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# Transmuted Mukherjee Islam Failure Model

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**Abstract:** In this paper, we have introduced Transmuted Mukherjee Islam Failure Model (TMIFM) by using the quadratic rank transmutation map (QRTM) studied by Shaw and Buckley (2007) to develop a Transmuted Mukherjee Islam Failure Model. The different characteristics as well as structural properties of this distribution are derived and the estimation of the model parameters are performed by maximum likelihood method. We have also derived expressions for the PDF of 1st order, nth order and rth order statistics.

**Keywords:** Transmuted Mukherjee Islam Failure Model, QRTM, Hazard Rate Function, Reliability Function, Order Statistics, Parameter Estimation.

## 1 Introduction

Transmuted distributions have been discussed dynamically in frequently occurring large scale experimental statistical data for model selection and related issues. In applied sciences such as environmental, medicine, engineering etc. modeling and analyzing experimental data are essential. There are several distributions which can be used to model such kind of experimental data. The procedures used in such a statistical analysis depend heavily on the assumed probability model or distributions. That is why the development of large classes of standard probability distributions along with relevant statistical methodologies has been expanded. However, there still remain many important problems where the real data does not follow any of the classical or standard probability models.

Mukherjee-Islam failure model is introduced by Mukherjee and Islam (1983). It is finite range distribution which is one of the most important property of it in recent time in reliability analysis. Its mathematical form is simple and can be handled easily, that is why, it is preferred to use over more complex distribution such as normal, weibull, beta etc.

A random variable X is said to have a transmuted distribution if its cumulative distribution function (cdf) is given by

$$F(x) = (1+\lambda)\, G(x) - \lambda G^2(x) \quad |\lambda|\leq 1 \qquad (1)$$

Where F(x) is the cdf of the transmuted distribution and G(x) is the cdf of the base distribution.

On differentiating (1) w.r.to x, which yields the pdf of the transmuted distribution as

$$f(x) = g(x)\,[1 + \lambda - 2\lambda G(x)] \qquad (2)$$

where f(x) and g(x) are the corresponding pdf of F(x) and G(x) respectively.

It might be noted that at $\lambda=0$, we have the base distribution of the base random variable.

## 2 Derivation of Transmuted Mukherjee Islam Failure Model

Let us consider the Mukherjee Islam Failure Model with probability density function (pdf)

$$g(x) = \frac{p}{\theta^p} x^{p-1}, \quad 0 < x < \theta, \quad p>0, \theta>0 \qquad (3)$$

and the cumulative distribution function (cdf)

$$G(x) = \left(\frac{x}{\theta}\right)^p \qquad 0 < x < \theta, \quad p>0, \theta>0 \qquad (4)$$

where $\theta$ is the scale parameter and p is the shape parameter.

Now using (4) in (1), we will get the cdf of Transmuted Mukherjee Islam Failure Model (TMFIM)

$$F(x) = \left(\frac{x}{\theta}\right)^p \left[1 + \lambda - \lambda\left(\frac{x}{\theta}\right)^p\right] \qquad (5)$$

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and (3) in (2), we will get the corresponding pdf of TMIFM

$$f(x) = \frac{p}{\theta^p} x^{p-1}\left[ 1 + \lambda - 2\lambda\left(\frac{x}{\theta}\right)^p \right] \quad (6)$$

## 3 Graphical representation of pdf and cdf of TMIFM

Fig. 1 and Fig. 2, shows the possible shapes of pdf and cdf of the TMIFM for different values of parameters p, θ and λ.

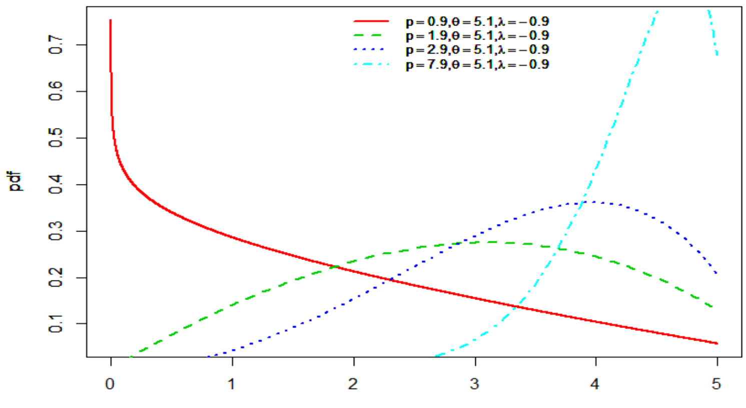


fig. 1: pdf plot of transmuted mukherjee islam failure model

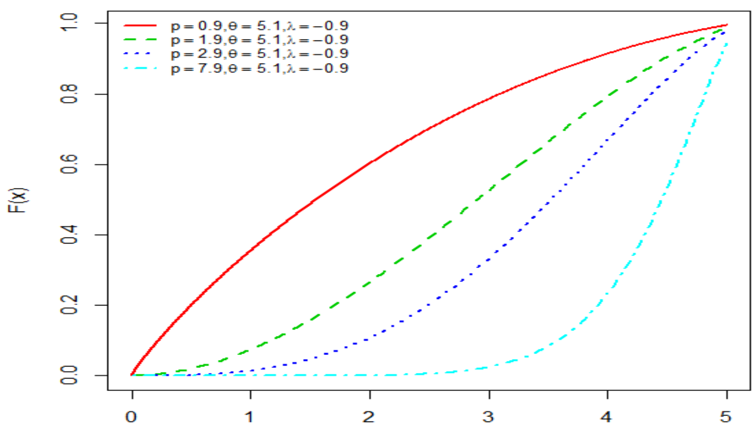


Fig. 2: CDF plot of Tranmuted Mukherjee Islam Failure model

## 4 Moments of TMIFM

The $r^{th}$order moment $E(x^r)$ of a Transmuted Mukherjee Islam Failure Model are computed as follows:

$$E(x^r) = \int_0^{\theta} x^r \ f(x)dx$$

$$= \int_0^{\theta} x^r \frac{p}{\theta^p} x^{p-1} \left[ 1 + \lambda - 2\lambda\left(\frac{x}{\theta}\right)^p \right] dx$$

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$$= \int_0^\theta x^{r+p-1} \frac{p(1+\lambda)}{\theta^p} dx - 2\lambda p \int_0^\theta \frac{x^{r+2p-1}}{\theta^{2p}} dx$$

$$= \frac{(1+\lambda)\, p\theta^r}{r+p} - \frac{2\lambda\, p\theta^r}{r+2p}$$

Then, the $r^{th}$ order moments of TMIFM is

$$E(x^r) = \frac{p\theta^r(r+2p-\lambda r)}{(r+p)(r+2p)} \quad (7)$$

Therefore, the expected value E(x) and variance V(x) of TMIFM is given by

$$E(X) = \frac{p\theta(1+2p-\lambda)}{(1+p)(1+2p)}$$

and $V(X) = E(X^2) - \{E(X)\}^2$

## 5 Moment Generating Function of TMIFM

Let X have a TMIFM, then the MGF of X is obtained by

$$M_X(t) = E(e^{tx}) = \int_0^\theta e^{tx} f(x)dx$$

$$= \int_0^\theta \left(1 + tx + \frac{(tx)^2}{2!} + \cdots\right) f(x)dx$$

$$= \int_0^\theta \sum_{r=0}^{\theta} \frac{t^r}{r!} x^r f(x)dx$$

$$= \sum_{r=0}^{\theta} \frac{t^r}{r!} E(x^r)$$

$$= \sum_{r=0}^{\theta} \frac{t^r}{r!} \frac{p\theta^r(r+2p-\lambda r)}{(r+p)(r+2p)}$$

## 6 Random Number Generation

We can generate the random numbers for the TMIFM, by uing the method of inversion, when the parameters p,θ and λ are known as follows

$$\left(\frac{x}{\theta}\right)^p \left[1 + \lambda - \lambda\left(\frac{x}{\theta}\right)^p\right] = u$$

where u ~ U(0,1) , this yields

$$x = \theta\left[\frac{(1+\lambda)+\sqrt{(1+\lambda)^2-4\lambda u}}{2\lambda}\right]^{\frac{1}{p}}$$

## 7 Parameter Estimation

Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample of size n from a Transmuted Mukherjee Islam Failure Model, then the likelihood function is given by

$$L = \frac{p^n}{\theta^{np}} \prod_{i=1}^{n} x_i^{p-1}\left[1 + \lambda - 2\lambda\left(\frac{x_i}{\theta}\right)^p\right].$$

Hence, the log-likelihood function becomes

$$\log L = n\log p - np\log\theta + (p-1)\log\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \log\left[(1+\lambda) - 2\lambda\left(\frac{x_i}{\theta}\right)^P\right] \quad (8)$$

Therefore, the maximum likelihood estimates of θ,λ and p which maximize (8) must satisfy the following normal equations

$$\frac{\partial \log L}{\partial \theta} = \frac{-np}{\theta} + \sum_{i=1}^{n} \frac{\left[2\lambda p \frac{x_i^p}{\theta^{P+1}}\right]}{\left[1+\lambda - 2\lambda\left(\frac{x_i}{\theta}\right)^p\right]} \quad (9)$$

$$\frac{\partial \log L}{\partial \lambda} = \sum_{i=1}^{n} \frac{\left[1-2\left(\frac{x_i}{\theta}\right)^p\right]}{\left[1+\lambda - 2\lambda\left(\frac{x_i}{\theta}\right)^p\right]} \quad (10)$$

$$\frac{\partial \log L}{\partial p} = \frac{n}{p} - n\log\theta + \sum_{i=1}^{n} \log x_i + \sum_{i=1}^{n} \frac{\left[-2\lambda\left(\frac{x_i}{\theta}\right)^p \log\left(\frac{x_i}{\theta}\right)\right]}{\left[1+\lambda - 2\lambda\left(\frac{x_i}{\theta}\right)^p\right]} \quad (11)$$

By solving the above nonlinear system of equations, We get the maximum likelihood estimator of three parameters $\hat{\mu} = (\hat{\theta}, \hat{\lambda}, \hat{p})$ . It is usually more convenient to use nonlinear optimization algorithms such as quasi-Newton or Newton-Raphson algorithms to numerically maximize the log-likelihood function (8). To compute the standard error and asymptotic confidence interval we use the usual large sample approximation in which the maximum likelihood estimators of μ can be treated as being approximately trivariate normal. Hence as $n \to \infty$ the asymptotic distribution of the MLE $(\hat{\theta}, \hat{\lambda}, \hat{p})$ is given

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by

$$\begin{pmatrix}\hat{\theta}\\ \hat{\lambda}\\ \hat{p}\end{pmatrix} \sim N\left[\begin{pmatrix}\hat{\theta}\\ \hat{\lambda}\\ \hat{p}\end{pmatrix}, \begin{pmatrix}\hat{U}_{11} & \hat{U}_{12} & \hat{U}_{13}\\ \hat{U}_{21} & \hat{U}_{22} & \hat{U}_{23}\\ \hat{U}_{31} & \hat{U}_{32} & \hat{U}_{33}\end{pmatrix}\right]$$

where $\hat{U}_{ij} = U_{ij|\mu}=\hat{\mu}$

Approximate two sided 100(1-α)% confidence intervals for θ, λ and p are given by

$\hat{\theta} \pm z_{\alpha/2}\sqrt{\hat{U}_{11}}$ , $\hat{\lambda} \pm z_{\alpha/2}\sqrt{\hat{U}_{22}}$ and $\hat{p} \pm z_{\alpha/2}\sqrt{\hat{U}_{33}}$

Where $z_\alpha$ is the upper α-th percentiles of the standard normal distribution. By using R, we can easily compute the standard errors, asymptotic confidence intervals, Hessian matrix and its inverse.

## 8 Reliability Analysis

### *8.1 Reliability Function*

The reliability function R(t), is defined by

$$R(t) = 1 - F(t)$$

The reliability function of transmuted mukherjee islam failure model is given by

$$R(t) = 1 - \left(\frac{t}{\theta}\right)^p\left[1 + \lambda - \lambda\left(\frac{t}{\theta}\right)^p\right]$$

### *8.2 Hazard Rate Function*

The hazard rate function h(t) is an important quantity characterizing life phenomenon and is defined by

$$h(t) = \frac{f(t)}{1-F(t)}$$

The hazard rate function for transmuted mukherjee islam failure model is given by

$$h(t) = \frac{pt^{p-1}\left[1 + \lambda - 2\lambda\left(\frac{t}{\theta}\right)^p\right]}{\theta^p - t^p\left[1 + \lambda - \lambda\left(\frac{t}{\theta}\right)^p\right]}$$

## 9 Order Statistics

Order statistics has a wider applications in the field of quality control, life testing and in several apects of statistical inference.

Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ denotes the order statistics of a random sample $X_1, X_2, \ldots, X_n$ from a contineous population with cumulative density function $F_X(x)$ and probability density function $f_X(x)$, then the pdf of $r^{th}$ order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!\,(n-r)!} f_X(x)(FX(x))^{r-1}(1 - FX(x))^{n-r} \quad (12)$$

Therefore, the pdf of the $^{rth}$ order statistics $X_{(r)}$ for a transmuted mukherjee islam failure model is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!\,(n-r)!}\frac{p}{\theta^p}x^{p-1}\left[1 + \lambda - 2\lambda\left(\frac{x}{\theta}\right)^p\right]\left[\left(\frac{x}{\theta}\right)^p\left\{1 + \lambda - \lambda\left(\frac{x}{\theta}\right)^p\right\}\right]^{r-1}$$
$$\left[1 - \left(\frac{x}{\theta}\right)^p\{1 + \lambda - \lambda\left(\frac{x}{\theta}\right)^p\}\right]^{n-r} \quad (13)$$

Using the equation (13), the pdf of the $n^{th}$ order transmuted mukherjee islam failure statistics $X_{(n)}$ is given by

$$f_{x(n)}(x) = \frac{np}{\theta^p}x^{p-1}\left[1 + \lambda - 2\lambda\left(\frac{x}{\theta}\right)^p\right]\quad\left[\left(\frac{x}{\theta}\right)^p\{1 + \lambda - \lambda\left(\frac{x}{\theta}\right)^p\}\right]^{n-1}$$

and the pdf of the $1^{st}$ order statistics $X_{(1)}$ is given by

$$f_{x(1)}(x) = \frac{np}{\theta^p}x^{p-1}\left[1 + \lambda - 2\lambda\left(\frac{x}{\theta}\right)^p\right]\left[1 - \left(\frac{x}{\theta}\right)^p\{1 + \lambda - \lambda\left(\frac{x}{\theta}\right)^p\}\right]^{n-1}$$

Note that λ = 0, yields the order statistics of the three parameter transmuted Mukherjee islam failure model.

## 10 Conclusion

In this paper, we have introduced a new distribution called the transmuted Mukherjee islam failure model. We investigate its different characteristics as well as structural properties. The subject distribution is generated by using the

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quadratic rank transmutation map and taking the Mukherjee islam failure model as base distribution. The parameters have been obtained using maximum likelihood technique. The advantage of this distribution is that the hazard rate function and reliability behavior of transmuted Mukherjee islam failure model shows that subject distribution can be used to model reliability data.

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