mail: d.b.panagiotakos@usa.net

γ is the location parameter. In the case of $\gamma = 0$ the pdf equation reduces to that of the two-parameter Weibull distribution. In the case of $\gamma = 0$ and $\lambda = 1$ the pdf called Standard Weibull distribution. The Weibull distribution is related with other probability distributions, i.e., exponential (for $\kappa = 1$). The value of κ is equal to the slope of the line in a probability plot. Different values of the shape parameter can have marked effects on the behavior of the distribution.

To the best of our knowledge, the repeatability of factors in factor analysis with or without rotation, when the variables are from skewed distributions, has not been studied in the literature. Thus, the purpose of this simulation study was to evaluate whether and how the rotation type may affect the repeatability of the extracted factors' loadings.

2 Methods

2.1 Setting

To test the research hypothesis, i.e., the repeatability of factors derived through factor analysis under various rotation methods, a data file with 10 variables of 1000 observations each, was created. This was conducted by simulating 1000 observations from Weibull distribution with scale parameter $\lambda = 1$ and shape parameters $\kappa = 0.5, 1, 1.5$, and 5. Then the same procedure was repeated 10 times having, thus, a 10×1000 matrix. In order to test the repeatability of factor analysis that will be applied later on the 10 variables, a random error from a Uniform distribution on $[-0.1, 0.1]$, $[-0.3, 0.3]$ and $[-0.5, 0.5]$ was added respectively to each element of the constructed matrices. This was done in order to simulate a real world scenario in which an observation of a measurement is made at a certain time point (i.e., reporting of consumption of certain food or food groups by an individual), and then, at a later, but short-time point (i.e., within few days), and under the same conditions, another observation is made for the same measurement. The introduction of random error in the initial measurements simulates the real-world case of measurement error in a variety of field surveys.

2.2 Factor analysis with and without rotation

Then, Factor Analysis was applied in order to identify common factors between the 10 variables group in each function separately. For each function, 10 new variables were created, the factors, which can be generally identified in a subjective way as some non-measurable variables. The loadings matrix was saved separately, to take account of the three cases: no rotation, orthogonal rotation and non-orthogonal rotation. The exact same procedure was applied to the matrices with added error.

2.3 Testing repeatability of Factor Analysis under different scenarios

After applying the previous steps, 4 matrices were created for each function: a matrix with loadings of the extracted factors before and 3 after the random error has been distributed $(0.1, 0.3$ and $0.5)$. Then, for each function, 3 new matrices were created, which were the matrices of the difference between the loadings resulting from the factorial analysis before the error was introduced and the loadings after the error was introduced. The purpose in this step was to find the percentage of differences in the loadings, before and after the random error input in the data that was less than a certain cut point (i.e., $< 10^{-3}$), under the Weibull functions. The selected cut point delimits (empirically) a convergence of the matrices. Thus, the higher the percent, the better the convergence and, consequently, the repeatability of the procedure. In addition, the empirical expectation of the Frobenius matrix norm, $\| L - L' \|_F = \left[\sum_{i,j} (1_{ij} - 1'_{ij})^2\right]^{1/2}$ was also calculated as a more global measure of showing how the input of random errors affect the repeatability of the results, under the Weibull functions. The procedure was repeated separately in case of not selecting rotation and in the case of orthogonal and non-orthogonal rotation.

2.4 Simulation Process

The last procedure was repeated 100 times in order to reduce bias that may presented due to creation of one only dataset. Each time the percentage of relative differences and the 95% Confidence Interval for the percentage was calculated, in order to measure to what extent the introduction of random error between multiple and independent measurements, with and without rotation, affects the repeatability of the procedure (i.e., the development of the dataset with 1000 cases and 10 variables). The 95% CIs of the differences of the factor's loadings that have been derived through the 100 simulation processes, before and after random error has been introduced, were calculated through the formula: [mean of differences $\pm 1,96\times$ Standard Error of differences].

3 Results

Tables 1 to 4 presents the results of the percentages of deviations of the loadings of the factorial analysis matrices, before and after the accidental error input into the data observations, based on 100 simulations from the Weibull distribution on a variety of forms based on the value of shape parameter κ. The percentages denote differences in factor loadings less than 10^{-3}; the higher the percent, the better the repeatability of the results. It is noted that if a rotation method was not chosen, the percentages were too small, meaning that the introduction of an error in the data caused significant deviations in factor's loadings, and consequently lower level of repeatability of the results. Moreover, as the error increased, the deviations also increased. Thus, the repeatability in each case is low and, in particular, even if the error was ± 0.1, it was found on average that only 0.01% (95%CI: $0.009\% - 0.011\%$) of the differences were less than 10^{-3} . This means that, all the loadings have been altered by the statistical process.

The orthogonal rotation, which was then applied as the most commonly used technique in order to improve the interpretation of the common factors, resulted in better results compared to the absence of rotation of axes. Nevertheless, it was obvious that when an error occurred in the data, deviations were increased. In particular, in the case that the error was ± 0.1 and varimax rotation was applied, it was found that, on average, only 18.84% (95%CI: $18.32\% - 19.37\%$) of the differences were less than 10^{-3}, while the percentage was reduced to 7.47% in the case that the error was ± 0.5 (95%CI: $7.26\% - 7.68\%$). Similar behavior has occurred in the case of Quartimax rotation with slightly lower rates.

Concerning the non-orthogonal rotation, the results were better than the previous procedures. When selecting promax rotation with a random error input of ± 0.1, on average, only 9% of the loadings were altered by the statistical process. In fact, this percentage remained slightly stable at introducing a larger error. Similar behavior was also observed in the case of Oblimin rotation with slightly lower percentages but also constant in the introduction of higher random errors.

In the case that the data were derived from other shape of Weibull distribution (***Table 2-4***), the behavior of the results was similar with the ones presented above, with the percentages being slightly smaller. It is important to note that as the shape parameter κ increased, the distribution became more symmetrical. Indeed, the results were very similar to the previous study [9] except in the case of non-rotation. As already mentioned in the case of non-rotation the percentages were surprisingly small.

Table 1: Percentages of deviations less than 10^{-3} of the loadings of the factorial analysis matrices, before and after the error input into the data from the Weibull distribution with $\lambda = 1$ and $\kappa = 0.5$, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non-orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error $= \pm 0.1$	0.01% (0.009%, 0.011%)	18.84% (18.32%, 19.37%)	12.54% (12.18%, 12.91%)	90.89% (90.67%, 91.13%)	82.74% (82.59%, 82.88%)
Error $= \pm 0.3$	0.0034% (0.0030%, 0.0038%)	7.47% (7.26%, 7.68%)	6.13% (5.93%, 6.32%)	89.19% (88.96%, 89.42%)	82.50% (82.35%, 82.64%)
Error $= \pm 0.5$	0.0024% (0.0021%, 0.0027%)	5.44% (5.26%, 5.61%)	4.80% (4.65%, 4.95%)	85.86% (85.63%, 86.08%)	82.09% (81.96%, 82.22%)

Table 2: Percentages of deviations less than 10^{-3} of the loadings of the factorial analysis matrices, before and after the error input into the data from the Weibull distribution with $\lambda = 1$ and $\kappa = 1$, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non-orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error $= \pm 0.1$	0.002% (0.0019%, 0.0030%)	5.05% (4.90%, 5.19%)	5.24% (5.08%, 5.40%)	82.65% (82.50%, 82.80%)	82.09% (81.97%, 82.22%)
Error $= \pm 0.3$	0.002% (0.0017%, 0.0023%)	4.19% (4.03%, 4.30%)	4.10% (3.97%, 4.22%)	82.06% (81.94%, 82.19%)	82.01% (81.89%, 82.13%)
Error $= \pm 0.5$	0.0019% (0.0016%, 0.0021%)	3.99% (3.87%, 4.12%)	3.71% (3.59%, 3.82%)	81.99% (81.87%, 82.11%)	81.97% (81.85%, 82.10%)

Table 3: Percentages of deviations less than 10^{-3} of the loadings of the factorial analysis matrices, before and after the error input into the data from the Weibull distribution with $\lambda = 1$ and $\kappa = 1.5$, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non-orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error $= \pm 0.1$	0.0019% (0.0016%, 0.0021%)	4.08% (3.94%, 4.22%)	4.60% (4.45%, 4.74%)	82.85% (82.69%, 83.00%)	81.90% (81.77%, 82.03%)
Error $= \pm 0.3$	0.0016% (0.0014%, 0.0019%)	4.05% (3.93%, 4.17%)	4.22% (4.08%, 4.35%)	81.98% (81.85%, 82.11%)	82.15% (82.02%, 82.27%)
Error $= \pm 0.5$	0.0019% (0.0017%, 0.0023%)	3.96% (3.83%, 4.08%)	3.78% (3.67%, 3.90%)	82.11% (81.98%, 82.23%)	81.02% (81.90%, 82.14%)

Table 4: Percentages of deviations less than 10^{-3} of the loadings of the factorial analysis matrices, before and after the error input into the data from the Weibull distribution with $\lambda = 1$ and $\kappa = 5$, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non-orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error $= \pm 0.1$	0.0018% (0.0015%, 0.0021%)	4.18% (4.23%, 4.26%)	4.08% (3.95%, 4.20%)	82.04% (81.92%, 82.17%)	82.03% (81.91%, 82.16%)
Error $= \pm 0.3$	0.0016% (0.0014%, 0.0019%)	3.93% (3.80%, 4.05%)	4.21% (4.08%, 4.34%)	82.05% (81.92%, 82.17%)	82.08% (81.96%, 82.20%)
Error $= \pm 0.5$	0.0019% (0.0016%, 0.0021%)	4.00% (3.87%, 4.13%)	4.06% (3.94%, 4.19%)	82.02% (81.90%, 81.14%)	82.02% (81.89%, 82.14%)

4 Discussion

In this work the influence of the factor's rotation in factor analysis on the repeatability of the extracted patterns was examined based on simulated skewed data; the main goal was to test the repeatability of the results derived through a commonly used pattern recognition methodology, i.e., factor analysis, and to reveal the best rotation method under various scenarios. From the results of the simulation studies performed here it was observed that when applying non-orthogonal rotation, and specifically the *promax* method, the results were more robust (i.e., repeatable) as compared to the orthogonal rotation. It is also important to note the fact that, when we did not apply any type of rotation, the results were much less repeatable and thus, it was not possible to generalize. Nevertheless, these results are unprecedented elements that must be further exploited and confirmed by empirical studies with real data.

An issue that has rarely been examined in the literature is the repeatability of the extracted patterns derived through the factor analysis. The importance of achieving repeatable and reliable information in both research and clinical practice is a fundamental issue in research methodology. Unfortunately, many researchers that use pattern recognition tools, like factor analysis, usually underestimate this important issue in the epidemiology studies. In this case, the problem becomes clear but, unfortunately, irreversible both in analyzing the data and in discussing the results of the study. In particular, if we consider that in fact the data that a health scientist has to manage, has a strong or moderate asymmetry, the study of Weibull distribution be the reason for productive discussion. The objective of factor rotation is to achieve the most parsimonious and simple structure possible through the manipulation of the factor pattern matrix. As previously mentioned, there are two broad classes of rotation, orthogonal and non-orthogonal, which have different underlying assumptions, but which share the common goal of simple structure [10]. Orthogonal rotation shifts the factors in the factor space maintaining 90° angles of the factors to one another to achieve the best simple structure. It is notable that any movement of the axes results in a new set of coordinates and, furthermore, in different numerical values of the observations, although, in a basic sense, the variates themselves remain unchanged. Any rotated factor solution explains exactly as much covariation in the data as the initial solution. What is attempted through rotation is a possible "simplification". Evidently, rotations by themselves cannot enhance interpretability if the data do not contain relevant relationships amongst the variables [11].

Generally, it is argued that employing a method of orthogonal rotation may be preferred over oblique rotation, due to better understanding the results in terms of interpretation. However, orthogonal rotations often do not honor a given researcher's view of reality as the researcher may believe that two or more of the extracted and retained factors are correlated. Secondly, orthogonal rotation of factor solutions may oversimplify the relationships between the variables and the factors and may not always accurately represent these relationships. In contrast, a non-orthogonal rotation follows the same rotation principles as an orthogonal rotation, but due to the factors not being independent, a 90° angle of rotation is not fixed between the axes [12]. An oblique rotation offers a better chance of finding simple structure, but at the price of complicating the interpretation.

The inconsistency between the methods of rotation is inevitable due to the different mathematical structure, but what really matters is how different they are. Specifically, Varimax made large loadings on a factor higher while the small loadings lower [13]. Respectively, Quartimax rotates the factors in order to maximize the squared loadings for each variable, so often generate a single factor [14]. On the other hand, Promax, of which name derives from Procrustean rotation, begins with orthogonal rotation, usually Varimax and provides the best structure using the lowest possible power loadings and the lowest correlation between the factors [15]. Promax is often the oblique rotation strategy of choice, as it is relatively easy to use, typically provides good solutions, and tends to generate more replicable results than the direct Oblimin rotations [12]. Oblimin results in higher eigenvalues, but diminished interpretability of the factors [7].

Consistent to our previous publication [9] it was revealed in the present study that when random error occurs, except for the case of non-orthogonal rotation, the results are not repeatable. This was confirmed by both approaches of the assessment of repeatability, "absolute" agreement (deviations of factors loadings less than 10^{-3}) or using the Frobenius matrix norms, with the exception of some cases. In the first approach this behavior was much more stronger, while in the second approach, whilst differences were observed, they were small. However, this can be attributed to the fact that "absolute" agreement gives a more qualitative and specific picture of the agreement between the loadings, than the matrix norms which give an overall assessment. Moreover, the lack of repeatability, interferes with the comparison of the methods of rotation, because if a method is not accurate, then it is not fruitful to compare it [16]. Therefore, it is proposed to focus on continuous estimation of repetitions by collecting data with repeated measurements. In a previous published paper [17] it was evaluated whether the selection of certain rotation type, under various scenarios of random error introduced in the initial variables, affects the repeatability of the patterns derived from the application of FA, based on simulated data from the Standard Normal distribution. According to the findings from the simulations, it was concluded that when rotation is needed to improve the interpretation of patterns derived through factor analysis, *promax* non-orthogonal rotation seems to produce more robust results. To the best of our knowledge, no study has been carried out before regarding data from asymmetric distributions, making this study unique in exploring the inherent properties of factor analysis as a robust pattern recognition tool on data derived. It has been discussed several times before that the distinction between orthogonal and oblique rotations is important for a better understanding of the extracted factor's structure. According to the findings of this simulation study, it is strongly concluded that when rotation is needed to improve the interpretation of patterns derived through factor analysis, non-orthogonal rotation- and particularly the *promax* method- seems to produce more robust results. The aforementioned results have to confirm the previous study with symmetrical distributions [9]. Certainly, the result from the findings should be made with conscious and further research is needed in order to proof them mathematically.

Conflict of interest

None to declare

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Appendix

Tables 5 to 8 presents the results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the data, based on 100 simulations from the Weibull distribution on a variety of forms based on the value of shape parameter κ.

Table 5: Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the data from the Weibull distribution with $\lambda = 1$ and $\kappa = 0.5$, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non-orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error $= \pm 0.1$	1.98 (1.94, 2.05)	2.98 (2.93, 3.04)	3.75 (3.72, 3.77)	2.92 (2.87, 2.96)	4.14 (4.12, 4.16)
Error $= \pm 0.3$	3.42 (3.38, 3.45)	3.52 (3.49, 3.54)	3.99 (3.97, 4.01)	3.22 (3.18, 3.26)	4.17 (4.15, 4.19)
Error $= \pm 0.5$	4.05 (4.03, 4.08)	4.01 (3.99, 4.03)	4.12 (4.10, 4.14)	3.72 (3.69, 3.75)	4.22 (4.21, 4.24)

Table 6: Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the data from the Weibull distribution with $\lambda = 1$ and $\kappa = 1$, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non-orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error $= \pm 0.1$	3.87 (3.85, 3.90)	4.14 (4.12, 4.15)	4.05 (4.03, 4.07)	4.15 (4.14, 4.17)	4.22 (4.21, 4.24)
Error $= \pm 0.3$	4.24 (4.23, 4.26)	4.22 (4.21, 4.24)	4.22 (4.21, 4.24)	4.23 (4.21, 4.24)	4.23 (4.22, 4.25)
Error $= \pm 0.5$	4.32 (4.30, 4.33)	4.23 (4.21, 4.24)	4.24 (4.22, 4.25)	4.24 (4.22, 4.25)	4.24 (4.22, 4.25)

Table 7: Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the data from the Weibull distribution with $\lambda = 1$ and $\kappa = 1.5$, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non-orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error $= \pm 0.1$	4.17 (4.15, 4.19)	4.17 (4.15, 4.19)	4.19 (4.17, 4.20)	4.13 (4.11, 4.15)	4.25 (4.23, 4.26)
Error $= \pm 0.3$	4.28 (4.26, 4.29)	4.25 (4.24, 4.27)	4.23 (4.22, 4.25)	4.24 (4.22, 4.25)	4.22 (4.20, 4.23)
Error $= \pm 0.5$	4.31 (4.29, 4.32)	4.24 (4.22, 4.25)	4.23 (4.22, 4.25)	4.22 (4.21, 4.24)	4.23 (4.22, 4.25)

Table 8: Results of the Frobenius matrix norms of deviations of the loadings of the factorial analysis matrices, before and after the error input into the data from the Weibull distribution with $\lambda = 1$ and $\kappa = 5$, under various rotation methods used (1000 simulations).

Random Error	None	Orthogonal Rotation of axes		Non-orthogonal rotation of axes	
		Varimax	Quartimax	Promax	Oblimin
Error = ± 0.1	4.29 (4.27, 4.30)	4.25 (4.23, 4.26)	4.24 (4.22, 4.26)	4.23 (4.21, 4.24)	4.23 (4.22, 4.25)
Error = ± 0.3	4.33 (4.31, 4.34)	4.25 (4.23, 4.26)	4.24 (4.23, 4.26)	4.23 (4.21, 4.24)	4.23 (4.21, 4.24)
Error = ± 0.5	4.31 (4.30, 4.33)	4.23 (4.22, 4.25)	4.23 (4.22, 4.25)	4.23 (4.22,4.25)	4.23 (4.22, 4.25)