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On A New Exponentiated Error Innovation Distributions: Evidence of Nigeria Stock Exchange

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Abstract: This paper compared new error innovation distribution in estimating volatility models. A new error innovation distribution called Exponentiated Generalized skewed student t distribution (EGSSTD) was developed and compared with the existing error distributions using an empirical dataset of daily returns of Nigeria Stock Exchange (NSE) index return from 2007 to 2017. The result of the stationarity Statistic shows that the data is stationary without transformation while the ARCH effect statistic using ADF statistic shows the presence of heteroscedasticity. The estimate of the volatility models were significant with probability values at 0.01 for the new error distribution and the existing distributions. The results obtained show that GARCH (1, 1) with EGSSTD error distribution performed better than the other models having the least AIC value. In terms of forecasting performance, GARCH (1, 1) with ESSTD error distribution outperformed other volatility models and error distributions with the least RSME.

Keywords: Volatility models, error distributions, exponentiated, stocks and ARCH effect

1 Introduction

Error distribution is one of the vital technique in estimating the parameters of any volatility models and hence Engle (1982) proposed the use of the normal distribution as an error distribution in estimating his proposed volatility model. This error distribution has gained more ground in the estimation of the volatility models, followed by the student t distribution proposed by Bollerslev (1987). However, due to the significant role played by the error distribution in estimating the parameters of heteroscedastic models (Su, 2010), six forms of error distributions have gained popularity in volatility modelling (Bali, 2007; Shamiri and Isa, 2009). These include normal distribution, Skewed normal distribution, student-t distribution, Skewed student –t distribution, generalized error distribution and Skewed generalized error distribution.
With the limitation found on the existing error innovation distributions especially in their inability to capture extreme values, heavy tailed etc., prompted us to propose a new error distribution from the distribution derived by Dikko and Agboola (2017b) called the Exponentiated Generalized Skewed Student t distribution (EGSSTD) to model some volatility models and estimate their parameters using this proposed error distribution and to compare it with existing error distributions in terms of fitness and forecasting evaluation using Nigeria Stock Exchange (NSE) index returns daily dataset. However, this paper tend proposed and check the significant of additional parameter to error innovation distribution in term of best fit over others error innovation distributions.

2 Literature review

Engle (1982) was the first to propose volatility models by coming up with heteroscedasticity model called the Autoregressive Conditional Heteroscedasticity (ARCH) model and Lamoureux and Lastrapes (1990) applied the ARCH model to a daily trading volume of stock market returns. Bollerslev (1986) proposed the Generalized ARCH model and Nelson (1991) proposed the Exponential GARCH models (EGARCH) to capture the leverage effect of financial time

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series. Agboola *et al.*, (2015) also worked on principle of parsimony in modelling time series using heteroscedasticity models with parsimonious volatility of ARCH, GARCH, EGARCH, and TGARCH and Power ARCH (PARCH) models. Dikko *et al.*, (2015) worked on modelling abrupt shift in time series using indicator variable on twelve volatility models
The ARCH model parameter were estimated using the normal error distribution proposed by Engle (1982) for volatility models and the model of GARCH proposed by Bollerslev (1986) also adopted this normal distribution for the error innovation and used the distribution to estimate volatility models. However, it was found that the error term being used does not follow a normal distribution and this led Bollerslev (1987) to use the student t distribution in the estimation of volatility models. Generalized Error Distribution (GED) was proposed for error innovation by Nelson (1991) in estimating EGARCH model. Skewed normal distribution was introduced by O'Hagan and Leonard (1976) to capture the Skewedness of the normal distribution. The skewed student t distribution of the error innovation was proposed by Hansen (1994). Skewed generalized error distribution was first used for error innovation by Theodossiou (1998) by adding a skewed parameter for the generalized error distribution to capture the skewedness of the GED.
Alberg *et al.*, (2006) also worked on the forecasting performance of four parsimony heteroscedasticity models such as GARCH (1, 1), EGARCH (1, 1), GJR-GRACH (1,1) and Asymmetric Power GARCH (APGARCH (1,1)) models with three error distributions; normal, student-t and Skewed student-t. The results showed that EGARCH (1, 1) models with Skewed student-t gave better results compared to the other models and distributions.
Shamiri and Isa (2009) also worked on modelling and forecasting of the Malaysian stock market by comparing three different types of GARCH models with six (3) error distributions. The GARCH models are the symmetric GARCH (1, 1), EGARCH (1, 1) and Non-linear GARCH (1, 1) model while the error distributions were normal, Skewed normal, Student-t, Skewed student-t, and generalized error distributions. In terms of models, EGARCH models performed better in volatility forecasting than other models while in error distributions, non-normal distributions outperformed the normal distribution.

3 Methodology

3.1 Computation of return series from price

Let R_{st} =ln $\left(\frac{P_t}{P_{t-1}}\right)$ **(??)**
where$P_t$and P_{t-1} are the present and previous closing prices and R_{st} the continuously compounded return series which is the natural logarithm of the simple gross return.

3.1.1 Stationary Test

Stationarity of the return series of the Augmented Dickey–Fuller (ADF) test is given as:
Let $x_t = \phi_1 x_{t-1}$ **(??)**

$$x_t - x_{t-1} = \phi_1 x_t - x_{t-1}$$

$$\Delta x_t = (\phi_i - 1)x_{t-1}$$

Series is stationary if there is no change
$\Rightarrow \phi_1 - 1 = 0$or $\phi_1 = 1$
Null hypothesis is $H_0 : \phi_1 = 1$
and alternative hypothesis is : $H_1 : \varphi_1 \neq 1$
The Test Statistic (t-ratio):$= \frac{\phi_1^n - 1}{std(\phi_1)} = \frac{\sum_{t=1}^T P_{t-1}e_t}{\sigma^2\sqrt{\sum_{t=1}^T P_{t-1}^2}}$ **(??)**
where$\phi_1 = \frac{\sum_{t=1}^T p_{t-1}p_t}{\sum_{t=1}^T p_{t-1}^2}and\hat{\sigma}^2 = \frac{\sum_{t=1}^T (p_t - \hat{\phi}_1 p_{t-1})^2}{T-1}$
$P_0 = 0$, T is the sample size and $\phi_1$for each Insurance stock.
The null hypothesis is rejected if the calculated value of t is greater than t critical value (Agboola, 015).

3.1.2 Test for ARCH effect

$$r_t = \phi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \tag{1}$$

after obtaining the residualse_t, the next step is regress the squared residual on a constant and its q lags as in the following equation:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + ... + \alpha_q e_{t-q}^2 + r_t \quad (2)$$

The null hypothesis, that there is no ARCH effect up to order q can be formulated as:

$$H_0 : \alpha_1 = ... = \alpha_q = 0 \quad (3)$$

against the alternative
$H_a : \alpha_i \neq 0$ for some i$\in \{1,..,m\}$ **(??)**

3.1.3 Some Volatility models

The ARCH (q) model proposed by Engle (1982) formulates volatility as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_q \varepsilon_{t-q}^2 + r_t \quad (4)$$

where $\alpha_i > 0$, for i=0, 1, 2... q are the parameters of the models
The GARCH (p, q) model was stated as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \varepsilon_{t-1}^2 + ... + \beta_p \varepsilon_{t-p}^2 + r_t \quad (5)$$

where $\alpha_i > 0$and$\beta_i > 0$ for all i and j
The EGARCH (p, q) model was proposed by Nelson (1991) formulate the volatility as follows: $\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left[\lambda \varepsilon_{t-i} + \gamma \left\{ \left|\varepsilon_{t-i}\right| - \sqrt{\frac{2}{\pi}} \right\} \right] + \sum_i^q \beta_j \ln(\sigma_{t-j}^2)$ **(??)**
$\alpha_0, \alpha_i, \gamma, \beta_j$ are the parameters of the model.
Threshold Generalized Autoregressive Conditional Heteroscedasticity (TGARCH) model
The Threshold GARCH model is similar to GJR-GARCH of Glosten *et al.*, (1993) stated as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i + \gamma_i N_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}{}^2 \quad (6)$$

$$\alpha_0, \alpha_i, \gamma_i, \beta_j \geq 0$$

where, N_{t-i} is an indicator for negative ε_{t-i} that is N_{t-i} is 1 if $\varepsilon_{t-i} < 0$ and 0

3.1.4 Model Selection

Akaike Information Criteria (AIC) is the most commonly used model selection criteria
$\text{AIC} = 2K - 2\ln(LL)$ **(??)**
where K is the number of parameters in the model and LL is the Log-Likelihood Function for the model.

3.1.5 Forecasting Evaluation

Evaluating the performance of different forecasting models plays a very important role in choosing the most accurate models. The most widely used evaluation measure is Root Mean Square Error (RMSE) given as:
RMSE $= \sqrt{\frac{\sum_{t=T+1}^{T+n} \left(\sigma^2{}_t - \hat{\sigma}^2{}_t\right)^2}{n}}$ **(??)**
where, n is the number of steps ahead, T is the sample size, $\hat{\sigma}_t$and σ_t are the square root of the conditional forecasted volatility and the realized volatility respectively.

3.1.6 Exponentiated Skewed student-t Distribution (ESSTD)

Dikko and Agboola (2017a) derived the Exponentiated skewed student t distribution as:
$z_t \sim ESSTD(u, \lambda)$,its PDF is given by;

$$g(x) = u\left\{\frac{1}{2}\left(1+\frac{x}{\sqrt{\lambda+x^2}}\right)\right\}^{u-1}\frac{\lambda}{2(\lambda+x^2)^{\frac{3}{2}}} \tag{7}$$

where $u > 0, \lambda > 0$ and
λ, is Skewed parameter and u is a shape parameters
where the error distribution of the Exponentiated Skewed student-t distribution is given as if the transformation function is given as: $\varepsilon_t = z_t\sigma_t$

$$g(\varepsilon_t, \alpha, \lambda) = \alpha\left\{\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right\}^{\alpha-1}\frac{\lambda}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{(\sigma_t^2)^{\frac{1}{2}}}\right) \tag{8}$$

Log-likelihood function of error innovation of Exponentiated Skewed student-t distribution

$$L(\theta) = \prod_{t=1}^{n} g(\varepsilon_t, \alpha, \lambda, \sigma_t) = L(\varepsilon_t, \alpha, \lambda, \sigma_t) = \prod_{t=1}^{n} g(\varepsilon_t;\theta)$$

where $\theta = (\alpha, \lambda, \sigma_t)$

$$= \prod_{t=1}^{n}\left[\alpha\left\{\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right\}^{\alpha-1}\frac{\lambda}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{(\sigma_t^2)^{\frac{1}{2}}}\right)\right] \tag{9}$$

$$= \alpha^n\lambda^n\prod_{t=1}^{n}\left[\left\{\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right\}^{\alpha-1}\frac{1}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{(\sigma_t^2)^{\frac{1}{2}}}\right)\right] \tag{10}$$

Taking the log likelihood function of the above equation

$$\begin{aligned} LL(\varepsilon_t;\theta) &= n\log(\alpha) + n\log(\lambda) - n\alpha\log 2 + \\ &(\alpha-1)\textstyle\sum_{t=1}^{n}\log\left(1+\frac{\varepsilon_t}{\sigma_t\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right) - \frac{3}{2}\sum_{t=1}^{n}\log\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right) - 0.5\times n\times\log\left(\sigma_t^2\right) \end{aligned} \tag{11}$$

where $u$is a shape parameter, λ is a skewed parameter and σ_t^2 are the specified volatility models with a vector parameters $\phi = (\omega, \alpha_1, \alpha_2, ..., \alpha_i, \beta_1, \beta_2, ..., \beta_j, \gamma, \delta)$.

4 Exponentiated Generalized Skewed student-t distribution

Dikko and Agboola (2017b) derived the Exponentiated Generalized skewed student t distribution as:

$$g(z_t, \alpha, v, \lambda) = \alpha v\left[1-\frac{1}{2}\left(1+\frac{z_t}{\sqrt{\lambda+z_t{}^2}}\right)\right]^{\alpha-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{z_t}{\sqrt{\lambda+z_t{}^2}}\right)\right]^{\alpha}\right\}^{v-1}\frac{\lambda}{2(\lambda+z_t{}^2)^{\frac{3}{2}}} \tag{12}$$

if the transformation function is given as :$\varepsilon_t = z_t\sigma_t$

$$g(z_t,\alpha,v,\lambda) = \alpha v\left[1-\tfrac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha-1} \left\{1-\left[1-\tfrac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha}\right\}^{v-1} \frac{\lambda}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{(\sigma_t^2)^{\frac{1}{2}}}\right) \tag{13}$$

Log-likelihood function of error innovation of Exponentiated Generalized Skewed student-t distribution

$$L(\theta) = \prod_{t=1}^{n} g(\varepsilon_t,\alpha,v,\lambda,\sigma_t) = L(\varepsilon_t,\alpha,v,\lambda,\sigma_t) = \prod_{t=1}^{n} g(\varepsilon_t;\theta) \tag{14}$$

where $\theta = (\alpha,v,\lambda,\sigma_t)$

$$= \prod_{t=1}^{n}\left[\alpha v\left[1-\tfrac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha-1} \left\{1-\left[1-\tfrac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha}\right\}^{v-1} \frac{\lambda}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{(\sigma_t^2)^{\frac{1}{2}}}\right)\right] \tag{15}$$

$$= \alpha^n v^n \lambda^n \prod_{t=1}^{n}\left[\left[1-\tfrac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha-1} \left\{1-\left[1-\tfrac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha}\right\}^{v-1} \frac{1}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{(\sigma_t^2)^{\frac{1}{2}}}\right)\right] \tag{16}$$

Taking the log likelihood function of the above equation

$$\begin{aligned} LL(\varepsilon_t;u,v,\lambda) &= n\log(\alpha)+n\log(v)+n\log(\lambda)-n\log(2) \\ &+(\alpha-1)\textstyle\sum_{t=1}^{n}\log\left(1-\tfrac{1}{2}\left(1+\frac{\varepsilon_t}{\sigma_t\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right) \\ &+(v-1)\textstyle\sum_{t=1}^{n}\log\left\{1-\left[1-\tfrac{1}{2}\left(1+\frac{\varepsilon_t}{\sigma_t\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{u}\right\} \\ &-\tfrac{3}{2}\textstyle\sum_{t=1}^{n}\log\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)-0.5\times n\times\log\left(\sigma_t^2\right) \end{aligned} \tag{17}$$

where $u,v$are shape parameters, λ is a skewed parameter and σ_t^2 are the specified volatility models with a vector parameters $\phi = (\omega,\alpha_1,\alpha_2,...,\alpha_i,\beta_1,\beta_2,...,\beta_j,\gamma,\delta)$.

4.0.7 Standardized Skewed student t-distribution

$$f(z_t,\mu,\sigma,v,\lambda) = \begin{cases} bc\left[1+\frac{1}{v-2}\left(\frac{b\left(\frac{z_t-\mu}{\sigma}\right)+a}{1-\lambda}\right)^2\right]^{-\frac{v+1}{2}} & ,z_t<-\frac{a}{b} \\ bc\left[1+\frac{1}{v-2}\left(\frac{b\left(\frac{z_t-\mu}{\sigma}\right)+a}{1+\lambda}\right)^2\right]^{-\frac{v+1}{2}} & ,z_t\geq-\frac{a}{b} \end{cases} \tag{18}$$

where v is the shape parameter with $2 < v < \infty$and λ is the Skewedness parameters with$-1 < \lambda < 1$, μ and σ^2 are the mean and variance of the Skewed student t-distribution.

$$a = 4\lambda c\left(\frac{v-2}{v-1}\right), b = 1+3\lambda^2 - a^2, c = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)\Gamma\left(\frac{V}{2}\right)}}$$

1. (a)**Standardized Skewed Generalized Error Distribution**

$$f(z_t/v,\varepsilon,\theta,\delta) = \frac{v}{2\theta\Gamma\left(\frac{1}{v}\right)}\exp\left[-\frac{|z_t-\delta|^v}{[1+sign(z_t-\delta)\varepsilon]^v\theta^v}\right] \tag{19}$$

$$\theta > 0, -\infty < z_t < \infty, v > 0, -1 < \varepsilon < 1, -\infty < z_t < \infty$$

where

$$\theta = \Gamma\left(\frac{1}{v}\right)^{0.5}\Gamma\left(\frac{3}{V}\right)^{-0.5}S(\varepsilon)^{-1},$$

$$\delta = 2\varepsilon S(\varepsilon)^{-1},$$

$$S(\varepsilon) = \sqrt{1+3\varepsilon^2-4A^2\varepsilon^2},$$

$$A = \Gamma\left(\frac{2}{v}\right)\Gamma\left(\frac{1}{v}\right)^{-0.5}\Gamma\left(\frac{3}{v}\right)^{-0.5}$$

where $v > 0$ is the shape parameter, ε is a Skewedness parameter with$-1 < \varepsilon < 1$.

1. (a)**Skewed Normal Distribution**

$$f(z_t) = \frac{1}{\sigma\pi}e^{-\frac{(z_t-\varepsilon)^2}{2\sigma^2}}\int_{-\infty}^{\alpha}\frac{z_t-\varepsilon}{\sigma}e^{-\frac{t^2}{2}}dt, -\infty < z_t < \alpha \tag{20}$$

where ε is the location, σ is the scale and α denotes the shape parameter.

1.**Results**

4.1 Empirical results

An empirical analysis of the NSE index returns was carried out on returns series. The obtained results as shown in Table 1 showed that the mean return series were positive, positive Skewed and high kurtosis for NSE index returns. The result of Jarque-Bera statistic revealed that the return series for NSE index returns is not normally distributed as the p-values were less than 1%.

Table 1: Descriptive Statistics of Nigeria Stock Exchange (NSE) index returns

Mean	0.0001
Std. Dev.	0.1283
Skewness	0.018767
Kurtosis	51.693
Jarque-Bera	2.6666e+05
Probability	0.001
Observations	3190

4.2 Stationary test

A test of stationarity was carried out using the Augmented Dickey- Fuller Test. The results obtained for NSE index returns showed that the Augmented Dickey- Fuller test statistic were all less than their critical values at 1% as shown in Table 2. Hence, there is no unit root. The return series were all stationary. Therefore, there is no need for transformation.

Table 2: Augmented Dickey-Fuller Test of stationarity (ADF) test of NSE index returns

Stocks	ADF Test Statistic	Comment
NSE	-23.62148	Stationary at level without transformation

1% critical = -3.432219

4.3 ARCH Effect Test

The results of F Statistic were significant at 1% for the NSE index returns which shows at different lag values that there is ARCH effect using Lagrange Multiplier test in Table 3 below.

Table 3: Lagrange Multiplier test of the presence of ARCH effect

	ARCH EFFECT	F-Statistic	p-value
NSE Returns	At lag 1-2	788.44	0.001
	At lag 1-5	443.82	0.001
	At lag 1-10	250.92	0.001

4.4 Estimates of the parameters of GARCH models and its extension based on Nigeria Stock Exchange (NSE) Index Returns

Table 4 present the parameter estimates of volatility models estimated at five (2) error distributions namely Skewed normal (SNORM), Skewed Student- distribution (SSTD) and Skewed generalized error distribution (SGED), Exponentiated skewed student t (ESSTD) and the proposed Exponentiated Generalized Skewed Student t distribution (EGSSTD) using returns from NSE. The result shows that the returns exhibit volatility clustering. This was concluded because the GARCH term was significant in most of the models considered ($p<0.05$) and ($p<0.01$) which means that small changes in volatility of returns tends to be followed by large changes in volatility while small changes in volatility tends to be followed by small changes in volatility. In terms of leverage effect which measures whether there is a negative relationship between asset returns and volatility, it was found to be significant in GARCH, GJR-GARCH, EGARCH, TGARCH and APARCH models estimated at the five (2) distributions of error innovation ($p<0.05$).

Table 4: Estimates of the parameters of GARCH models and its extension based on Nigeria Stock Exchange (NSE) index returns using new classes of error distributions

Model	**Error Distributions**	ω	α_1	β_1	γ_1	δ	Skew	Shape	Shape (u)	Shape (v)
GARCH (1,1)	SSTD	5.674 x10^{-10}	3.749 x 10^{-01} ***	1.00 x 10^{-08} **			1.2460**	2.000***		
	SNORM	7.875 x10^{-03}*	1.952 x10^{-01} ***	6.864 x10^{-01}***			1.078**			
	SGED	2.091 x 10^{-06}***	1.259 x 10^{-01}***	8.624 x 10^{-01}***			1.052***	1.173***		
	ESSTD	0.16676	-0.24515	1.11683			0.00596		8.5634	
	EGSSTD	0.3524	-0.1544	0.1211			1.1310		1.0001	4.4720
GJR-GARCH (1,1)	SSTD	0.0000	0.3923	0.0412	0.8992		1.1428	2.0173**		
	SNORM	0.000006	0.1054***	0.9104***	-0.0577**		0.9710**			
	SGED	0.0000001**	0.0500**	0.9000**	0.4569**		1.0000**	2.0000**		
	ESSTD	0.29080	0.0081	0.00008	-0.87643		0.4571		9.8900**	
	EGSSTD	-0.26038	1.76415	-1,76416	-1.11784		0.09977		0.35710	5.35710
EGARCH (1,1)	SSTD	- 9.9999**	0.00008**	0.673004**	0.000081**		0.9208**	2.0100**		
	SNORM	- 1.3039**	-0.0323**	0.8153**	0.3605**		1.0718**			
	SGED	- 1.1784**	0.0078**	0.8962**	0.0185**		1.0001**	0.2323**		
	ESSTD	0.094465	0.338697	0.26803	0.00105		0.04258		1.99821*	
	EGSSTD	0.0999	-0.01834	0.11887	1.16372		0.10659		2.6023	4.71788
TGARCH (1,1)	SSTD	0.0000	0.13448**	0.06967*			1.03096	2.01715*		
	SNORM	0.000004	0.0743**	0.9213*			0.9526			
	SGED	0.00001	0.0500	0.9000			1.0000	2.0000		
	ESSTD	4.95819	-1.11642	3.90917			5.56328		4.60107**	
	EGSSTD	0.8531	0.8356	0.5472	-0.2327		1.0000		4.8610	2.0250
APARCH (1,1)	SSTD	0.0000	0.1348**	0.16862*	-0.91683	2.000***	1.0075**	2.01706*		
	SNORM	0.000006	0.0801**	0.9072	-0.2285	2.000**	0.9787			
	SGED	0.000001*	0.0500	0.9000	0.0500	2.000**	1.0000	2.0000		
	ESSTD	0.05931	1.6709	0.1976	-1.646	-1.9872	0.2794		4.2030	
	EGSSTD	1.4640	1.1761	0.1885	-0.0896	2.000**	1.0000		1.7910	4.6540

* at 5%, ** at 1% and *** at 10% significant

4.5 Fitness and Model selection of GARCH models and its extension based on Nigeria Stock Exchange (NSE) index returns

The performance of GARCH model and its extension estimated at four (1) error distributions namely; Exponentiated skewed student t , skewed student-t distribution, skewed normal and skewed Generalized error distribution were compared with that of the proposed distributions. Table 5 shows the result of the fitness and model selection based on log- likelihood and Akaike Information Criteria (AIC) of GARCH, GJR-GARCH, EGARCH, TGARCH and APARCH models. The EGSSTD error distribution outperformed better on GARCH (1,1), GJR-GARCH (1,1), EGARCH (1,1) and TARCH (1,1) while ESSTD outperformed other error distribution on APARCH (1,1) model. The Exponentiated Generalized Skewed Student-t error distribution (EGSSTD) was found to outperformed other error distributions except for ESSTD for APARCH model and as well the four volatility models as revealed by its largest log- likelihood and least value of Akaike Information Criteria (AIC) for the NSE index returns while ESSTD outperformed SSTD, SNORM, SGED and EGSSTD error distribution on APARCH model. Furthermore, the results of the AIC based on models selection shows that TARCH (1,1) with EGSSTD performed better than the other models with the least AIC value.

Table 5: Shows the result of the fitness and model selection based on log- likelihood and Akaike Information Criteria (AIC) of Nigeria Stock Exchange (NSE) index Returns

Model	**Error Distributions**	**LL**	**AIC**
GARCH (1,1)	SSTD	12848.89	-10.724
	SNORM	5830.812	-4.8650
	SGED	8476.708	-6.4909
	ESSTD	166000.899	-36.03949
	EGSSTD	**120000.1230**	**-37.39049**
GJR-GARCH (1,1)	SSTD	12412.8	-10.360
	SNORM	6023.987	-5.0255
	SGED	4617.117	-3.8498
	ESSTD	411000.8465	-37.85270
	EGSSTD	**163000.2064**	**-38.0030**
EGARCH (1,1)	SSTD	10765.72	-8.9843
	SNORM	5853.97	-4.8835
	SGED	14583.32	-7.1691
	ESSTD	45000.3303	-33.42885
	EGSSTD	**83000.1079**	**-36.65319**
TGARCH (1,1)	SSTD	12087.42	-10.089
	SNORM	6004.817	-5.0103
	SGED	4038.42	-3.3674
	ESSTD	29200.5547	-32.56388
	EGSSTD	**160000.22621**	**-39.96586**
APARCH (1,1)	SSTD	12470.73	-10.408
	SNORM	6026.297	-5.0297
	SGED	4041.617	-3.3692
	ESSTD	**408100.134**	**-37.8385**
	EGSSTD	120000.32182	-37.39049

* at 5%, ** at 1% and *** at 10% significant

5 Bolded values are the highest value of likelihood function and the least value of AIC

AIC=2p-2*Ln(LL):- Ln is the natural logarithm

5.1 Forecasting Evaluation performance of estimated GARCH model and its extension on Nigeria Stock Exchange (NSE)

Table 6 shows the forecasting performance of the estimated models using Root Mean Square Error (RMSE). Model with the smallest RMSE was considered to be most suitable for forecasting performance of GARCH model and its extension. Hence, forecasting performance of the models, show that GARCH (1, 1) with ESSTD outperformed other volatility models because it has the smallest RMSE value.

Table 6: Forecasting Evaluation of GARCH models and its extension based on Nigeria Stock Exchange (NSE) index returns

Model	**Error**	**RMSE**
GARCH (1,1)	SSTD	0.0710
	SNORM	5.7584
	SGED	0.4135
	ESSTD	**0.0006**
	EGSSTD	0.00486
GJR-GARCH (1,1)	SSTD	0.071097
	SNORM	1.009956
	SGED	0.05132
	ESSTD	**0.035432**
	EGSSTD	0.423558
EGARCH (1,1)	SSTD	0.07109
	SNORM	4.2396
	SGED	**0.00101**
	ESSTD	0.00600
	EGSSTD	0.008407
TGARCH (1,1)	SSTD	0.07084
	SNORM	6.8924
	SGED	0.0708
	ESSTD	**0.00451**
	EGSSTD	0.00512
APARCH (1,1)	SSTD	0.071097
	SNORM	6.9715
	SGED	2.6000
	ESSTD	**0.04653**
	EGSSTD	0.171824

RMSE- Root Mean Square Error, Bolded values are the least Root Mean Square Error (RMSE)

6 Perspective

In the article, we are able to compare various error innovation distributions with new error distribution in estimating the parameters of some volatility models. The empirical result shows a positive returns in the mean, high kurtosis and positive skewness and the ADF test validate the result of the Jarque-Bera Statistic that the return is stationary at level stated without transformation. The data also shows evidence of ARCH effect and the parameters estimation shows most coefficients in the models were significant at 0.01. From the results obtained shows on model selection using the AIC, TARCH (1, 1) with EGSSTD performed better than the other models with the least AIC value. And based on models forecasting performance, GARCH (1, 1) outperformed with ESSTD error distribution outperformed other error distributions. From this article, the new error distribution proposed EGSSTD outperformed others error distributions with that of the existing error distributions. In this paper, it was discovered the effect of shape parameter to an error innovation distribution. The proposed EGSSTD has one additional parameter to that of ESSTD and from the results of best fit using the statistic of AIC values, it shows the significant of the additional parameter of EGSSTD error innovation distribution over ESSTD error innovation distribution and the existing error distributions.

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