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A Ratio and Ratio Exponential Estimator for Finite Population Mean in Case of Post-Stratification

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Abstract: This paper studies a ratio and ratio exponential estimator for population mean. The suggested estimator has been differentiated with unbiased estimator and ratio exponential estimator for population mean. An empirical study analyse the qualities of the propound estimator over the existing estimators.

Keywords: Ratio and ratio type exponential estimator, Post-stratification, Bias, MSE.

1 Introduction

Application of stratified sampling involves the availability of the information on number of units of the various strata besides the sampling frame for each stratum. Many times number of units in the different strata are available but list of units to identity the individual unit may not be available. Such circumstances need the use of post-stratification technique instead of stratified random sampling.

In post stratification technique, first a sample of required size is drawn from the population using SRS and then stratified it using stratification factor.

Ratio and product estimators for population mean were discussed in post- stratification by [3]. [1] ratio exponential estimator was studied in post-stratification by [6]. Above work motivate the authors to study ratio and ratio type exponential estimator in post- stratification.

The unbiased estimator for population mean is explained as.

$$\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h \tag{1}$$

[3] defined [2] classical ratio type estimator in post-stratification, as

$$\hat{\bar{Y}}_{PS}^{R} = \bar{y}_{PS}\left(\frac{\bar{X}}{\bar{x}_{PS}}\right) \tag{2}$$

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Where $\overline{X} = \sum_{h=1}^{L} W_h \overline{X}_h$, $\overline{y}_{PS} = \sum_{h=1}^{L} W_h \overline{y}_h$ and $\overline{x}_{PS} = \sum_{h=1}^{L} W_h \overline{x}_h$.

Up to the fda, the bias and MSE statement of $\hat{\overline{Y}}_{PS}^{R}$ are defined as

$$B\left(\hat{\overline{Y}}_{PS}^{R}\right) = \left(\frac{1}{n} - \frac{1}{N}\right) \frac{1}{\overline{X}} \sum_{h=1}^{L} W_h \left(R_1 S_{xh}^2 - S_{yxh}\right) \tag{3}$$

and

$$MSE\left(\hat{\overline{Y}}_{PS}^{R}\right) = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h \left(S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}\right) \tag{4}$$

Where $R_1 = \dfrac{\overline{Y}}{\overline{X}}$.

[6] defined [1] ratio type exponential estimator as

$$\hat{\overline{Y}}_{PS}^{RE} = \overline{y}_{PS} \exp\left(\frac{\overline{X} - \overline{x}_{PS}}{\overline{X} + \overline{x}_{PS}}\right). \tag{5}$$

Up to the fda, the bias and MSE assertion of $\hat{\overline{Y}}_{PS}^{RE}$ are respectively given by

$$B\left(\hat{\overline{Y}}_{PS}^{RE}\right) = \left(\frac{1}{n} - \frac{1}{N}\right) \frac{1}{\overline{X}} \sum_{h=1}^{L} W_h \left(\frac{3}{8} R_1 S_{xh}^2 - \frac{1}{2} S_{yxh}\right) \tag{6}$$

and

$$MSE\left(\hat{\overline{Y}}_{PS}^{RE}\right) = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h \left(S_{yh}^2 + \frac{1}{4} R_1^2 S_{xh}^2 - R_1 S_{yxh}\right). \tag{7}$$

2 Suggested Estimator

[4] studied ratio and ratio estimator in stratified sampling using details on two auxiliary variables as, both positively correlated with the study variate as $\hat{\overline{Y}}_{RR}^{st} = \overline{y}_{st} \left(\dfrac{\overline{X}}{\overline{x}_{st}}\right)\left(\dfrac{\overline{Z}}{\overline{z}_{st}}\right)$, Motivated by [4], a ratio and ratio exponential estimator for population mean as

$$\hat{\overline{Y}}_{S} = \overline{y}_{PS} \exp\left(\frac{\sum_{h=1}^{L} W_h \overline{X}_h - \sum_{h=1}^{L} W_h \overline{x}_h}{\sum_{h=1}^{L} W_h \overline{X}_h + \sum_{h=1}^{L} W_h \overline{x}_h}\right) \exp\left(\frac{\sum_{h=1}^{L} W_h \overline{Z}_h - \sum_{h=1}^{L} W_h \overline{z}_h}{\sum_{h=1}^{L} W_h \overline{Z}_h + \sum_{h=1}^{L} W_h \overline{z}_h}\right) \tag{8}$$

$\hat{\overline{Y}}_{S}$ can also be expressed as

$$\hat{\bar{Y}}_S = \bar{y}_{PS} \exp\left(\frac{\bar{X} - \bar{x}_{PS}}{\bar{X} + \bar{x}_{PS}}\right) \exp\left(\frac{\bar{Z} - \bar{z}_{PS}}{\bar{Z} + \bar{z}_{PS}}\right) \tag{9}$$

We write $\bar{y}_h = \bar{Y}_h(1+e_{0h})$, $\bar{x}_h = \bar{X}_h(1+e_{1h})$ and $\bar{z}_h = \bar{Z}_h(1+e_{2h})$ such that

$$E(e_{0h}) = E(e_{1h}) = E(e_{2h}) = 0,$$

$$E(e_{0h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right) C_{yh}^2, \quad E(e_{1h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right) C_{xh}^2,$$

$$E(e_{2h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right) C_{zh}^2, \quad E(e_{0h}e_{1h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right) \rho_{yxh} C_{yh} C_{xh},$$

$$E(e_{0h}e_{2h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right) \rho_{yzh} C_{yh} C_{zh},$$

and

$$E(e_{1h}e_{2h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right) \rho_{xzh} C_{xh} C_{zh}.$$

The suggested estimator $\hat{\bar{Y}}_S$ can be explicit in terms of $e_i^{'}s$ as

$$\hat{\bar{Y}}_S = \bar{Y}(1+e_0) \exp\left(\frac{-e_1}{2+e_1}\right) \exp\left(\frac{e_2}{2+e_2}\right). \tag{10}$$

where $e_0 = \frac{\sum_{h=1}^{L} W_h \bar{Y}_h e_{0h}}{\bar{Y}}$, $e_1 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h e_{1h}}{\bar{X}}$, $e_2 = \frac{\sum_{h=1}^{L} W_h \bar{Z}_h e_{2h}}{\bar{Z}}$

such that $E(e_0) = E(e_1) = E(e_2) = 0$ and

$$E(e_0^2) = \frac{1}{\bar{Y}^2}\left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h S_{yh}^2,$$

$$E(e_1^2) = \frac{1}{\bar{X}^2}\left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h S_{xh}^2, \quad E(e_2^2) = \frac{1}{\bar{Z}^2}\left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h S_{zh}^2,$$

$$E(e_0 e_1) = \frac{1}{\bar{Y}\,\bar{X}}\left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h S_{yxh}, \quad E(e_0 e_2) = \frac{1}{\bar{Y}\,\bar{Z}}\left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h S_{yzh}$$

$$E(e_1e_2)=\frac{1}{\bar{X}\,\bar{Z}}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{xzh}\,.$$

Up to the fda, the bias and MSE of the suggested estimator are obtained as

$$B\left(\hat{\bar{Y}}_S\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left[\frac{3}{8}\left(\frac{R_1S_{xh}^2}{\bar{X}}+\frac{R_2S_{zh}^2}{\bar{Z}}\right)-\frac{1}{2}\left(\frac{S_{yzh}}{\bar{Z}}+\frac{S_{yxh}}{\bar{X}}\right)+\frac{1}{4}\frac{R_1S_{xzh}}{\bar{Z}}\right] \tag{11}$$

and

$$MSE\left(\hat{\bar{Y}}_S\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(S_{yh}^2+\frac{R_1^2S_{xh}^2}{4}+\frac{R_2^2S_{zh}^2}{4}-R_1S_{yxh}+R_2S_{yzh}+\frac{1}{2}R_1R_2S_{xzh}\right) \tag{12}$$

where $R_2=\dfrac{\bar{Y}}{\bar{Z}}$.

3 Efficiency Comparison

we have made comparison of the suggested estimator $\hat{\bar{Y}}_S$ with the existing estimators such as unbiased estimator $\bar{y}_{PS}$, ratio estimator $\hat{\bar{Y}}_{PS}^{R}$ and exponential estimator $\hat{\bar{Y}}_{PS}^{RE}$. Variance of unbiased estimator in post-stratification is expressed as

$$V(\bar{y}_{PS})=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{yh}^2\,. \tag{13}$$

It follows from (4), (7), (12) and (13) that the suggested ratio and ratio type exponential estimator $\hat{\bar{Y}}_S$ is high efficient than

i) $\bar{y}_{PS}$ if

$$\sum_{h=1}^{L}W_h\left(\frac{1}{4}R_1^2S_{xh}^2+\frac{1}{4}R_2^2S_{zh}^2-R_1S_{yxh}-R_2S_{yzh}+\frac{1}{2}R_1R_2S_{xzh}\right)<0\,, \tag{14}$$

ii) $\hat{\bar{Y}}_{PS}^{R}$ if

$$\sum_{h=1}^{L}W_h\left(\frac{1}{4}R_2^2S_{zh}^2-\frac{3}{4}R_1^2S_{xh}^2+R_1S_{yxh}-R_2S_{yzh}+\frac{1}{2}R_1R_2S_{xzh}\right)<0\,, \tag{15}$$

iii) $\hat{\bar{Y}}_{PS}^{RE}$ if

$$\sum_{h=1}^{L}W_h\left(\frac{1}{4}R_2^2S_{zh}^2-R_2S_{yzh}+\frac{1}{2}R_1R_2S_{xzh}\right)<0\,. \tag{16}$$

4 Empirical Study

A data set is considered to compare suggested estimator with other considered estimators.

Population [Source: Murthy (1967), p. 228]

y : Output

x : Fixed capital

z : Number of workers

Constant	Stratum I	Stratum II
N_h	05	05
n_h	4	4
$\bar{Y}_h$	1925.80	3115.60
$\bar{X}_h$	214.40	333.80
$\bar{Z}_h$	51.80	60.60
S_{yh}	615.92	340.38
S_{xh}	74.87	66.35
S_{zh}	0.75	4.84
S_{yxh}	39360.68	22356.50
S_{yzh}	411.20	1536.00
S_{xzh}	38.08	287.92

Table 4.1: Percent relative efficiencies of $\bar{y}_{PS}$, $\hat{\bar{Y}}_{PS}^{R}$, $\hat{\bar{Y}}_{PS}^{RE}$ and $\hat{\bar{Y}}_{S}$ with respect to $\bar{y}_{PS}$.

Estimator	$\bar{y}_{PS}$	$\hat{\bar{Y}}_{PS}^{R}$	$\hat{\bar{Y}}_{PS}^{RE}$	$\hat{\bar{Y}}_{S}$
PREs	100.00	239.87	355.69	475.451

5 Conclusion

Section 3 gives the situations under which the suggested estimator $\hat{\bar{Y}}_{S}$ has higher efficient than other considered estimators.

Expression (14), (15) and (16) are conditions under which the suggested estimator $\hat{\bar{Y}}_{S}$ would has less MSE in comparison to $\bar{y}_{PS}$, $\hat{\bar{Y}}_{PS}^{R}$ and $\hat{\bar{Y}}_{PS}^{RE}$.

Table 4.1 exhibits the performance of different considered estimators empirically. This table shows that the suggested estimator $\hat{\bar{Y}}_S$ has maximum PRE in comparison to other estimators.

Thus suggested estimator $\hat{\bar{Y}}_S$ is preferred for estimation of population mean if conditions obtained in section 3 are satisfied.

References

[1] S. Bahl, and R. K Tuteja, Ratio and product type exponential estimators. Information &optimization science., 12**(**1**)**, 159-163, 1991.

[2] W.G Cochran, The estimation of yields of cereal experiments by sampling for ratio of grain to total produce. Journal of Agricultural Sci., 30, 262-275, 1940.

[3] F.A Ige, and T.P Tripathi, Estimation of population mean using post-stratification & auxiliary information. Abacus., 18(2), 265-276, 1989.

[4] N. Koyuncu and C. Kadilar, Ratio and product estimators in stratified random sampling. Jour. Statist. Plang. & Inf.,139(8), 2552-2558, 2009.

[5] M. N Murthy, Sampling Theory and Methods, Statistical Publishing Society. Calcutta, India, (1967).

[6] R. Tailor, R. Tailor, and S. Chouhan, Improved ratio- and product- type exponential estimators for population mean in case of post-stratification. Communications in Statistics- Theo. & Meth.,46(21), 10387-10393, 2017.