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System Identification with Fractional-order Models: A Comparative Study with Different Model Structures

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Abstract: The use of fractional order models are growing in the research field of modeling. However, there is no attempt to compare different fractional order models. In this paper, a comparison of different fractional model structures is presented with a simulation of various systems. The various model structures cover classical model, classical model with zero, commensurate, and non-commensurate fractional model. The results of fractional model structures are also compared with an integer order model structure. Simulation results show that non-commensurate fractional model is performing better than the other fractional and integer model structures.

Keywords: System identification, fractional calculus, non-commensurate fractional model, commensurate fractional model, model structures, DaISy (Database for System Identification).

1 Introduction

Use of fractional calculus in the domain of engineering and science is increasing gradually [1,2,3,4,5]. In the control system, it is mainly used for designing controller and process modeling [1,6]. A process model is useful for prediction, soft sensor, control, monitoring, optimization, etc. [7,8]. Mainly, there are two approaches for process modeling: first principle method and empirical method. A first principle-based model can be developed using different equations like mass and energy equations for the most of the systems. In the empirical approach, a model is developed based on input/output data. Selection of model structure is always a challenging task in empirical method. In the fractional model structures, it becomes even more difficult task as compared to integer order model.

In literature, a classical and commensurate fractional model structure are used by many researchers [6,9,10,11,12,13,14,15]. However, there is no much attention from the research community on non-commensurate fractional model structure for system identification. In this paper, different fractional model structures are compared with different systems. Various models include classical, commensurate, non-commensurate fractional, and integer model structure.

A six different plants are used for simulation. These plants are excited with various signals like pseudo-random binary sequence (PRBS), step input, sinusoidal signal, colored signal, etc. Excitation of the system is critical for system identification to capture dynamics of the system. Four plants are referred from a database of system identification (DaISy), which includes different systems such as process control systems, mechanical systems, and thermal systems.

Fractional model structures are more stable than the integer order model structure [16]. The stability of fractional order system is presented in this paper. The stable model structure has more advantages than the unstable model structure in the analysis and design of the control system.

The fractional calculus has a property of short and long memory. The fractional operator needs infinite memory for implementation. However, it is not possible in the real-time implementation. The fractional operator is approximated in the operating frequency range. Oustaloup recursive approximation method is used in approximation of fractional operator for the most of the applications [17].

In Section 2, a basic of fractional calculus and fractional order system is presented briefly. Section 3 describes for system identification of the fractional model structure. Methodology to estimate various parameters for fractional model

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structures is discussed in Section 4. Results and discussion are presented in Section 5. Finally, conclusions are commented in Section 6 followed by references.

2 Fractional Calculus and Fractional Order System

2.1 Fractional calculus

Fractional calculus is a theory of derivatives and integers of real numbers. It induced the notation for integer order and n fold integration. The fractional calculus is not very prevalent in the research field in spite of its three centuries old as conventional calculus is. From a couple of decades, researchers have applied fractional calculus in different areas of science and engineering (control system, signal processing, modeling of physical systems, etc.) [2,5,18,19,20,21].

There are many definitions of fractional calculus: starting from n-fold definitions to other different variations related to definitions. The following definitions of fractional calculus are used widely in the area of control system [22].

2.1.1 Grunwald-Letnikov Definition

It is defined as

$$ {}_aD_t^{\alpha} = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{r=0}^{\left[\frac{t-a}{h}\right]} (-1)^r \binom{n}{r} f(t-rh), \tag{1}$$

where $[\frac{t-a}{h}]$ is an integer part, h is the step size for differentiation, n is an integer which satisfies the condition $(n-1) \leq \alpha \leq n$, and t and a are the boundaries of differentiation.

2.1.2 Riemann-Liouville Definition

It is defined as the following

$$ {}_aD_t^{\alpha} = D^n J^{n-\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \tag{2}$$

where n is an integer which fulfils the constraint $(n-1) \leq \alpha \leq n$, α is a real number, J is the integral operator, and t and a are the boundaries of integration. For an example, if α is 0.78, then n would be two as $0 \leq 0.78 \leq 1$.

2.1.3 M. Caputo Definition

It is given by

$$ {}_aD_t^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \tag{3}$$

where n is an integer which fulfils the constraint $(n-1) \leq \alpha \leq n$, α is a real number, and t and a are the boundaries of integration.

2.2 Fractional order linear system

A typical fractional differentiation equation is expressed by

$$ a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \cdots + a_0 D^{\alpha_0} y(t) = b_m D^{\beta_n} u(t) + b_{m-1} D^{\beta_{n-1}} u(t) + \cdots + b_0 D^{\beta_0} u(t), \tag{4}$$

where D^{α} is denote the fractional differentiation order with time limits of 0 to t, a_n, a_{n-1}, $\cdots$, a_0, b_m, b_{m-1}, $\cdots$, b_0 are real constants and $\alpha_n, \alpha_{n-1}, \cdots, \alpha_0$, $\beta_m, \beta_{m-1}, \cdots, \beta_0$ are a positive real number, $y(t)$ is the output of the system, and $u(t)$ is the input of the system.

The Eq. 4 can be transformed into frequency domain using Laplace transfer using following property [23],

$$L\{D^{\gamma}x(t)\} = s^{\gamma}X(s);\ \text{if x(t) =0 } \forall t < 0. \tag{5}$$

By using the above property, it can be presented in the transfer function as follows:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1}s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + s_{n-1}s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}}, \tag{6}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{Q(s^{\alpha})}{P(s^{\alpha})}. \tag{7}$$

Eq. (6) can be called commensurate order system if α_k and β_k is an arithmetical progression with the same difference. Mathematically,

$$\alpha_k = k \times \alpha; \text{where k=0,1,2,} \cdots, n$$

$$\beta_k = k \times \alpha; \text{where k=0,1,2,} \cdots, m$$

and the value of α is between 0 to 1. The stability of fractional order system can be obtained using Riemann surface shown in Fig. 1.

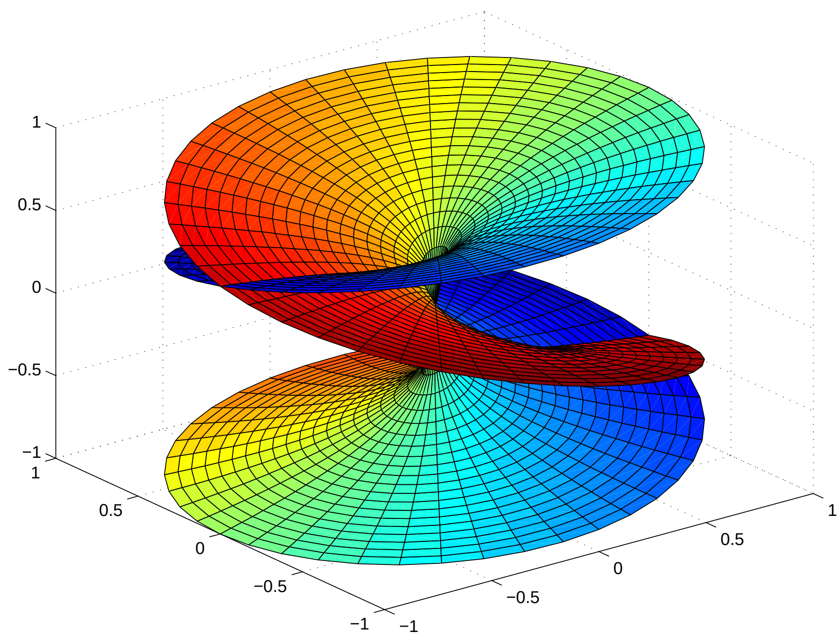


Fig. 1: Riemann surface for $s^{1/3}$.

3 System Identification

3.1 Introduction

The system identification deals with developing a suitable model from input/output data. System identification is not a just tool for getting the model from the gathered data. However, it should be used with good experimental design and a prior knowledge if available. The more details of system identification can be found in [7,8].

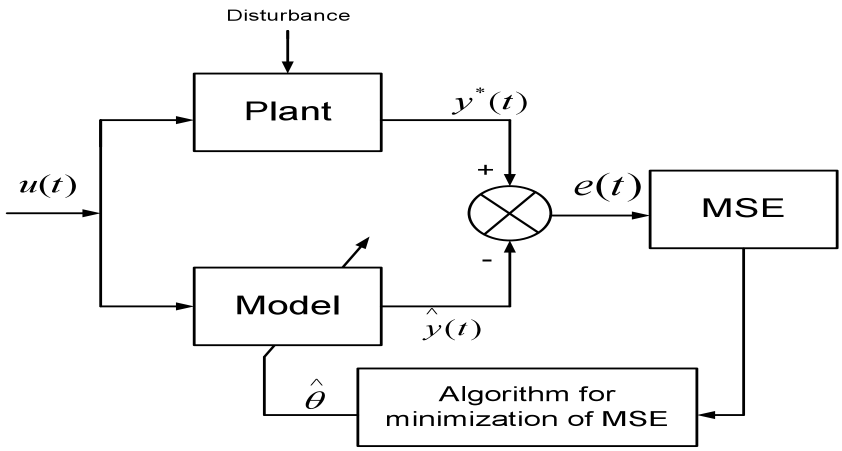


Fig. 2: Block diagram of output-error method [34].

A fractional model structure was used in system identification by Le Lay in his Ph.D. thesis for frequency domain system identification [24]. Time domain system identification using fractional model was started by the Ph.D. theses of Le Lay, J. Lin and O. Cois [24,25,26]. Mainly, two classes of methods are used in literature: equation error-based method and output error based methods. There are many real systems, which can be represented by the fractional model. It includes electrode-electrolyte polarization [27], diffusion systems: electro-chemistry, heat transfer and electromagnetism [10,28], Nuclear reactor [29], charge estimation of lead acid battery [11], Semi-infinite thermal system [6], Thermal diffusion in a wall [13], Ultra-capacitors or super-capacitors [30], fractional order chaotic systems [31], modeling for HIV infection with drug therapy effect [32], boost converter [33], etc.

3.2 Output error method for system identification

The parameters of different model structures can be estimated by output error method [34]. Assuming that $\hat{\theta}$ is required to be an estimation of the exact parameter θ. The data set for identification is composed of N dataset points $\{u_n, y_n^*\}$ with sampling time T_c. y_n^* represents noise measurement of the exact output y_n. The general block diagram of output error method for system identification is shown in Fig. 2.

The output predication error,

$$\varepsilon_n = y_n^* - \hat{y}_n(u_n, \hat{\theta}). \tag{8}$$

The optimal value of $\hat{\theta}$ can be obtained by minimized of the mean square predication error which is given by,

$$J = \sum_{n=1}^{N} \varepsilon_n^2. \tag{9}$$

Most of the time $\hat{y}_n$ is nonlinear. In this situation , the Marquardt's algorithm [35] can be used which gives a robust convergence for parameters estimation. This algorithm estimates parameters by following recursive formula [7],

$$\theta_{i+1} = \theta_i - \left\{ \left[J''_{\theta\theta} + \lambda I \right]^{-1} J'_{\theta} \right\}_{\hat{\theta}=\theta_i}, \tag{10}$$

where

$$J'_{\theta} = -2 \sum_{n=1}^{N} \varepsilon_n \sigma_n \theta_i : \text{Gradient}$$

$$J''_{\theta\theta} \approx 2 \sum_{n=1}^{N} \sigma_{n,\theta_i} \sigma_{n,\theta_i}^T \text{Hessian}$$

λ is the monitoring parameters.

$$\sigma_{n,\theta_i} = \frac{\partial \hat{y_n}}{\partial \theta_i} \text{ Output sensitivity function. .}$$

The mean squared error (MSE) is the mean of the squares of the error, that is, the difference between measured and prediction values. Mathematically, it is represented by,

$$MSE = \frac{1}{N}(y_n^* - \hat{y^*_n}^2), \tag{11}$$

where N is the total number of samples.

The normalized mean square error (NMSE) is a performance indicator for the overall deviations between measured and prediction values. Mathematically, it is represented by,

$$NMSE = \frac{\overline{y_n^* - \hat{y^*_n}}^2}{\overline{y_n^*}\,\overline{\hat{y^*_n}}}, \tag{12}$$

where the overbar represents the mean over the sampling points.

4 Methodology

For comparing different fractional model, six different plants are used for simulation. These systems are taken from [36, 37]. Plant 1 is a DC motor system [36], where plant 2 is higher order system [37]. Plant 3 to plants 6 are referred from a database of system identification (DaISy), which is used by many researchers in the field of system identification [38,39, 40,41]. In case of plant 1 and plant 2, data are gathered using Simulink model from system transfer function. Different plants used for simulation are summarized in Table 1.

Table 1: Different plants used for simulation.

Plant	Descriptions	Remark(s)
P_1	Second order system (DC-Motor)	Ref. [42]
P_2	Higher order system	Ref. [37]
P_3	Hair dryer system	DaISy
P_4	Ball beam system	DaISy
P_5	Wing flutter system	DaISy
P_6	Heating system	DaISy

Detail of various plants as follows:

$$P_1(s) = \frac{K}{(Js+b)(Ls+R)+K^2}, \tag{13}$$

where J is the moment of inertia (0.01 $kg.m^2$), b is the motor viscous friction constant (0.1 $N.m.s.$), L is the electrical inductance (0.5 H), R is the electric resistance (1 Ω), and K is the motor torque constant (0.01 $N.m/Amp$). The modeling of DC motor for fractional calculus can be found in [43].

$$P_2(s) = \frac{1}{s^4+4s^3+6s^2+4s+1}. \tag{14}$$

Plant 3 detail: Hair dryer system

– Inputs: voltage of the heating device
– Outputs: output air temperature
– Sampling time: 0.1 sec
– Description: laboratory setup. Air is fanned through a tube and heated at the inlet.

Plant 4 detail: Ball bear system

– Inputs: angle of the beam
– Sampling time: 0.1 sec
– Outputs: position of the ball

Plant 5 detail: Wing flutter system

–Inputs: the input is highly colored.
–Sampling time: 0.1 sec

Plant 6 detail: Heating system

–Inputs: input drive voltage
–Output:temperature (deg. C)
–Sampling time: 2.0 sec
–Description: The data of expriment is a single-input single-output heating system. The input drives a 300 Watt Halogen lamp, suspended several inches above a thin steel plate.

The excitation of different plants is PRBS signal, step input, sinusoidal signal, colored signal, etc. The input and output plots for different plants are shown in Figs. 3 and 4. These different types of signal captures different dynamics of the system. A dataset is divided into two sets: 50 % used for identification and 50 % used for model validation.

Following fractional model structures are covered for simulation:

–Model 1 (classical fractional model)

$$G_1(s) = \frac{K}{s^\alpha + a}. \tag{15}$$

In Eq. (15), the following parameters are estimated in this paper [10]

$$\theta^T = [k \quad \alpha \quad a]$$

The range of a and k are between 0 to 100, and α is between 0 to 2.

–Model 2 (classical fractional model with zero)

$$G_2(s) = \frac{b_0 s^\beta + b_1}{a_0 s^\alpha + a_1}; \alpha \geq \beta. \tag{16}$$

In Eq. (16), the following parameters are estimated

$$\theta^T = [b_0 \quad b_1 \quad \beta \quad a_0 \quad a_1 \quad \alpha]$$

The range of a_0, a_1, b_0 and b_1 are between 0 to 100, and α and β are between 0 to 2 with $\alpha \geq \beta$.

–Model 3 (commensurate order)

$$G_3(s) = \frac{K}{a_0 s^\beta + a_1 s^\alpha + a_2}. \tag{17}$$

In Eq. (17), the value of β is an integer multiple of α. So, the above model structure is commensurate order. Here, the value of β is taken twice of α ($\beta = 2\alpha$). The range of a_0, a_1, a_2 and k are between 0 to 100, and the value of α is between 0 to 1.

–Model 4 (Non-commensurate order)

$$G_4(s) = \frac{b_0 s^\gamma + b_1}{a_0 s^\beta + a_1 s^\alpha + a_2}. \tag{18}$$

In Eq. (18), the value of α is not an integer multiple of β. So, the above model structure is non-commensurate order. The range of b_0, b_1, a_0, a_1 and a_2 are between 0 to 100, the value of α and γ are between 0 to 1, and the value of β is between 1 to 2.

–Model 5 (Integer order)

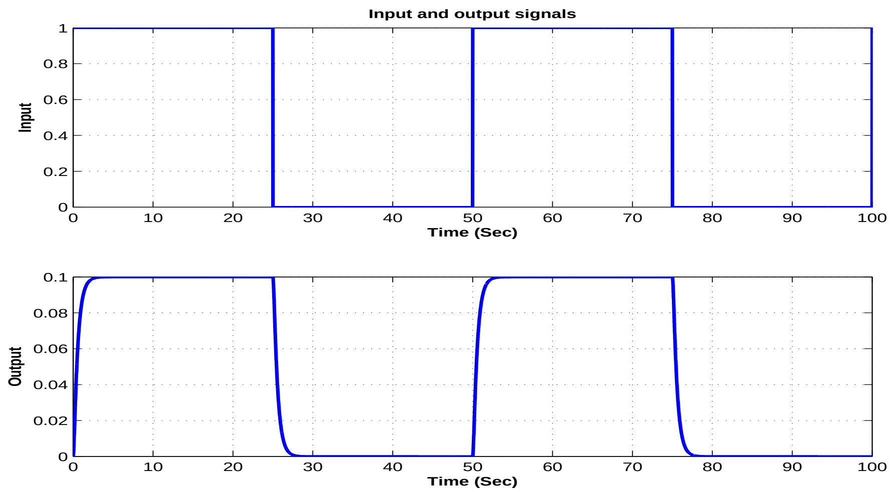
Assuming the plant dynamics is of higher order, we have used this model structures.

$$G_5(s) = \frac{b_0 s + b_1}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}. \tag{19}$$

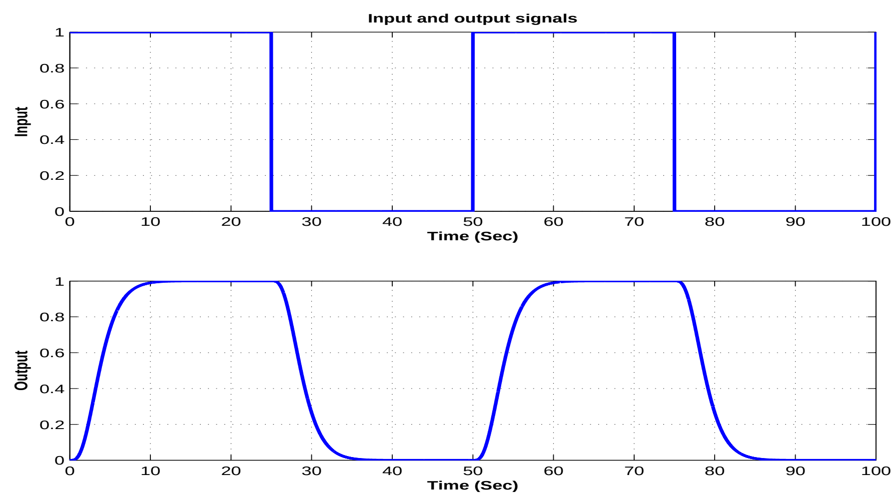
The parameters (b_0, b_1, a_0, a_1, a_2 and a_3) of Eq. (19) are in the range of 0 to 100 and estimation by same approach as used for fractional model.

The summary of different model structure is given in Table 2 with number of parameters to be estimated.

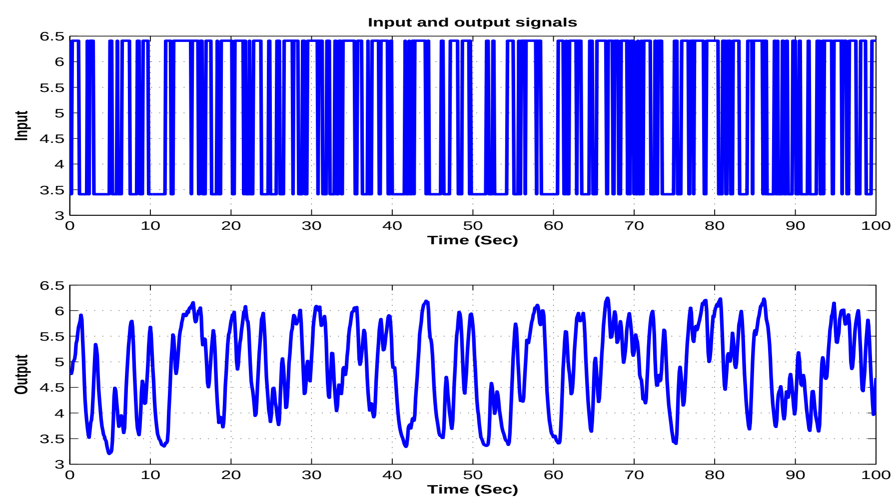
The parameters of the various model structure are estimated using optimization toolbox. The steps for obtaining parameters of different models follow:



(a) Plant 1

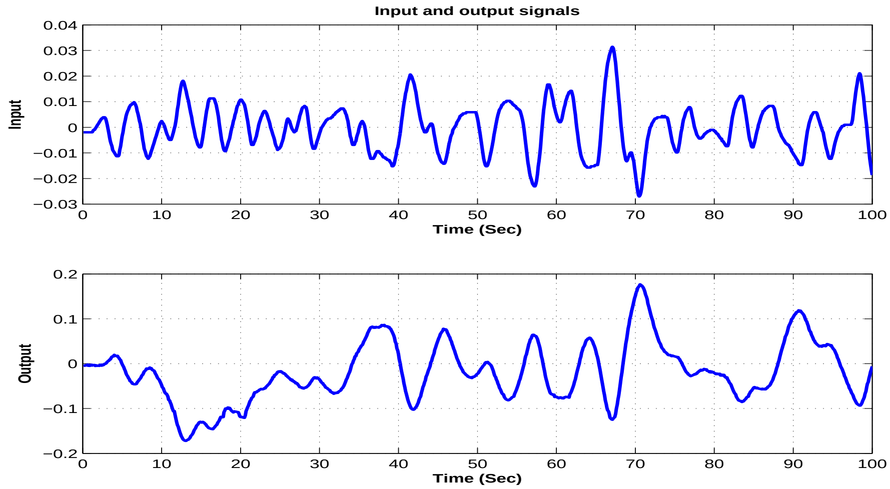


(b) Plant 2

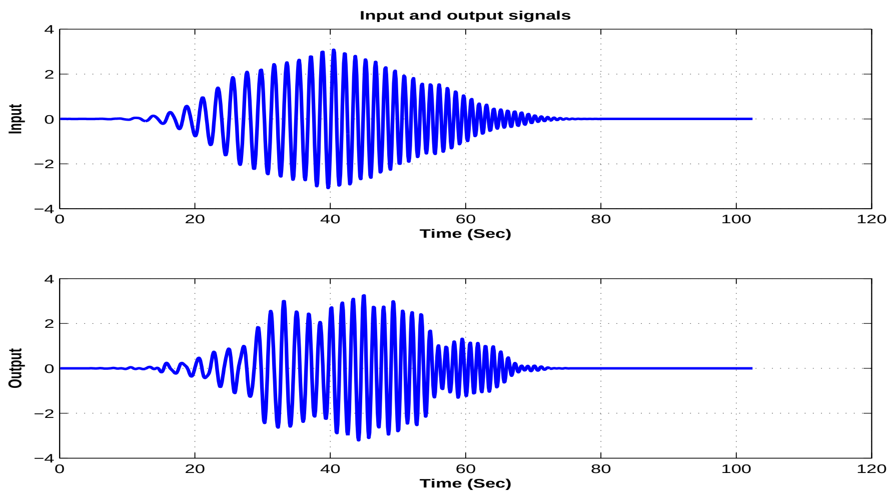


(c) Plant 3

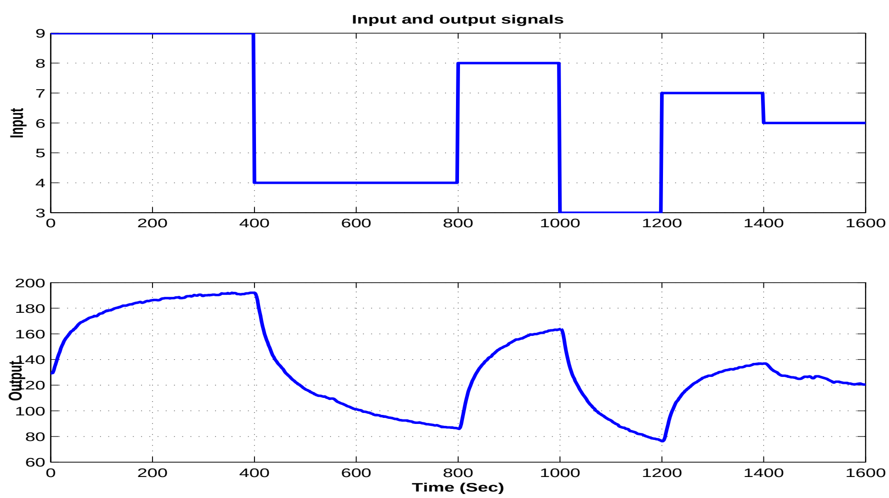
Fig. 3: Input and output signal for plant 1 to plant 3.



(a) Plant 4



(b) Plant 5



(c) Plant 6

Fig. 4: Input and output signal for plant 4 to plant 6.

Table 2: Number of parameters to be estimate in various model structure.

Sr. No	Model Structure	No of Parameters
1	$G_1(s) = \frac{K}{s^\alpha + a}$	3
2	$G_2(s) = \frac{b_0 s^\beta + b_1}{a_0 s^\alpha + a_1}$	6
3	$G_3(s) = \frac{K}{a_0 s^\beta + a_1 s^\alpha + a_2}$	5
4	$G_4(s) = \frac{b_0 s^\gamma + b_1}{a_0 s^\alpha + a_1 s^\beta + a_2}$	8
5	$G_5(s) = \frac{b_0 s + b_1}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$	6

1.Experiment is carried out to generate data using Simulink.
2.Dataset is collected using experiment or referred from a database of system identification (DaISy).
3.Dataset is divided into two parts: 50 % for parameter identification and 50 % for model validation.
4.Using identification dataset, model outputs are calculated for selected model structure.
5.Error index function (MSE/NMSE) is minimized in the search domain using optimization method (Genetic algorithm).
6.The estimated parameters are validated using validation dataset.

5 Results and Discussions

The above method, which is described in the previous section, is applied to six plants. For every model structure, the value of MSE and NMSE is calculated. For model structure 1, which is a classical fractional model, the different parameters are estimated. Results are tabulated in Table 3. The value of NMSE does not depend on the range of input. However, the value of MSE depends on the range of input. In estimation theory, datasets are mostly normalized before using for system identification.

Table 3: Results for model structure 1.

Plant	K	α	a	MSE	$NMSE$
P_1	0.1339	1.0831	1.3415	4.0915E-6	0.0120
P_2	0.2006	1.1431	0.1993	0.00426	0.0199
P_3	1.6223	1.3296	1.6853	0.3496	0.3998
P_4	0.0100	0.0170	0.3460	0.0053	4.0201
P_5	0.5745	0.0001	1.7293	1.5837	20.1734
P_6	2.9862	0.6392	0.1227	218.5861	0.1083

Six parameters are estimated for model structure 2. Results for this model structure are tabulated in Table 4. A commensurate model structure (model structure 3) is also used for comparison of different model structures. In this simulation, the value of β is taken twice the value of α, where the range of α is between 0 to 1. Results are presented in Table 5. This model structure gives better results than model structure 1 and 2.

A non-commensurate model structure is also used for identification of plants dynamics. In the non-commensurate model, the value of α and β are not in the multiple of an integer. Results are provided in Table 6. This model structure gives much better results than other model structures. However, more parameters are required to be estimated in this model structure. An integer model is also developed using the same method as used for the fractional model structure. In this model structure, there are six parameters need to be estimated. The results are tabulated in Table 7.

Finally, step response for different plants is shown in Figs. 5, 6, 7, 8, 9 and 10. It demonstrates that fractional models capture more dynamics expect for plant 4. For plant 1 and plant 6, the model structure 3 (commensurate model) is giving minimum MSE and NMSE. As mention earlier, plant 2 is higher-order model system of 4^{th} order. So, integer order model gives better results in this case. However, non-commensurate results are closed to integer order model. For plant 3 and plant 5, the non-commensurate model gives good results. For plant 4, integer and fractional order model structure are

Table 4: Results for model structure 2.

Plant	b_0	b_1	β	a_0	a_1	α	*MSE*	*NMSE*
P_1	0.0849	0.1687	0.0104	1.6536	2.5612	1.3051	1.4669e-05	0.0051
P_2	5.9361e-05	0.35922	1.0336	2.1581	0.3686	1.1950	0.00358	0.0165
P_3	0.0100	1.7961	0.0265	1.0417	1.8751	1.3342	0.3478	0.3858
P_4	0.3819	0.7872	0.0105	8.6821	0.0100	1.9999	0.0049	11.1552
P_5	0.2331	0.0129	1.1722	0.0600	1.2012	1.3697	0.1022	0.0647
P_6	4.8057	2.9773	0.6463	2.6610	0.1294	0.8130	261.8012	0.1138

Table 5: Results for model structure 3.

Plant	K	a_0	a_1	a_2	α	*MSE*	*NMSE*
P_1	0.1366	0.3896	0.5739	1.3505	0.7932	1.7055e-06	0.0007
P_2	0.6109	0.8417	3.6960	1.3538	0.5968	0.00139	0.0068
P_3	0.9320	0.3683	0.2503	0.9560	0.7850	0.3300	0.3641
P_4	0.1075	1.4980	0.3017	0.4875	0.9899	0.0048	7.2427
P_5	0.2235	0.0255	0.2948	0.7375	4.3e-04	1.5838	20.0248
P_6	0.8586	1.609e-04	0.1847	0.0343	0.5480	197.1101	0.1016

Table 6: Results for model structure 4.

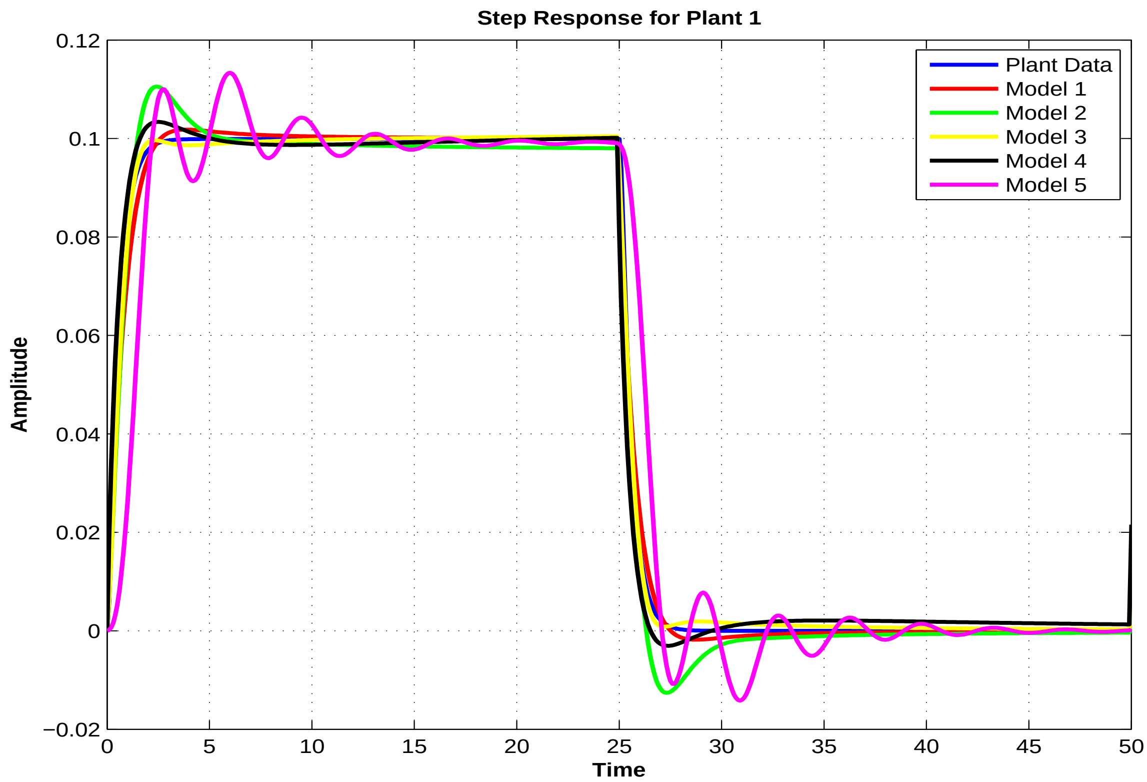
Plant	b_0	b_1	γ	a_0	a_1	a_2	α	β	*MSE*	*NMSE*
P_1	0.2586	0.0959	0.7787	2.4208	1.7031	0.9188	0.6962	1.6394	1.5831e-05	0.0158
P_2	8.8503e-04	0.1779	0.9252	0.3192	1.1121	0.1734	0.7806	1.5934	0.00161	0.0082
P_3	1.0694	0.9238	0.0460	1.2103	0.4455	1.8496	0.7955	1.9999	0.3133	0.3446
P_4	2.8643	0.0	0.9904	1.4109	0.5078	3.4839	0.9961	1.2168	0.0052	29.4533
P_5	0.1387	7.1585e-05	0.2459	0.1646	0.0368	0.8610	0.1619	1.8623	0.0866	0.0548
P_6	2.8061	2.2568	0.0983	1.1856	0.0745	0.1399	0.5510	1.9997	199.6909	0.1006

not mapping dynamics of the system. The validation of obtained models are carried out and their responses are plotted in Figs. 11 and 12. The obtained parameters also give good results with validation dataset, which is not used during the identification process.

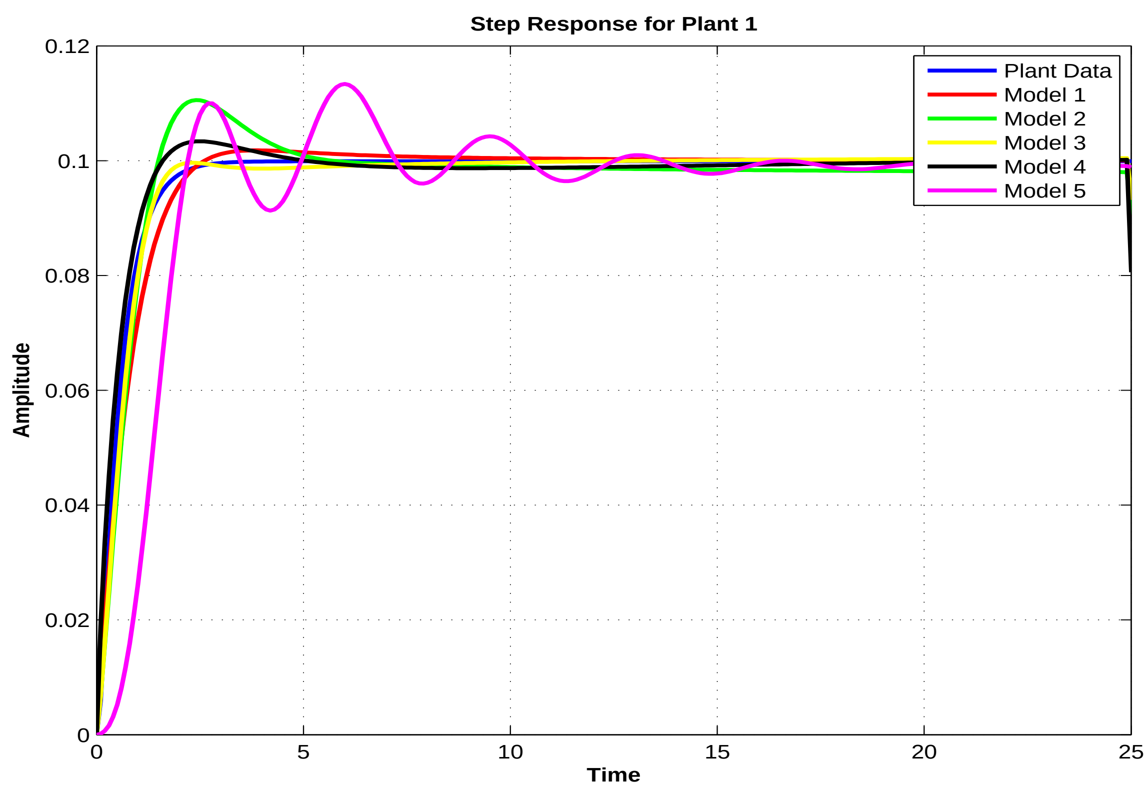
6 Conclusions and future directions

In this paper, different fractional model structures are compared with various systems. A non-commensurates fractional model gives better results regarding MSE and NMSE. Stability of different fractional model structures is also presented briefly. A comparison is also made with an integer order model structure, which is identified by system identification toolbox. Fractional model structures are capable to captures dynamics of many systems. However, the presence of the fractionality in the system needs to cross check before selection of fractional model.

A non-commensurates fractional model can be explored more in details, which should work with initial conditions. System identification toolbox may be extended for user defined model structure including fractional model.

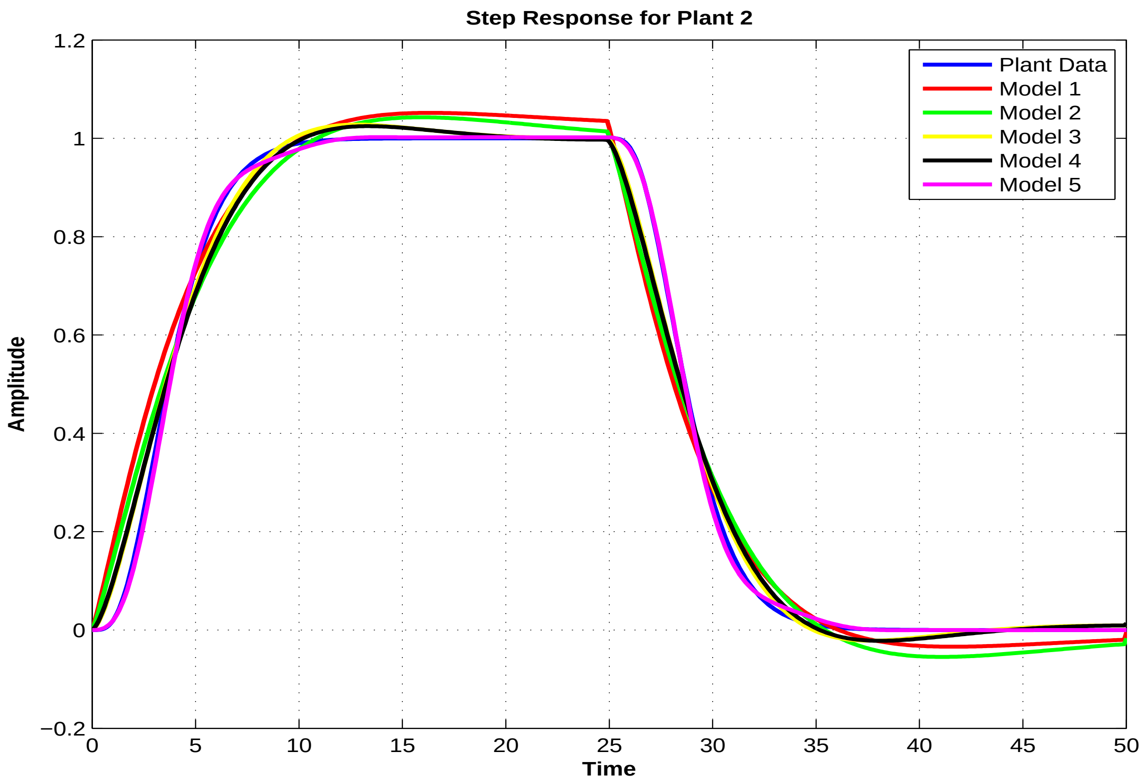


(a) Actual

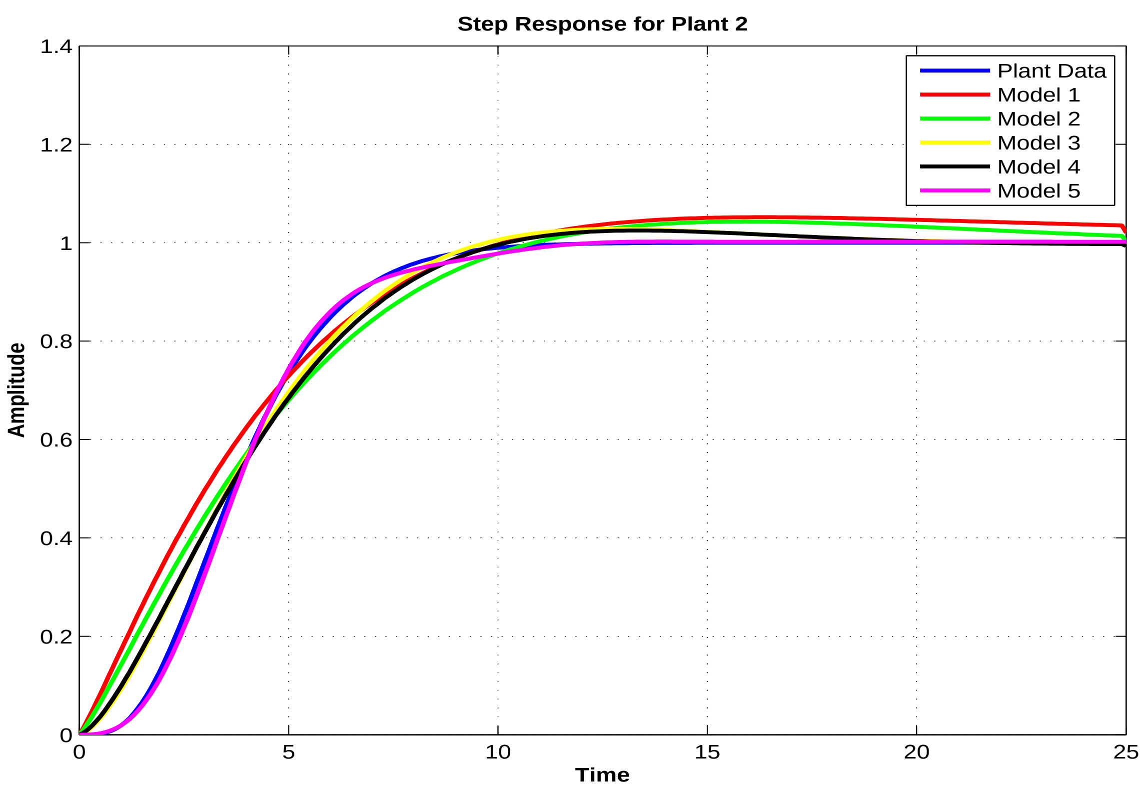


(b) Zoomed

Fig. 5: Step response for plant 1.

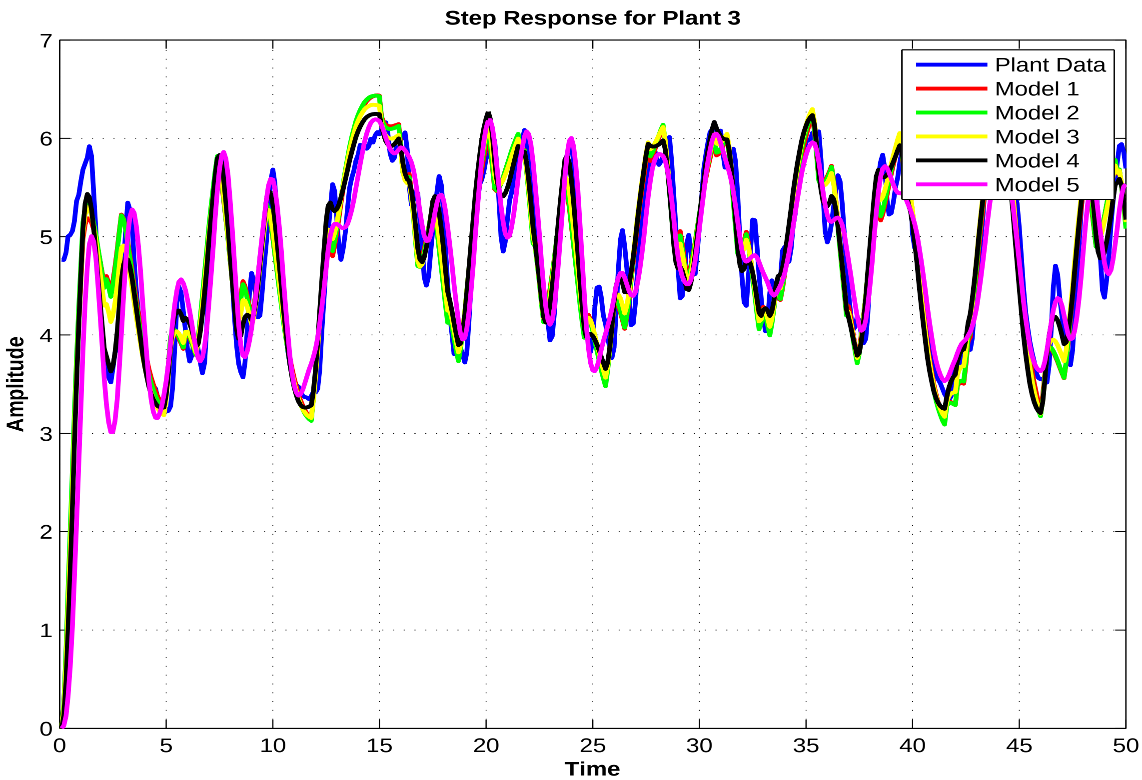


(a) Actual

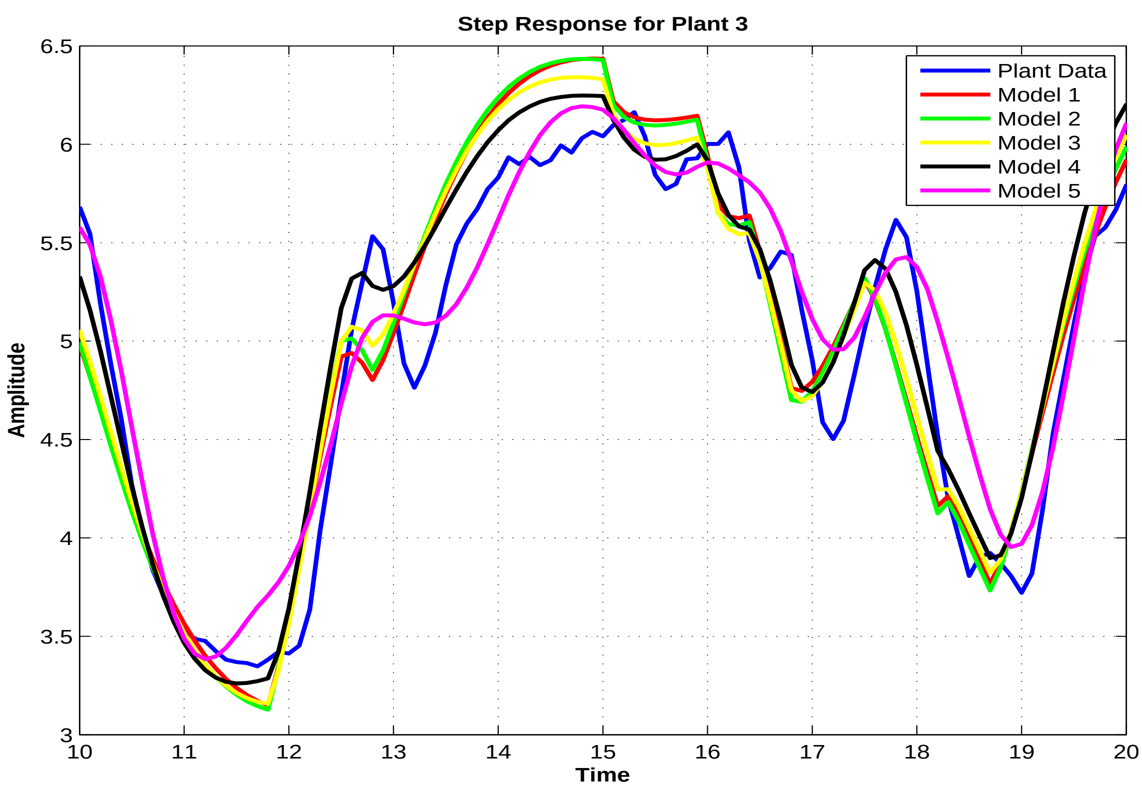


(b) Zoomed

Fig. 6: Step response for plant 2.

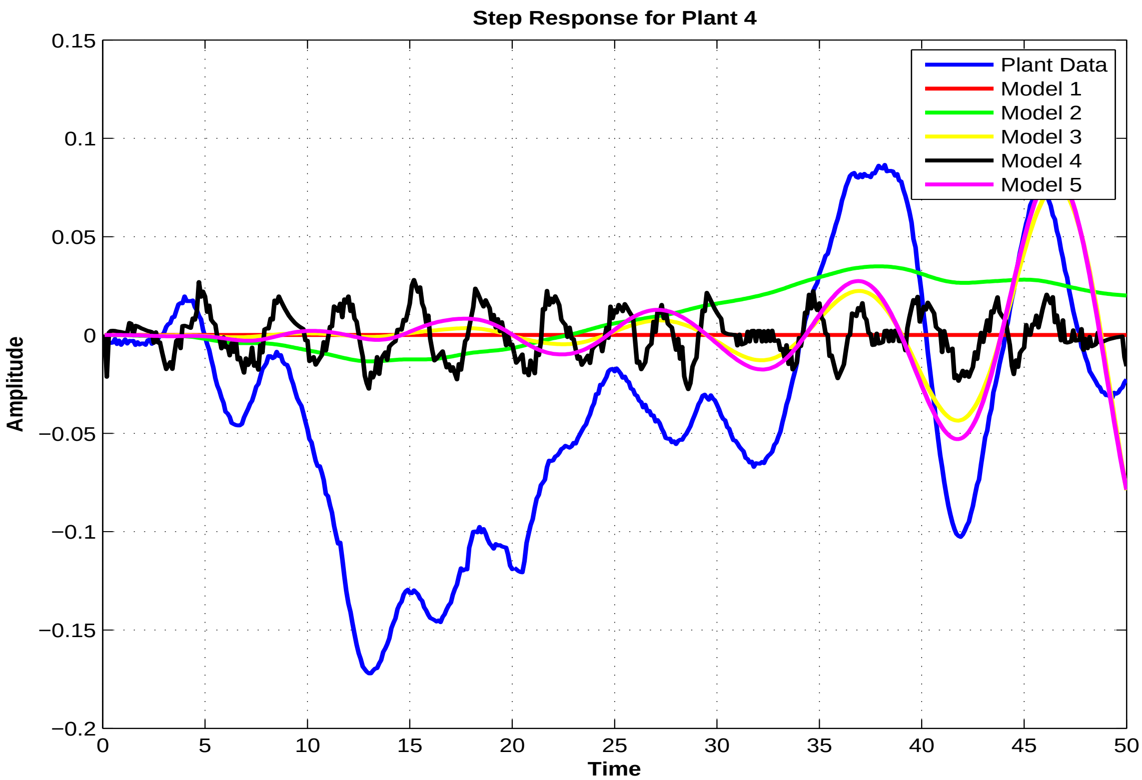


(a) Actual

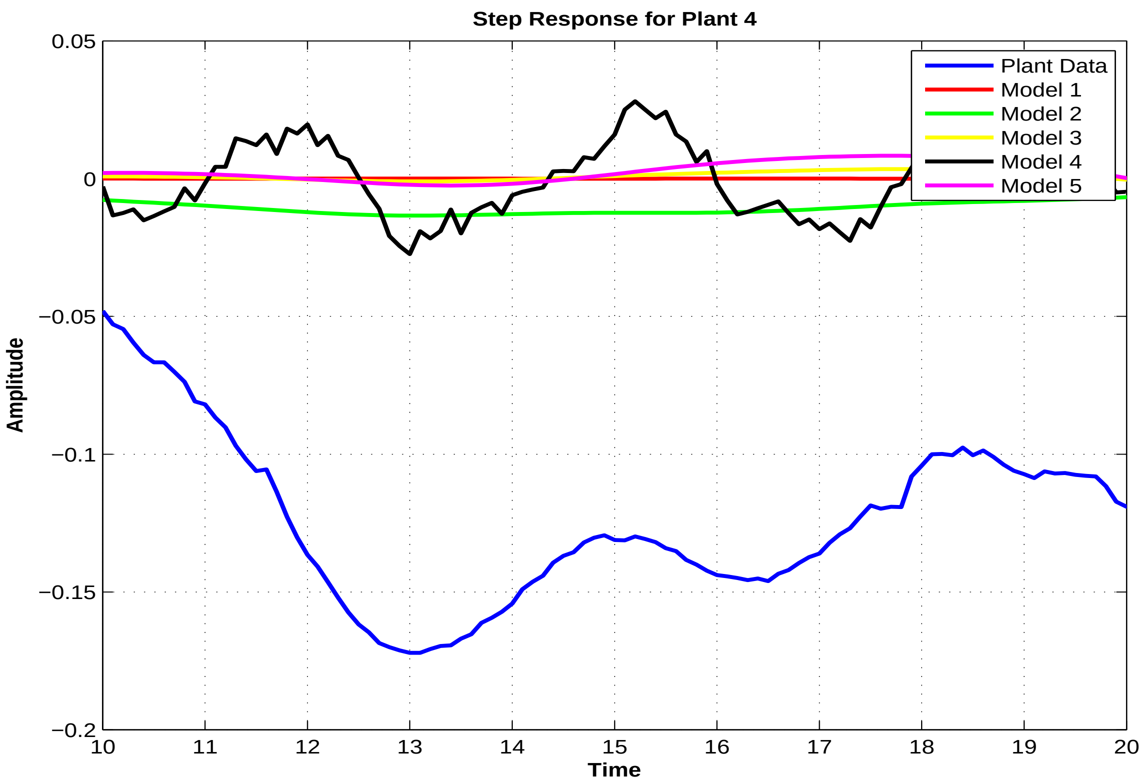


(b) Zoomed

Fig. 7: Step response for plant 3.

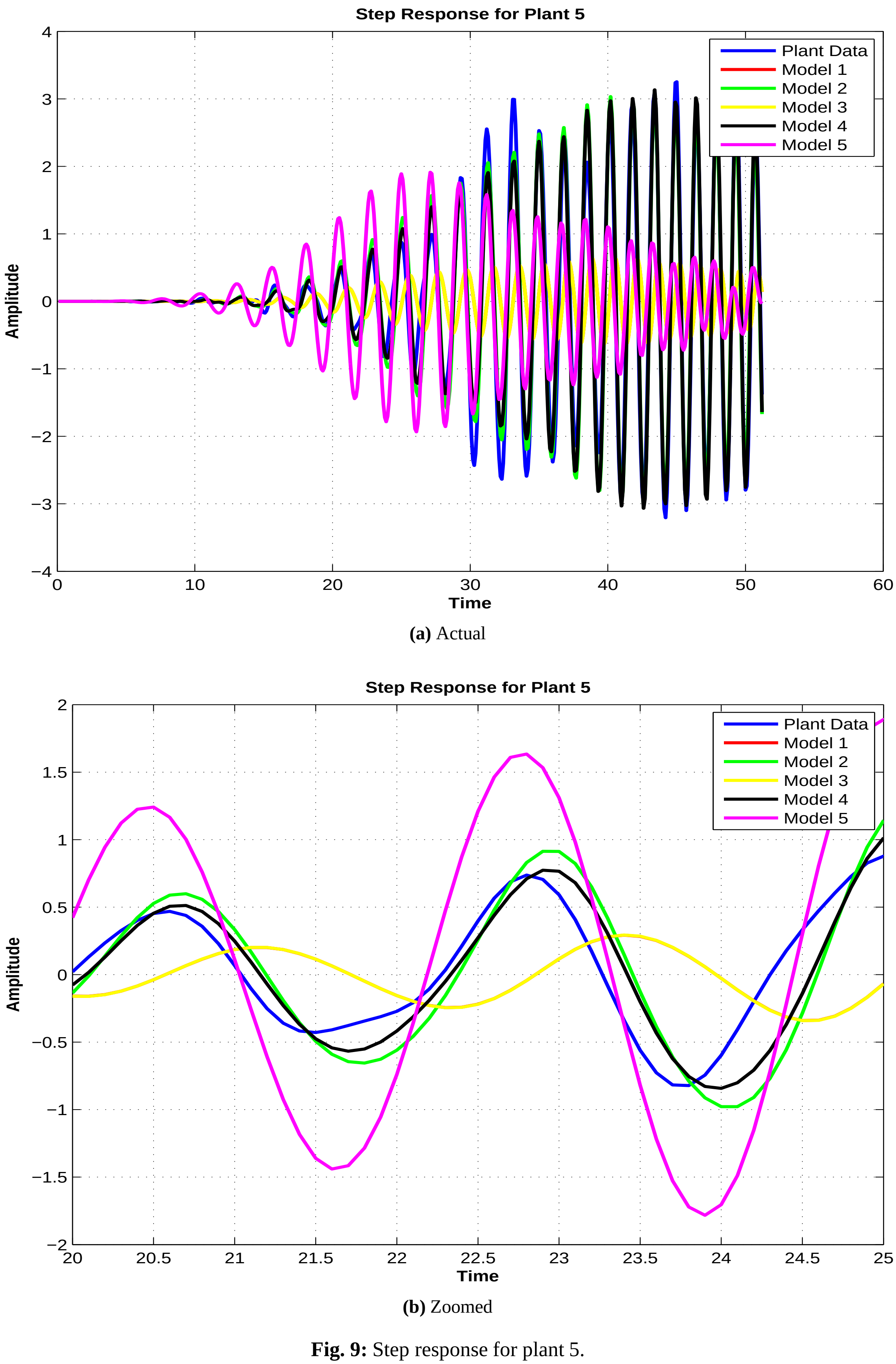


(a) Actual



(b) Zoomed

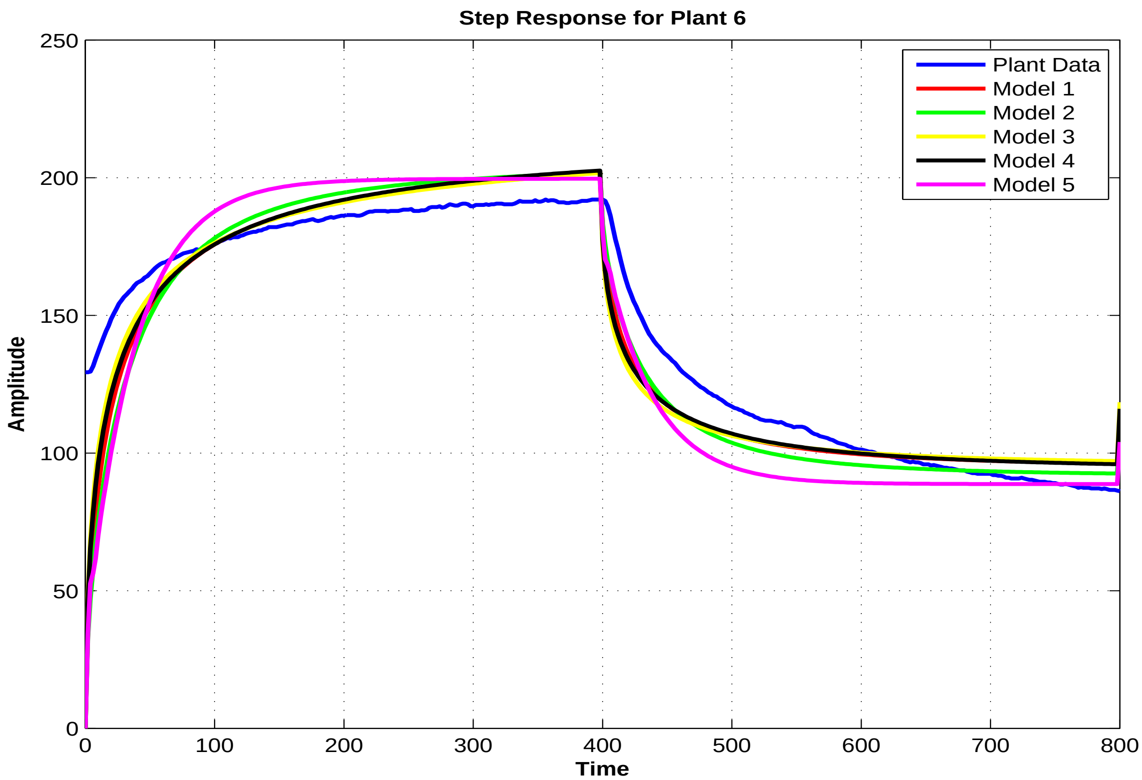
Fig. 8: Step response for plant 4.



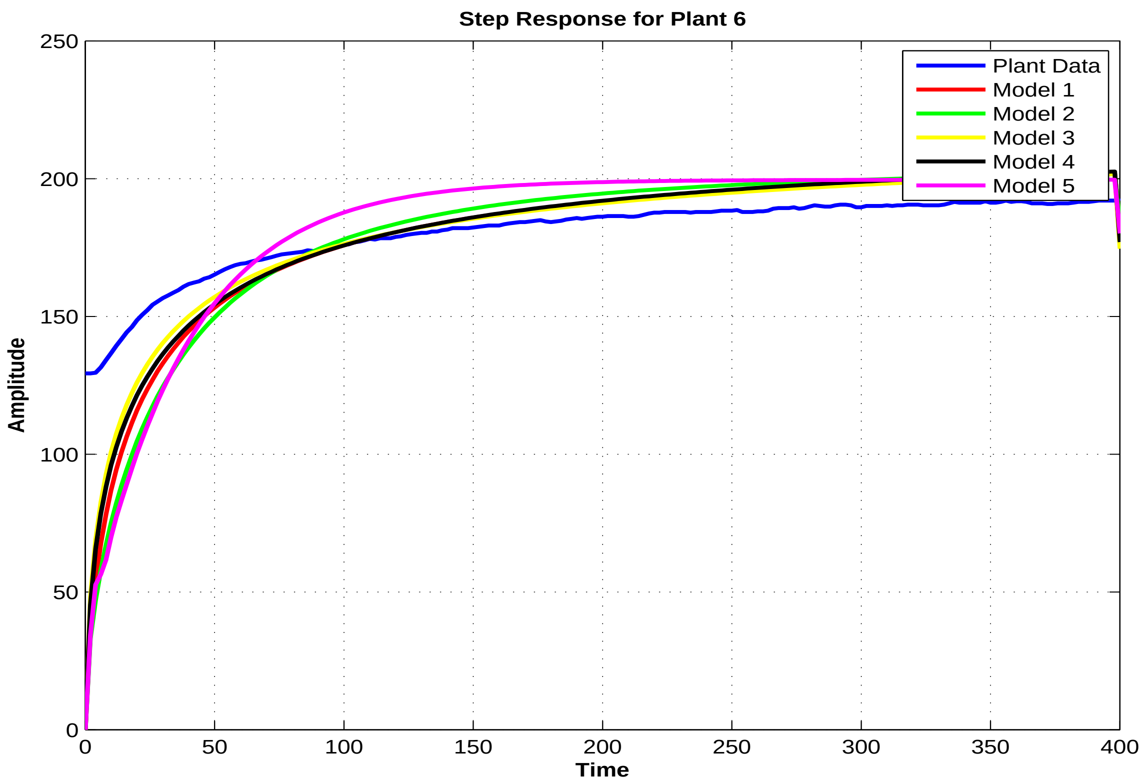
(a) Actual

(b) Zoomed

Fig. 9: Step response for plant 5.

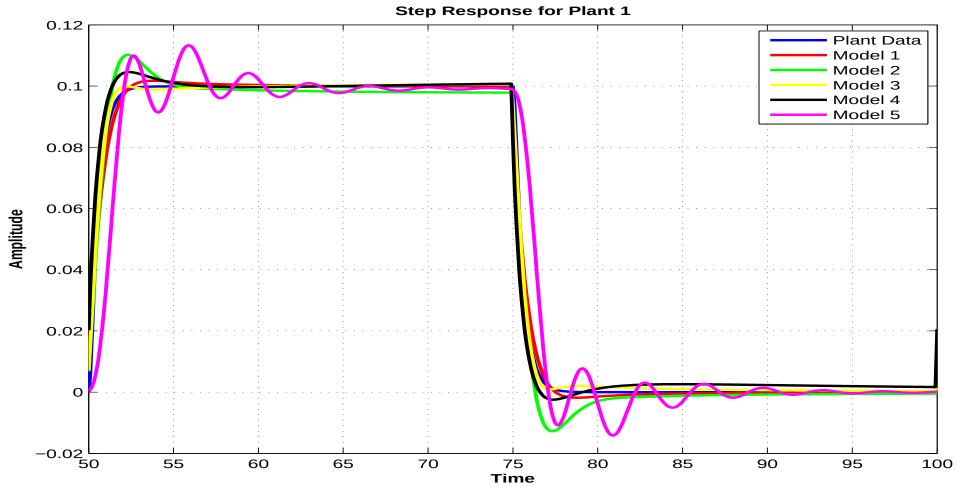


(a) Actual

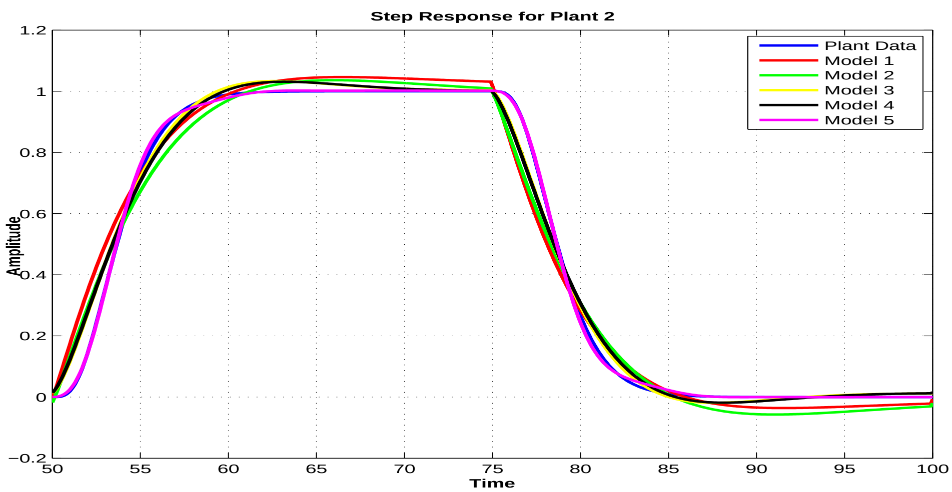


(b) Zoomed

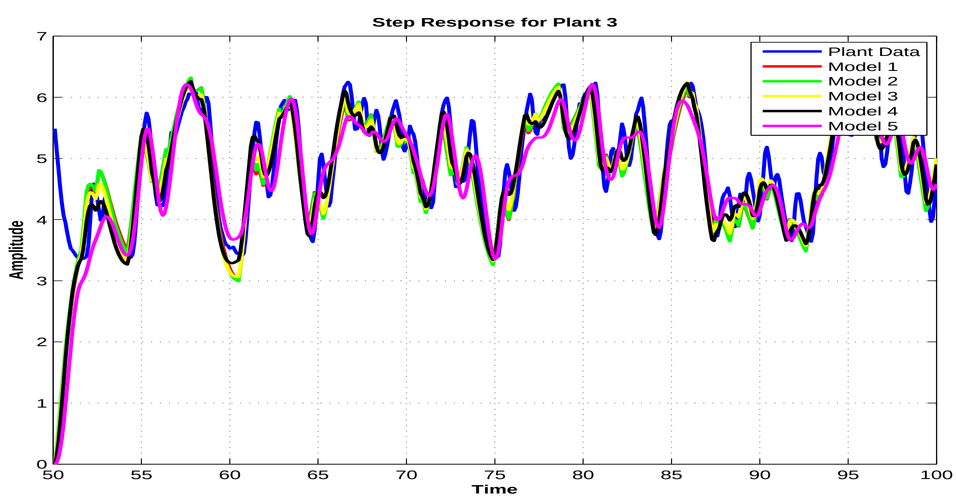
Fig. 10: Step response for plant 6.



(a) Plant 1

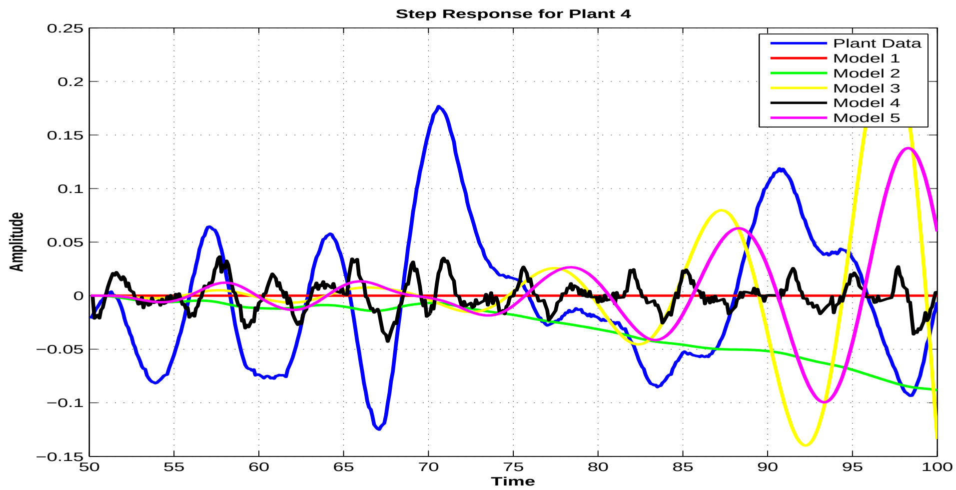


(b) Plant 2

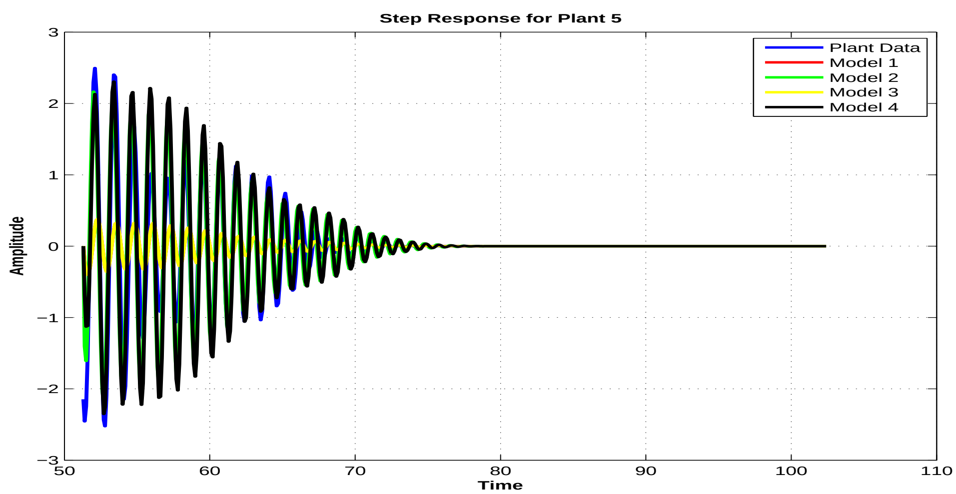


(c) Plant 3

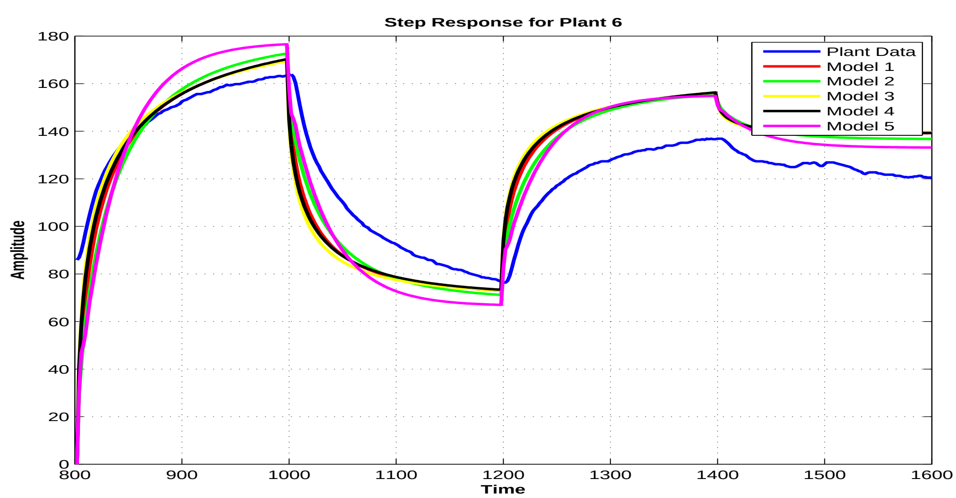
Fig. 11: Model validations for plant 1 to plant 3.



(a) Plant 4



(b) Plant 5



(c) Plant 6

Fig. 12: (continued) Model validations for plant 4 to plant 6.

Table 7: Results for integer model structure 5.

Plant	b_0	b_1	a_3	a_2	a_1	a_0	*MSE*	*NMSE*
P_1	0.1824	0.1169	1.0268	3.7431	3.0562	1.1795	1.2784e-04	0.0847
P_2	0.0565	0.2959	1.7065	2.0587	1.2499	0.2953	6.4765e-05	0.0011
P_3	9.9057	1.0238	1.3453	10.9092	10.8039	1.0626	1.9484	1.2793
P_4	0.0290	0.3449	1.3443	1.6814	0.3304	0.6289	0.0048	5.6742
P_5	12.8342	19.6285	1.4254e-04	3.7377	0.4532	3.8767	0.9667	1.7431
P_6	5.1563	0.9057	0.3425	0.0641	1.5008	0.0408	345.5185	0.1251

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