Quantum Physics Letters
An International Journal

http://dx.doi.org/10.18576/qpl/070104

Theoretical Study of Dust Acoustic Solitary Waves Interaction in a Strongly Coupled Dusty Plasma with Nonextensive Electrons and Ions

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Received: 7 Jun. 2017, Revised: 21 Feb. 2018, Accepted: 28 Feb. 2018
Published online: 1 Apr. 2018

Abstract: The head-on collision between two dust acoustic solitary waves in an unmagnetized strongly coupled dusty plasma with dust grains of negative charge and nonextensive ions and electrons are studied through the extended Poincaré-Lighthill-Kuo approach. Two Korteweg-de Vries equations are derived and accordingly two solitary wave solutions are obtained. In addition, an analytical expression for the phase shift due to the collision is derived. The nonextensivity effect of both ions and electrons on the characteristics of the head-on collision and the resulting phase shift due to the collision is studied. It is found that the characteristics of the head-on collision and the resulting phase shift strongly depend on the nonextensive parameter, the ratio of ion to electron densities as well as the ratio of ion to electron temperatures. The obtained results from this study can be used to understand the solitary waves interaction that may occur in plasma with dust impurities situations.

Keywords: Dusty Plasma; Solitary Waves Interaction; Poincaré-Lighthill-Kuo Perturbation Technique; Korteweg-de Vries Equation; Nonextensive Distribution

1 Introduction

In the last few years, the study of the characteristics of a strongly coupled dusty (complex) plasma had a great deal of interest because of its importance in industrial plasma applications, in laboratory plasmas as well as astrophysical plasmas. In a strongly coupled dusty plasma, the inertia is produced by the mass of the charged dust grains while the restoring force is produced by the pressures of the inertialess plasma particles (ions and electrons) [1]. The existence of the inertial dust particles in plasma can excite new collective modes (e.g., dust acoustic (DA) waves, dust ion acoustic (DIA) waves, etc) and nonlinear coherent structures (e.g., DA and DIA solitary waves) in the dusty plasma medium [2] and [3].

Many researchers are studied the properties and characteristics of DA solitary and shock waves in a strongly dusty plasma. For instance, Alinejad and Mamun [4] studied the nonlinear characteristics of the DA solitary waves in an inhomogeneous strongly coupled dusty plasma with negatively charged dust particles are correlated strongly with each other and Maxwellian electrons and ions. They are found that the dark solitary waves only are propagated in such dusty plasma medium. Also, the properties of DA shock waves in a strongly coupled unmagnetized dusty plasma with charged dust particles as well as Boltzmann Distributed ions and electrons has been studied by Shukla and Mamun [5]. Most of these studies on DA Solitary waves are concerned with the Maxwellian distributions where the microscopic interactions and memories are short ranged. But in the systems where the long range interactions are existed, the nonextensive distribution is more convenient than the Maxwellian distribution. It was first proposed by Renyi [6] and then by Tsallis [7] and observed in many astrophysics and cosmological environments [8].

On the other hand, the interaction between two solitary waves is one of the most important problems in plasma physics. Such interaction may undergo different two ways. The first one is the overtaking collision which occurs when the two solitary waves move in the same direction and can be studied by using the inverse scattering method [9]. The second one is the head-on collision which occurs when the two solitary waves move

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in opposite directions causing a change in the trajectories of motion and phase shifts after the collision. The head-on collision can be studied by using the extended Poincare-Lighthill-Kuo (PLK) method [10]. This method has many applications in physics such as in plasma physics [11], in Boes-Einstein condensates [12] and in nonlinear lattice dynamics [13]. Many authors used this method to study the solitary waves interaction in different dusty plasma systems. For example, El-Labany et al. [14] studied the properties of the head-on collision of two colliding DA solitary waves in an adiabatic dusty plasma with variable dust particles of negative charged, two temperature Maxwellian ions and Maxwellian electrons in the existence of an external magnetic field. Also, Jaiswal et al. [15] investigated the propagation properties of two interacting DA solitary waves in a strongly coupled dusty plasma with charged dust grains and thermal ions and electrons. In their paper, they are found that the resulting phase shift due to the collision is affected by the compressibility of the strongly coupled dusty plasma medium.

The aim of this study is to investigate the effect of nonextensive electrons and ions on the nonlinear properties of the DA solitary waves interaction (via the head-on collision between them) in a strongly coupled unmagnetized dusty plasma with negatively charged dust particles. For this purpose, the extended PLK method has been used and a couple of Korteweg de-Vries equations are derived. In addition, expressions for the trajectories and the resulting phase shifts of the colliding DA solitary waves are deduced and the effect of the nonextensivity of ions and electrons on the phase shifts and on the characteristics of head-on collision is also discussed. The manuscript is organized as follows. In Sec. 2, the model equations governing our strongly coupled dusty plasma system are presented. In Sec. 3, two Korteweg–de Vries (KdV) equations and phase shifts are derived while our results are discussed in section 4 and a brief conclusion is given in Sec. 5.

2 Basic Equations

In order to study the interaction between two DA solitary waves (the case of head-on collision), we consider an unmagnetized strongly coupled dusty plasma system with negatively charged inertial dust fluid and inertialess q-nonextensive distributed electrons and ions. Due to high temperatures and smaller electric charges of both electrons and ions, they are assumed to be weakly coupled while dust grains are strongly coupled as a result of their lower temperature and larger electric charge. In general, strong coupling effects in a phenomenological manner is takes into account by introducing visco-elastic effects and a modified compressibility. Here, we retain the compressibility effect and neglect dissipative effects arising from viscosity and dust neutral collisions. The neglect of dissipative effects is a valid approximation in the so called "kinetic regime" when $\omega\tau_m >> 1$, where ω is the mode frequency and τ_m is the relaxation time. Thus, the dynamics of the nonlinear DA waves in the given dusty plasma system are governed by the well-known generalized hydrodynamic model [16]

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d\, u_d)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} - \frac{\partial \phi}{\partial x} + \frac{\mu_d}{n_d}\frac{\partial n_d}{\partial x} = 0, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} - n_d - n_e + n_i = 0, \tag{3}$$

where n_d is the dust grain number density, u_d is the dust fluid velocity, ϕ is the electrostatic potential, n_e is the electron number density and n_i is the ion number density. The following normalization $x \to x/\lambda_{Dd}$, $t \to t\ \omega_{pd}$, $n_d \to n_d/n_{d0}$, $u_d \to u_d/C_d$, $\phi \to e\phi/K_B T_i$, $n_e \to n_e/Z_d n_{d0}$ and $n_i \to n_i/Z_d n_{d0}$ have been applied into Eqs.(1-3). $\lambda_{Dd} = \left(K_B T_i/4\pi Z_d n_{d0} e^2\right)^{1/2}$ is the dust Debye length with K_B, n_{d0} and e being the Boltzmann constant, the unperturbed dust grain number density and the magnitude of the electron charge, respectively. $\omega_{pd} = \left(4\pi Z_d^2 n_{d0} e^2/m_d\right)^{1/2}$ is the dust plasma frequency with m_d being the dust grain mass and $C_d = (K_B T_i Z_d/m_d)^{1/2}$ is the DA speed. The contribution due to the compressibility μ appears in terms of μ_d in Eq. 3 where $\mu_d = \mu T_d/Z_d T_i$. T_d, T_i and Z_d refer to the dust temperature, the ion temperature and the number of electrons residing on the surface of the negatively charged dust grains, respectively. The compressibility μ, [16] is defined as

$$\mu = 1 + \frac{u(\Gamma)}{3} + \frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial \Gamma}, \tag{4}$$

in which Γ is the Coulomb coupling parameter and $u(\Gamma)$ is a measure of the excess internal energy of the system and can be expressed as $u(\Gamma) = -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81$.

The normalized number densities of q-nonextinsive distributed electrons and ions, [17] and [18] can be expressed as

$$n_e = \mu_e \left(1 + (q_e - 1)\,\sigma_i \phi\right)^{\frac{q_e+1}{2(q_e-1)}}, \tag{5}$$

$$n_i = \mu_i\ \left(1 - (q_i - 1)\,\phi\right)^{\frac{q_i+1}{2(q_i-1)}}, \tag{6}$$

where $\mu_e = n_{e0}/Z_d n_{d0}$ and $\mu_i = n_{i0}/Z_d n_{d0}$ and $\sigma_i = T_i/T_e$ with T_e is the temperature of electrons. By using the quasineutrality condition, $n_{eo} = n_{io} - Z_d n_{do}$, one can write $\mu_e = 1/(\delta - 1)$ and $\mu_i = \delta/(\delta - 1)$ where δ is the ratio of equilibrium ion to electron densities. The real number parameter $q_{e,i}$ stands for strength of nonextensivity. In the limiting case ($q_{e,i} \to 1$) Eqs. (5) and (6) reduces to the well known Maxwellian distribution.

3 KdV Equations and Phase Shifts

Now, let us assume that there are two opposite propagating solitary waves R and L which are far apart from each other in the initial states where one of them propagates in the positive x direction (solitary wave R) and the other is in the negative x direction (solitary wave L). After some time they interact and collide with each other and then depart. Also, we assume that the solitary waves have small amplitudes $\sim \varepsilon$ (where ε is a smallness formal perturbation parameter measuring the strength of nonlinearity, i.e., $0 < \varepsilon << 1$) and the interaction between two solitary waves is weak. Therefore, it is expected that the collision will be quasielastic and will only cause shifts of the post collision trajectories (phase shift). Here, we are interested to investigate the dynamics of these solitary waves in the presence of superthermality effect in the strongly coupled dusty plasma. In order to analyze the effects of collision, the extended PLK perturbation method is used [19]. Following the extended PLK perturbation method, the dependent variables should be expanded as

$$n_d = 1+\varepsilon^2 n_{d1}+\varepsilon^3 n_{d2}+\varepsilon^4 n_{d3}+..., \quad (7)$$
$$u_d = \varepsilon^2 u_{d1}+\varepsilon^3 u_{d2}+\varepsilon^4 u_{d3}+..., \quad (8)$$
$$\phi = \varepsilon^2\phi_1+\varepsilon^3\phi_2+\varepsilon^4\phi_3+..., \quad (9)$$

while the independent variables are expressed as

$$\xi = \varepsilon(x-\lambda t)+\varepsilon^2 P_0(\xi,\ \tau)+..., \quad (10)$$
$$\eta = \varepsilon(x+\lambda t)+\varepsilon^2 Q_0(\eta,\ \tau)+..., \quad (11)$$
$$\tau = \varepsilon^3 t, \quad (12)$$

where ξ and η refer to the trajectories of the two solitary waves R and L, respectively. The wave velocity λ and the variables P_0 and Q_0 are to be determined. From Eqs. (10-12), one gets

$$\frac{\partial}{\partial x} = \varepsilon\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)+\varepsilon^3\left(P_{0\eta}\frac{\partial}{\partial\xi}+Q_{0\xi}\frac{\partial}{\partial\eta}\right)+..., \quad (13)$$
$$\frac{\partial}{\partial t} = \varepsilon\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)+\varepsilon^3\left(\frac{\partial}{\partial\tau}+\lambda P_{0\eta}\frac{\partial}{\partial\xi}-\lambda Q_{0\xi}\frac{\partial}{\partial\eta}\right)+..., \quad (14)$$
$$\frac{\partial^2}{\partial x^2} = \varepsilon^2\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)^2+\varepsilon^4\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\left(P_{0\eta}\frac{\partial}{\partial\xi}+Q_{0\xi}\frac{\partial}{\partial\eta}\right)$$
$$+\varepsilon^4\left(P_{0\eta}\frac{\partial}{\partial\xi}+Q_{0\xi}\frac{\partial}{\partial\eta}\right)\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)+..., \quad (15)$$

where $P_{0\eta} = \partial P_0/\partial\eta$ and $Q_{0\xi} = \partial Q_0/\partial\xi$. Then, by substituting Eqs. (7-15) into Eqs. (1-6) and equating the quantities of equal power of ε, one gets a set of coupled equations for each order of ε. At the lowest order of ε, one obtains

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{d1}+\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)u_{d1} = 0, \quad (16)$$
$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)u_{d1}-\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\phi_1+\mu_d\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{d1} = 0, \quad (17)$$
$$n_{d1}+\frac{1}{2}\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]\phi_1 = 0. \quad (18)$$

Solving Eqs. (16-18) gives

$$\phi_1 = \Phi_1(\xi,\ \tau)+\Phi_2(\eta,\ \tau), \quad (19)$$
$$n_{d1} = -\frac{1}{2}\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]\left[\Phi_1(\xi,\ \tau)+\Phi_2(\eta,\ \tau)\right], \quad (20)$$
$$u_{d1} = -\frac{1}{\lambda}\left(1+\frac{1}{2}\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]\mu_d\right)$$
$$\times\left[\Phi_1(\xi,\ \tau)-\Phi_2(\eta,\ \tau)\right], \quad (21)$$

and with the solvability condition, i.e., the condition to obtain a uniquely defined n_{d1} and u_{d1} from Eqs. (16-18) when ϕ_1 is given by Eq. (19)], the phase velocity λ is found to be as

$$\lambda = \left[\frac{2}{\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)}+\mu_d\right]^{1/2} \quad (22)$$

It is clear that the phase velocity λ depends obviously on the nonextensive parameters q_e and q_i.

The unknown functions $\Phi_1(\xi,\ \tau)$ and $\Phi_2(\eta,\ \tau)$ in Eqs.(19-21) will be determined later from the higher orders. These two functions represent two solitary waves (one travels to the right, $\Phi_1(\xi,\ \tau)$, and the other travels to the left, $\Phi_2(\eta,\ \tau)$). At the next order, we obtain

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{d2}+\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)u_{d2} = 0, \quad (23)$$
$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)u_{d2}-\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\phi_2+\mu_d\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{d2} = 0, \quad (24)$$
$$n_{d2}+\frac{1}{2}\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]\phi_2 = 0. \quad (25)$$

By simply inspection, the structure of the above system of equations is similar to that of the lowest order, then the solutions can be written in the same manner as

$$\phi_2 = \Psi_1(\xi,\ \tau)+\Psi_2(\eta,\ \tau), \quad (26)$$
$$n_{d2} = -\frac{1}{2}\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]\left[\Psi_1(\xi,\ \tau)+\Psi_2(\eta,\ \tau)\right], \quad (27)$$
$$u_{d2} = -\frac{1}{\lambda}\left(1+\frac{1}{2}\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]\mu_d\right)$$
$$\times\left[\Psi_1(\xi,\ \tau)-\Psi_2(\eta,\ \tau)\right]. \quad (28)$$

The inclusion of the next higher order leads to the following

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{d3}+\left(\frac{\partial}{\partial\tau}+\lambda P_{0\eta}\frac{\partial}{\partial\xi}-\lambda Q_{0\xi}\frac{\partial}{\partial\eta}\right)n_{d1} +\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)u_{d1}+\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{d1}u_{d1} +\left(P_0\frac{\partial}{\partial\xi}+Q_0\frac{\partial}{\partial\eta}\right)u_{d1}=0, \tag{29}$$

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)u_{d3}+\left(\frac{\partial}{\partial\tau}+\lambda P_{0\eta}\frac{\partial}{\partial\xi}-\lambda Q_{0\xi}\frac{\partial}{\partial\eta}\right)u_{d1} +u_{d1}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)u_{d1}-\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\phi_3 -\left(P_0\frac{\partial}{\partial\xi}+Q_0\frac{\partial}{\partial\eta}\right)\phi_1+\mu_d\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{d3} -\mu_d n_{d1}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{d1}+\mu_d\left(P_0\frac{\partial}{\partial\xi}+Q_0\frac{\partial}{\partial\eta}\right)n_{d1}=0, \tag{30}$$

$$\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)^2\phi_1-n_{d3}-\frac{1}{2}\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]\phi_3 -\frac{1}{8}\left[\mu_e(q_e+1)(-q_e+3)\sigma_i^2-\mu_i(q_i+1)(-q_i+3)\right]\phi_1^2=0. \tag{31}$$

From Eqs. (29-31), we can deduce

$$\lambda\frac{\partial^2 u_{d3}}{\partial\xi\partial\eta}=\frac{\rho}{2\lambda}\left(\frac{\partial\Phi_1}{\partial\tau}+\alpha\Phi_1\frac{\partial\Phi_1}{\partial\xi}+\beta\frac{\partial^3\Phi_1}{\partial\xi^3}\right) +\frac{\rho}{2\lambda}\left(\frac{\partial\Phi_2}{\partial\tau}-\alpha\Phi_2\frac{\partial\Phi_2}{\partial\eta}-\beta\frac{\partial^3\Phi_2}{\partial\eta^3}\right) +\left(\rho P_{o\eta}+\gamma\Phi_2\right)\frac{\partial^2\Phi_1}{\partial\xi^2} -\left(\rho Q_{o\xi}+\gamma\Phi_1\right)\frac{\partial^2\Phi_2}{\partial\eta^2}, \tag{32}$$

where

$$\alpha=\frac{1}{2\rho}\left(-\frac{3\rho^2}{\lambda}+\frac{1}{4}\lambda\mu_d\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]^2 -\frac{1}{2}\lambda\frac{\mu_e(q_e+1)(-q_e+3)\sigma_i^2-\mu_i(q_i+1)(-q_i+3)}{\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)}\right),$$

$$\beta=\frac{\lambda}{\rho\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]},$$

$$\gamma=\frac{-1}{4\lambda}\left(\frac{1}{2}\lambda\frac{\mu_e(q_e+1)(-q_e+3)\sigma_i^2-\mu_i(q_i+1)(-q_i+3)}{\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)} -\frac{1}{4}\lambda\mu_d\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]^2-\frac{\rho^2}{\lambda}\right),$$

$$\rho=1+\frac{1}{2}\left[\mu_e(q_e+1)\sigma_i+\mu_i(q_i+1)\right]\mu_d. \tag{33}$$

Upon integrating Eq. (32) with respect to ξ and η, one gets

$$\lambda u_{d3}=\frac{\rho}{2\lambda}\left(\frac{\partial\Phi_1}{\partial\tau}+\alpha\Phi_1\frac{\partial\Phi_1}{\partial\xi}+\beta\frac{\partial^3\Phi_1}{\partial\xi^3}\right)d\eta +\frac{\rho}{2\lambda}\int\left(\frac{\partial\Phi_2}{\partial\tau}-\alpha\Phi_2\frac{\partial\Phi_2}{\partial\eta}-\beta\frac{\partial^3\Phi_2}{\partial\eta^3}\right)d\xi +\int\int\left(\rho\frac{\partial P_o}{\partial\eta}+\gamma\Phi_2\right)\frac{\partial^2\Phi_1}{\partial\xi^2}d\xi d\eta -\int\int\left(\rho\frac{\partial Q_o}{\partial\xi}+\gamma\Phi_1\right)\frac{\partial^2\Phi_2}{\partial\eta^2}d\xi d\eta. \tag{34}$$

The first term in the right hand side of Eq. (34) will be proportional to η because the integrand is independent of η and the second term will be proportional to ξ because the integrand is independent of ξ. Hence, they are secular terms and must be eliminated to avoid spurious resonances. Accordingly, we obtain the following two KdV equations

$$\frac{\partial\Phi_1}{\partial\tau}+\alpha\Phi_1\frac{\partial\Phi_1}{\partial\xi}+\beta\frac{\partial^3\Phi_1}{\partial\xi^3}=0, \tag{35}$$

$$\frac{\partial\Phi_2}{\partial\tau}-\alpha\Phi_2\frac{\partial\Phi_2}{\partial\eta}-\beta\frac{\partial^3\Phi_2}{\partial\eta^3}=0, \tag{36}$$

for Φ_1 and Φ_2 with the same coefficients of nonlinear and dispersion α and β, respectively.

In addition, the third and fourth terms in Eq. (34) are not secular terms in this order but they will be secular in the next order. Hence, we get

$$\rho\frac{\partial P_o}{\partial\eta}+\gamma\Phi_2=0, \tag{37}$$

$$\rho\frac{\partial Q_o}{\partial\xi}+\gamma\Phi_1=0. \tag{38}$$

The effect of the nonextensive parameters q_e and q_i is clearly involved in the nonlinear coefficient α, the dispersion coefficient β and also in the coefficients γ and ρ.

Eqs. (35) and (36) represent the two side traveling wave KdV equations in the reference frames of ξ and η, respectively. Such KdV equations have the following solitary wave solutions

$$\Phi_1=\Phi_R Sech^2\left[\sqrt{\frac{\alpha\Phi_R}{12\beta}}\left(\xi-\frac{1}{3}\alpha\Phi_R\tau\right)\right], \tag{39}$$

$$\Phi_2=\Phi_L Sech^2\left[\sqrt{\frac{\alpha\Phi_L}{12\beta}}\left(\eta+\frac{1}{3}\alpha\Phi_L\tau\right)\right], \tag{40}$$

where $\Phi_R=3u_0/\alpha$ and $\Phi_L=3U_0/\alpha$ are the amplitudes of the two solitary waves R and L, respectively with velocities u_0 and U_0. Inserting Eqs. (39) and (40) into Eqs. (37) and

(38), the phase changes due to the collision are given by

$$P_0(\eta,\ \tau) = -\frac{\gamma}{\rho}\sqrt{\frac{12\beta\Phi_L}{\alpha}} \times\left\{\tanh\left[\sqrt{\frac{\alpha\Phi_L}{12\beta}}\left(\eta+\frac{1}{3}\alpha\Phi_L\tau\right)\right]+1\right\}, \quad (41)$$

$$Q_0(\xi,\ \tau) = -\frac{\gamma}{\rho}\sqrt{\frac{12\beta\Phi_R}{\alpha}} \times\left\{\tanh\left[\sqrt{\frac{\alpha\Phi_R}{12\beta}}\left(\xi-\frac{1}{3}\alpha\Phi_R\tau\right)\right]-1\right\}. \quad (42)$$

The trajectories of the two solitary waves for weak head-on interactions in the presence of strong coupling and nonextensivity effects are

$$\xi = \varepsilon(x-\lambda t)-\varepsilon^2\frac{\gamma}{\rho}\sqrt{\frac{12\beta\Phi_L}{\alpha}} \times\left\{\tanh\left[\sqrt{\frac{\alpha\Phi_L}{12\beta}}\left(\eta+\frac{1}{3}\alpha\Phi_L\tau\right)\right]+1\right\}+O(\varepsilon^3), \quad (43)$$

$$\eta = \varepsilon(x+\lambda t)-\varepsilon^2\frac{\gamma}{\rho}\sqrt{\frac{12\beta\Phi_R}{\alpha}} \times\left\{\tanh\left[\sqrt{\frac{\alpha\Phi_R}{12\beta}}\left(\xi-\frac{1}{3}\alpha\Phi_R\tau\right)\right]-1\right\}+O(\varepsilon^3). \quad (44)$$

In order to obtain the phase shifts due to a head-on collision of the two solitary waves R and L, we assume that they are asymptotically far from each other at the initial time ($t=-\infty$), i.e., solitary wave R is at ($\xi=0$, $\eta=-\infty$) while solitary wave L is at ($\eta=0$, $\xi=+\infty$). Then after the collision ($t=+\infty$), solitary wave R is far to the right of solitary wave L, i.e., solitary wave R is at ($\xi=0$, $\eta=+\infty$) while solitary wave L is at ($\eta=0$, $\xi=-\infty$). Using Eqs. (43) and (44), the corresponding phase shifts ΔR and ΔL follow

$$\Delta R = \varepsilon(x-\lambda t)|_{\xi=0,\eta=+\infty}-\varepsilon(x-\lambda t)|_{\xi=0,\eta=-\infty}, \quad (45)$$

$$\Delta L = \varepsilon(x+\lambda t)|_{\eta=0,\xi=-\infty}-\varepsilon(x+\lambda t)|_{\eta=0,\xi=+\infty}, \quad (46)$$

by which the phase shift in the solitary waves R and L can be expressed as

$$\Delta R = 2\varepsilon^2\frac{\gamma}{\rho}\sqrt{\frac{12\beta\Phi_L}{\alpha}}, \quad (47)$$

$$\Delta L = -2\varepsilon^2\frac{\gamma}{\rho}\sqrt{\frac{12\beta\Phi_R}{\alpha}}. \quad (48)$$

Since the solitary wave R travels to the right and the solitary wave L travels to the left, one can observe from Eqs. (47) and (48) that as a result of the collision each solitary wave has a positive phase shift in its traveling direction.

4 Results and Discussion

In the present study, the head-on collision between two DA solitary waves in a strongly coupled dusty plasma containing negatively charged dust particles and nonextensive distributed electrons and ions has been investigated. For this purpose, the extended PLK method has been used and a couple of KdV equations are derived, Eq.(35) and Eq.(36), and consequently two solitary wave solutions are obtained, Eq.(39) and Eq.(40). The nonextensivity effect of both electrons and ions on both the amplitude and the width of the two colliding DA solitary waves R (the wave which moves to right) and L (the wave which moves to left) has been plotted as in Figs. (1-4). From Figs.(1) and (2), one can see that the amplitudes Φ_R and Φ_L of the solitary waves R and L, respectively increase with increasing the nonextensivity of electrons and ions q_e and q_i, respectively. In contrast, Figs.(3) and (4) indicate the decreasing of the widths W_R and W_L of the solitary waves with increasing q_e and q_i, respectively. It means that the collision between the DA solitary waves will be more faster when the nonextensivity effect increases. In addition, the effect of the nonextensivity of electrons and ions (q_e and q_i) and the temperature ratio of ions to electrons (σ_i) on the nonlinear coefficient (α) has been studied and plotted in Figs.(5) and (6). It is seen from thses figures that the nonlinear coefficient α decreases when the values of q_e, q_i and σ_i increase.

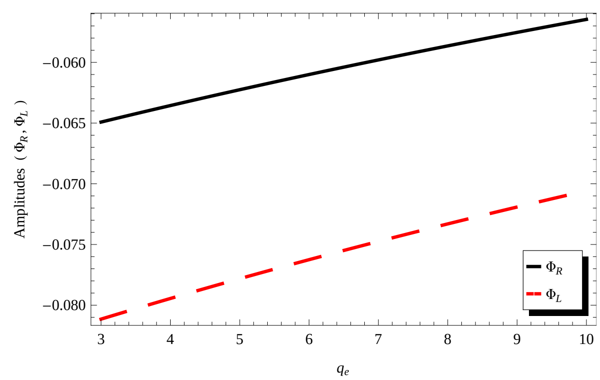

Fig. 1: Variation of the amplitudes Φ_R and Φ_L of the solitary waves R and L with the nonextensive parameter of electrons q_e for $\mu_d = 10$, $\sigma_i = 0.1$, $\delta = 1.4$ and $q_i = 2$.

As a result of the head-on collision between the solitary waves, the trajectories of the motion are changed and hence phase shafts are produced. To investigate the effect of the amplitude Φ_L and the width W_L of the solitary wave L, the nonextensive parameters (q_e and q_i), the temperature ratio of ions to electrons σ_i and the ratio of equilibrium ion to electron densities δ on the phase shift ΔR of the solitary wave R, Figs.(7-11) are plotted. It is shown from Figs.(7) and (8) that the phase shift ΔR decreases with increasing the nonextensivity parameters q_e and q_i while it increases with increasing the amplitude Φ_L. On the other hand in Figs.(9) and (10), it is illustrated

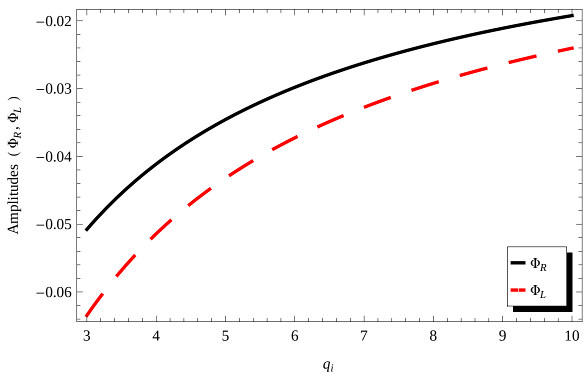

Fig. 2: Variation of the amplitudes Φ_R and Φ_L of the solitary waves R and L with the nonextensive parameter of ions q_i for $\mu_d = 10$, $\sigma_i = 0.1$, $\delta = 1.4$ and $q_e = 2$.

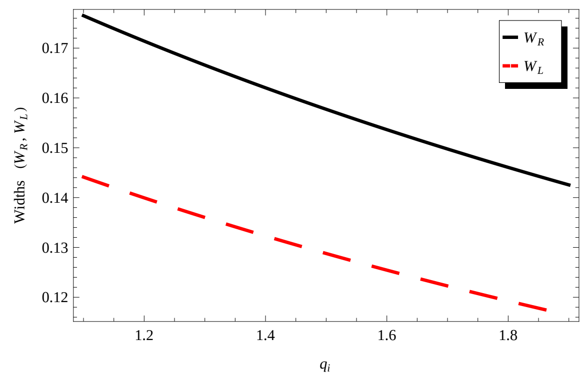

Fig. 4: Variation of the widths W_R and W_L of the solitary waves R and L with the nonextensive parameter of ions q_i for $\mu_d = 10$, $\sigma_i = 0.5$, $\delta = 1.2$ and $q_e = 2$.

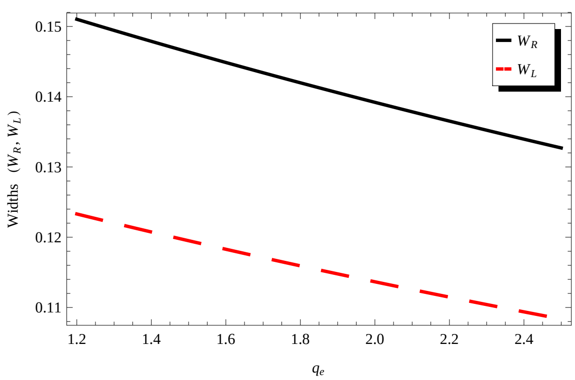

Fig. 3: Variation of the widths W_R and W_L of the solitary waves R and L with the nonextensive parameter of electrons q_e for $\mu_d = 10$, $\sigma_i = 0.5$, $\delta = 1.2$ and $q_i = 2$.

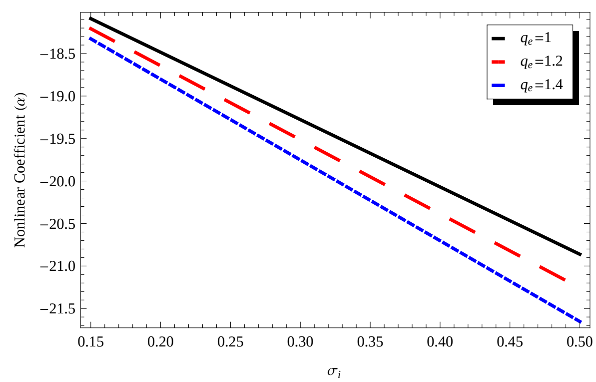

Fig. 5: Variation of the nonlinear coefficient α with the ratio of the ion to electron temperatures σ_i at different values of the nonextensive parameter of electrons q_e for $\mu_d = 10$, $\delta = 1.4$ and $q_i = 2$.

that the phase shift ΔR decreases with increasing the width W_L of the solitary wave L. Finally in Fig.(11), it is found that the phase shift ΔR decreases with increasing σ_i whereas it increases with increasing the values of δ.

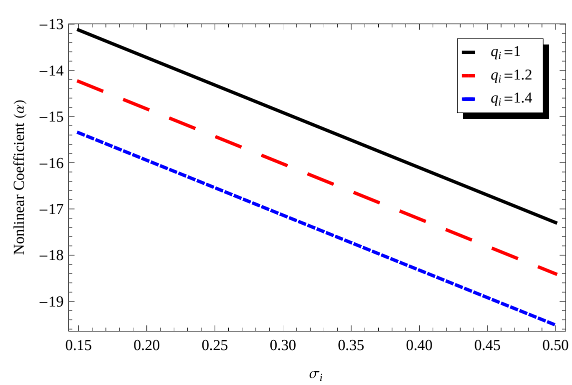

Fig. 6: Variation of the nonlinear coefficient α with the ratio of the ion to electron temperatures σ_i at different values of the nonextensive parameter of ions q_i for $\mu_d = 10$, $\delta = 1.4$ and $q_e = 2$.

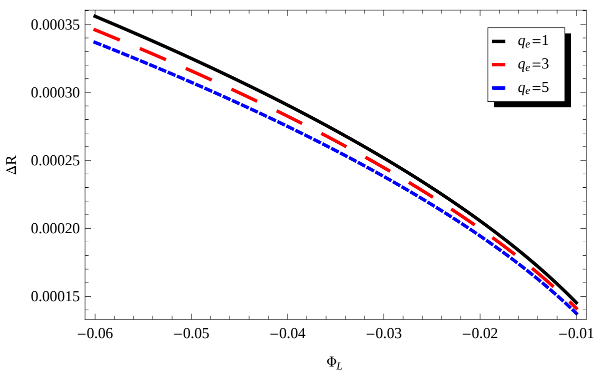

Fig. 7: Variation of the phase shift ΔR of the solitary wave R with the amplitude Φ_L of the solitary wave L at different values of the nonextensive parameter of electrons q_e for $\varepsilon = 0.1$, $\mu_d = 10$, $\sigma_i = 0.1$, $\delta = 1.06$ and $q_i = 2$.

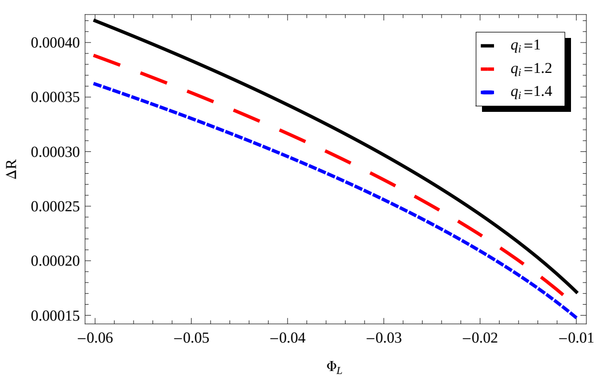

Fig. 8: Variation of the phase shift ΔR of the solitary wave R with the amplitude Φ_L of the solitary wave L at different values of the nonextensive parameter of ions q_i for $\varepsilon = 0.1$, $\mu_d = 10$, $\sigma_i = 0.1$, $\delta = 1.06$ and $q_e = 2$.

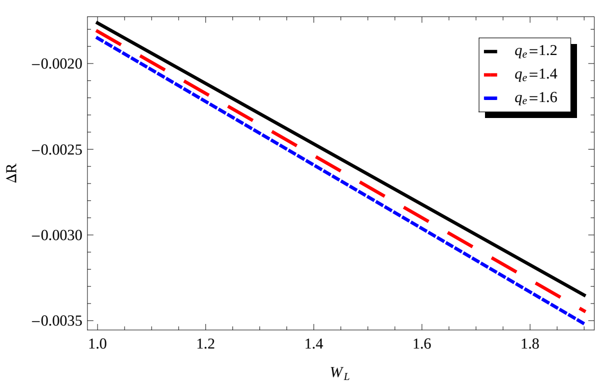

Fig. 9: Variation of the phase shift ΔR of the solitary wave R with the width W_L of the solitary wave L at different values of the nonextensive parameter of electrons q_e for $\varepsilon = 0.1$, $\mu_d = 10$, $\sigma_i = 0.5$, $\delta = 0.06$ and $q_i = 12$.

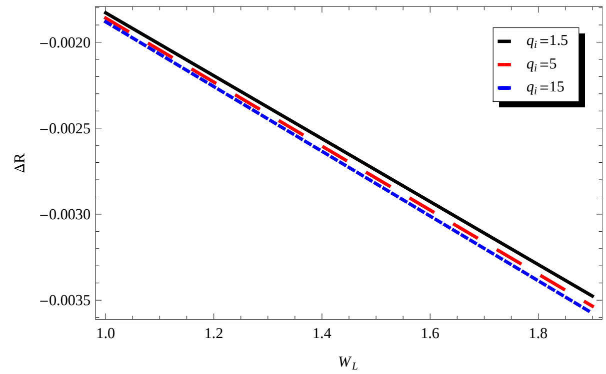

Fig. 10: Variation of the phase shift ΔR of the solitary wave R with the width W_L of the solitary wave L at different values of the nonextensive parameter of ions q_i for $\varepsilon = 0.1$, $\mu_d = 10$, $\sigma_i = 0.05$, $\delta = 1.6$ and $q_e = 2$.

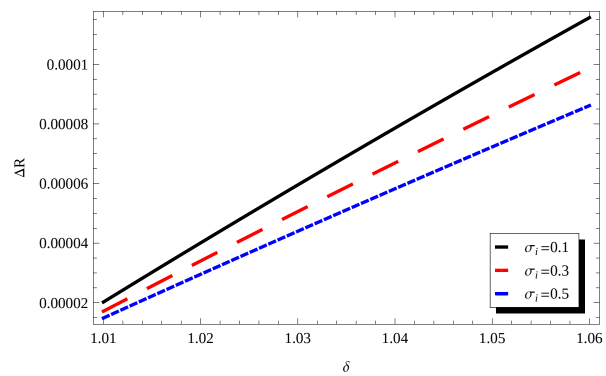

Fig. 11: Variation of the phase shift ΔR of the solitary wave R with the ratio of equilibrium ion to electron densities δ at different values of the ratio of the ion to electron temperatures σ_i for $\varepsilon = 0.1$, $\mu_d = 10$, $q_e = 2$ and $q_i = 2$

5 Summary

In summary, the nonextensive effect of electrons and ions on the properties of the head-on collision between two DA solitary waves and the resulting phase shift in a strongly coupled dusty plasma has been discussed and found that it fasts the collision between them and decreases the resulting phase shift. The present results may be helpful in understanding the properties and characteristics of the head-on collision between solitary waves that may occur in astrophysical as well as laboratory plasmas.

Acknowledgement

The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

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