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Ratio Estimators for Estimating Population Mean in Simple Random Sampling Using Auxiliary Information

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Abstract: The present study was taken in consideration to propose new modified ratio estimators for estimating population mean in simple random sampling using the auxiliary information of non-conventional location parameters such as Tri-Mean, Mid-Range Hodges-Lehmann with coefficient of kurtosis and coefficient of skewness. The properties associated with the proposed estimators are assessed by mean square error and bias. For illustration an empirical study is provided, which confirms that our proposed estimators are more efficient than the existing estimators.

Keywords: Non-conventional location parameters, Skewness, Kurtosis, Ratio estimators, Efficiency.

1 Introduction

In sample surveys, auxiliary information on the finite population under study is quite often available from previous experience, census or administrative databases. The sampling theory describes a wide variety of techniques/ methods for using auxiliary information to improve the sampling design and to obtain more efficient estimators like Ratio, Product and Regression estimators. Ratio estimators, improves the precision of estimate of the population mean or total of a study variable by using prior information on auxiliary variable X which is correlated with the study variable Y. Over the years the ratio method of estimation has been extensively used because of its intuitive appeal and the computational simplicity.

The classical Ratio estimator for the population mean $\bar{Y}$ of the study variable Y is defined as:

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X}, \text{ Where } \hat{R} = \frac{\bar{y}}{\bar{x}}$$

Where $\bar{y}$ sample mean of the study variable Y and $\bar{x}$ is the sample mean of the auxiliary variable X . It is assumed that the population mean $\bar{X}$ of the auxiliary variable X is known. The bias and mean squared error of $\hat{\bar{Y}}_R$ to the first degree of approximation are given below

$$B(\hat{\bar{Y}}_R) = \frac{(1-f)}{n}\bar{Y}(C_x^2 - C_x C_y \rho)$$

$$MSE(\hat{\bar{Y}}_R) = \frac{(1-f)}{n}\bar{Y}^2(C_y^2 + C_x^2 - 2C_x C_y \rho)$$

After that lot of modifications has been made on the classical ratio estimator proposed by Cochran [1] by using the various characteristics of the auxiliary variable either at design or estimation or at both the stages. Further improvements are also achieved on the classical ratio estimator by introducing a large number of modified ratio estimators with the use of known parameters like, coefficient of variation, coefficient of kurtosis, coefficient of skewness and population correlation coefficient. For more detailed discussion one may refer to Cochran [2], Kadilar and Cingi [3] [4], Koyuncu and Kadilar [5], Murthy [6], Prasad [7], Rao [8], Singh [11], Singh and Tailor [12] [13], Singh *et al.*[14], Sisodia and Dwivedi [15], Upadhyaya and Singh [20], Robson [9] and Yan and Tian [22].

Further, Subramani and Kumarapandiyan [16] had taken initiative by proposing modified ratio estimator for estimating the population mean of the study variable by using the population deciles of the auxiliary variable.

Recently Subzar *et al.* [19] had proposed some estimators using population deciles and correlation coefficient of the auxiliary variable, also Subzar *et al.* [18] had proposed some modified ratio type estimators using the quartile deviation and population deciles of auxiliary variable and Subzar *et al.* [17] had also proposed an efficient class of

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estimators by using the auxiliary information of population deciles, median and their linear combination with correlation coefficient and coefficient of variation.

In this paper we have envisaged a new class of improved ratio type estimators for estimation of population mean of the study variable using the information of non-conventional location parameters and their linear combination with coefficient of skewness and coefficient of kurtosis. Let $G = \{G_1, G_2, G_3, ..., G_N\}$ be a finite population of N distinct and identifiable units. Let y and x denotes the study variable and the auxiliary variable taking values y_i and x_i respectively on the i^{th}unit ($i = 1, 2, ..., N$). Before discussing about the proposed estimators, we will mention the estimators in Literature using the notations given in the next sub-section.

1.1 Notation

N	Population size
n	Sample size
$f = n/N$	Sampling fraction
Y	Study variable
X	Auxiliary variable
$\bar{X}, \bar{Y}$	Population means
$\bar{x}, \bar{y}$	Sample means
x, y	Sample totals
S_x, S_y	Population standard deviations
S_{xy}	Population covariance between variables
C_x, C_y	Population coefficient of variation
ρ	Population correlation coefficient
$B(.)$	Bias of the estimator
$MSE(.)$	Mean square error of the estimator
$\hat{Y}_i$	Existing modified ratio estimator of $\bar{Y}$
$\hat{Y}_{pj}$	Proposed modified ratio estimator of $\bar{Y}$
β_2	Population kurtosis
β_1	Population skewness

$TM = \dfrac{Q_1 + 2Q_2 + Q_3}{4}$ Tri-Mean

$HL = median((X_j + X_k)/2, 1 \le j \le k \le N)$

Hodges-Lehmann estimator

$MR = \dfrac{X_{(1)} + X_{(N)}}{2}$ Population mid-range

Subscript

i For existing estimators

j For proposed estimators.

2 Estimators in Literature

Kadilar and Cingi [3] suggested ratio type estimators for the population mean in the simple random sampling using some known auxiliary information on coefficient of kurtosis and coefficient of variation. They showed that their suggested estimators are more efficient than traditional ratio estimator in the estimation of the population mean.

Kadilar & Cingi [3] estimators are given by

$$\hat{Y}_1 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X},$$

$$\hat{Y}_2 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + C_x)} (\bar{X} + C_x),$$

$$\hat{Y}_3 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_2)} (\bar{X} + \beta_2),$$

$$\hat{Y}_4 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + C_x)} (\bar{X}\beta_2 + C_x),$$

$$\hat{Y}_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \beta_2)} (\bar{X}C_x + \beta_2),$$

The biases, related constants and the MSE for Kadilar and Cingi [3] estimators are respectively as follows:

$$B(\hat{Y}_1) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_1^2, \qquad R_1 = \frac{\bar{Y}}{\bar{X}}$$

$$MSE(\hat{Y}_1) = \frac{(1-f)}{n} (R_1^2 S_x^2 + S_y^2 (1-\rho^2)),$$

$$B(\hat{Y}_2) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_2^2, \quad R_2 = \frac{\bar{Y}}{(\bar{X} + C_x)}$$

$$MSE(\hat{Y}_2) = \frac{(1-f)}{n} (R_2^2 S_x^2 + S_y^2 (1-\rho^2)),$$

$$B(\hat{Y}_3) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_3^2, \quad R_3 = \frac{\bar{Y}}{(\bar{X} + \beta_2)}$$

$$MSE(\hat{Y}_3) = \frac{(1-f)}{n} (R_3^2 S_x^2 + S_y^2 (1-\rho^2)),$$

$$B(\hat{Y}_4) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_4^2, \quad R_4 = \frac{\bar{Y}}{(\bar{X}\beta_2 + C_x)}$$

$$MSE(\hat{Y}_4) = \frac{(1-f)}{n} (R_4^2 S_x^2 + S_y^2 (1-\rho^2)),$$

$$B(\hat{Y}_5) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_5^2, \quad R_5 = \frac{\bar{Y}}{(\bar{X}C_x + \beta_2)}$$

$$MSE(\hat{Y}_5) = \frac{(1-f)}{n} (R_5^2 S_x^2 + S_y^2 (1-\rho^2)).$$

Kadilar and Cingi [4] developed some modified ratio estimators using known value of coefficient of correlation, kurtosis and coefficient of variation as follows:

$$\breve{Y}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho),$$

$$\breve{Y}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho),$$

$$\breve{Y}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x),$$

$$\breve{Y}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho),$$

$$\breve{Y}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2).$$

The biases, related constants and the MSE for Kadilar and Cingi [4] estimators are respectively given by

$$B(\breve{Y}_6) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_6^2, \quad R_6 = \frac{\bar{Y}}{\bar{X}+\rho}$$

$$MSE(\breve{Y}_6) = \frac{(1-f)}{n}(R_6^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\breve{Y}_7) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_7^2, \quad R_7 = \frac{\bar{Y}C_x}{\bar{X}C_x+\rho}$$

$$MSE(\breve{Y}_7) = \frac{(1-f)}{n}(R_7^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\breve{Y}_8) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_8^2, \quad R_8 = \frac{\bar{Y}\rho}{\bar{X}\rho+C_x}$$

$$MSE(\breve{Y}_8) = \frac{(1-f)}{n}(R_8^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\breve{Y}_9) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_9^2, \quad R_9 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho}$$

$$MSE(\breve{Y}_9) = \frac{(1-f)}{n}(R_9^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\breve{Y}_{10}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{10}^2, \quad R_{10} = \frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$$

$$MSE(\breve{Y}_{10}) = \frac{(1-f)}{n}(R_{10}^2S_x^2+S_y^2(1-\rho^2)).$$

3 Improved Ratio Estimators

Motivated by the mentioned estimators in Section 2, we propose a new class of efficient ratio type estimators using the linear combination of non-conventional location parameters with coefficient of skewness and coefficient of kurtosis. We use here non-conventional location parameters, as they are not affected by extreme values and thus these estimators would perform better than the existing estimators even if in the presence of outliers in the population and the proposed estimators are as follows:

$$\breve{Y}_{p1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+TM)}(\bar{X}\beta_1+TM),$$

$$\breve{Y}_{p2} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+MR)}(\bar{X}\beta_1+MR),$$

$$\breve{Y}_{p3} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+HL)}(\bar{X}\beta_1+HL),$$

$$\breve{Y}_{p4} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+TM)}(\bar{X}\beta_2+TM),$$

$$\breve{Y}_{p5} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+MR)}(\bar{X}\beta_2+MR),$$

$$\breve{Y}_{p6} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+HL)}(\bar{X}\beta_2+HL),$$

The bias, related constant and the MSE for the first proposed estimator can be obtained as follows:
MSE of this estimator can be found using Taylor series method defined as

$$h(\bar{x},\bar{y}) \cong h(\bar{X},\bar{Y}) + \frac{\partial h(c,d)}{\partial c}\Big|_{\bar{X},\bar{Y}}(\bar{x}-\bar{X}) + \frac{\partial h(c,d)}{\partial d}\Big|_{\bar{X},\bar{Y}}(\bar{y}-\bar{Y}) \quad (3.1)$$

Where $h(\bar{x},\bar{y}) = \hat{R}_{p1}$ and $h(\bar{X},\bar{Y}) = R.$

As shown in Wolter [21], (2.1) can be applied to the proposed estimator in order to obtain MSE equation as follows:

$$\hat{R}_{p1} - R \cong \frac{\partial((\bar{y}+b(\bar{X}-\bar{x}))/(\bar{x}\beta_1+TM))}{\partial\bar{x}}\Big|_{\bar{X},\bar{Y}}(\bar{x}-\bar{X}) + \frac{\partial((\bar{y}+b(\bar{X}-\bar{x}))/(\bar{x}\beta_1+TM))}{\partial\bar{y}}\Big|_{\bar{X},\bar{Y}}(\bar{y}-\bar{Y})$$

$$\cong -\left(\frac{\bar{y}}{(\bar{x}\beta_1+TM)^2} + \frac{b(\bar{X}\beta_1+TM)}{(\bar{x}\beta_1+TM)^2}\right)\Big|_{\bar{X},\bar{Y}}(\bar{x}-\bar{X}) + \frac{1}{(\bar{x}\beta_1+TM)}\Big|_{\bar{X},\bar{Y}}(\bar{y}-\bar{Y})$$

$$E(\hat{R}_{p1}-R)^2 \cong \frac{(\bar{Y}\beta_1+B(\bar{X}\beta_1+TM))^2}{(\bar{X}\beta_1+TM)^4}V(\bar{x}) - \frac{2(\bar{Y}\beta_1+B(\bar{X}\beta_1+TM))}{(\bar{X}\beta_1+TM)^3}Cov(\bar{x},\bar{y}) +$$

$$\frac{1}{(\bar{X}\beta_1+TM)^2}V(\bar{y})$$

$$\cong \frac{1}{(\bar{X}\beta_1+TM)^2}\left\{\frac{(\bar{Y}\beta_1+B(\bar{X}\beta_1+TM))^2}{(\bar{X}\beta_1+TM)^2}V(\bar{x})\right.$$

$$\left.-\frac{2(\bar{Y}\beta_1+B(\bar{X}\beta_1+TM)}{(\bar{X}\beta_1+TM)}Cov(\bar{x},\bar{y})+V(\bar{y})\right\}$$

Where $B=\frac{s_{xy}}{s_x^2}=\frac{\rho s_x s_y}{s_x^2}=\frac{\rho s_y}{s_x}$. Note that we omit the difference of $(E(b)-B)$

$$MSE(\bar{y}_{p1})=(\bar{X}\beta_1+TM)^2E(\hat{R}_{p1}-R)^2$$

$$\cong \frac{(\bar{Y}\beta_1+B(\bar{X}\beta_1+TM))^2}{(\bar{X}\beta_1+TM)^2}V(\bar{x})-\frac{2(\bar{Y}\beta_1+B(\bar{X}\beta_1+TM))}{(\bar{X}\beta_1+TM)}$$

$$Cov(\bar{x},\bar{y})+V(\bar{y})$$

$$\cong \frac{\bar{Y}^2\beta_1+2B(\bar{X}\beta_1+TM)\bar{Y}\beta_1+B^2(\bar{X}\beta_1+TM)^2}{(\bar{X}\beta_1+TM)^2}$$

$$V(\bar{x})-\frac{2\bar{Y}\beta_1+2B(\bar{X}\beta_1+TM)}{(\bar{X}\beta_1+TM)}Cov(\bar{x},\bar{y})+V(\bar{y})$$

$$\cong \frac{(1-f)}{n}\left\{\begin{matrix}\left(\frac{\bar{Y}^2\beta_1}{(\bar{X}\beta_1+TM)^2}+\frac{2B\bar{Y}\beta_1}{(\bar{X}\beta_1+TM)}+B^2\right)\\ S_x^2-\left(\frac{2\bar{Y}\beta_1}{(\bar{X}\beta_1+TM)}+2B\right)S_{xy}+S_y^2\end{matrix}\right\}$$

$$\cong \frac{(1-f)}{n}(R^2S_x^2+2BRS_x^2+B^2S_x^2-2RS_{xy}-2BS_{xy}+S_y^2)$$

$$MSE(\bar{y}_{p1})\cong \frac{(1-f)}{n}(R^2S_x^2+2R\rho S_xS_y+\rho^2S_y^2-$$

$$2R\rho S_xS_y-2\rho^2S_y^2+S_y^2$$

$$\cong \frac{(1-f)}{n}(R^2S_x^2-\rho^2S_y^2+S_y^2)$$

$$\cong \frac{(1-f)}{n}(R^2S_x^2+S_y^2(1-\rho^2))$$

Similarly, the bias is obtained as

$$Bias(\bar{y}_{p1})\quad \cong \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_1^2$$

Thus the bias and MSE of the proposed estimator is given below:

$$B(\hat{\bar{Y}}_{p1})=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_1^2,\quad R_1=\frac{\bar{Y}\beta_1}{\bar{X}\beta_1+TM}$$

$$MSE(\hat{\bar{Y}}_{p1})=\frac{(1-f)}{n}(R_1^2S_x^2+S_y^2(1-\rho^2)),$$

Similarly, the bias, constant and the mean square error can be found using the Taylor series method and is given as below:

$$B(\hat{\bar{Y}}_{p2})=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_2^2,\quad R_2=\frac{\bar{Y}\beta_1}{\bar{X}\beta_1+MR}$$

$$MSE(\hat{\bar{Y}}_{p2})=\frac{(1-f)}{n}(R_2^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p4})=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_4^2,\quad R_4=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+TM}$$

$$MSE(\hat{\bar{Y}}_{p4})=\frac{(1-f)}{n}(R_4^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p5})=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_5^2,\quad R_5=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+MR}$$

$$MSE(\hat{\bar{Y}}_{p5})=\frac{(1-f)}{n}(R_5^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p6})=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_6^2,\quad R_6=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+HL}$$

$$MSE(\hat{\bar{Y}}_{p6})=\frac{(1-f)}{n}(R_6^2S_x^2+S_y^2(1-\rho^2)),$$

4 Efficiency Comparisons

Comparisons with existing ratio estimators

From the expressions of the MSE of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators as follows:

$$MSE(\hat{\bar{Y}}_{pj})\le MSE(\hat{\bar{Y}}_i),$$

$$\frac{(1-f)}{n}(R_{pj}^2S_x^2+S_y^2(1-\rho^2)\le \frac{(1-f)}{n}(R_i^2S_x^2+S_y^2(1-\rho^2)),$$

$$R_{pj}^2S_x^2\le R_i^2S_x^2,$$

$$R_{pj}\le R_i,$$

Where $j=1,2,....,6$ and $i=1,2,....,10$.

5 Applications

The performances of the proposed ratio estimators are evaluated and compared with the mentioned ratio estimators in Section 2 by using the data of the two natural populations. For the population I and II we use the data of

Singh and Chaudhary [10] page 177. We apply the proposed and existing estimators to these data sets and the data statistics of these populations are given in Table 1.
From Table 2, we observe that the proposed estimators are more efficient than all of the estimators in literature as their Bias, Constant and Mean Square error are much lower than the existing estimators.
The percentage relative efficiency (PRE) of the proposed estimators (p), with respective to the existing estimators (e), is computed by

$$PRE = \frac{MSE\,of\ Existing\ Estimator}{MSE\,of\ propoesd\,estimator} \times 100$$

These PRE values are given in Table 3 and Table 4 for the population I and Population II respectively. From these tables, it is clearly evident that the proposed estimators are quiet efficient with respect to the estimators in literature

Table 1: Characteristics of these populations.

Parameters	**Population 1**	**Population 2**
N	34	34
n	20	20
$\bar{Y}$	856.4117	856.4117
$\bar{X}$	199.4412	208.8823
ρ	0.4453	0.4491
S_y	733.1407	733.1407
C_y	0.8561	0.8561
S_x	150.2150	150.5059
C_x	0.7531	0.7205
β_2	1.0445	0.0978
β_1	1.1823	0.9782
TM	165.562	162.25
MR	320	284.5
HL	184	190

Table 2a: The Statistical Analysis of the Estimators for these Populations.

Estimators	**Population I**			**Population II**		
	Constant	**Bias**	**MSE**	**Constant**	**Bias**	**MSE**
$\overset{J}{Y}_1$	4.294	10.0023	17437.7	4.100	9.1539	16673.5
$\overset{J}{Y}_2$	4.278	9.9272	17373.3	4.086	9.0911	16619.6
$\overset{J}{Y}_3$	4.272	9.8983	17348.6	4.098	9.1454	16666.1
$\overset{J}{Y}_4$	4.279	9.9303	17376.0	3.960	8.5387	16146.6
$\overset{J}{Y}_5$	4.264	9.8646	17319.8	4.097	9.142	16663.3
$\overset{J}{Y}_6$	4.284	9.9578	17399.5	4.091	9.1147	16639.9
$\overset{J}{Y}_7$	4.281	9..9432	17387.1	4.088	9.0995	16626.9

$\hat{Y}_{8}$	4.258	9.8348	17294.2	4.069	9.0149	16554.4
$\hat{Y}_{9}$	4.285	9.9597	17401.1	4.011	8.7630	16338.7
$\hat{Y}_{10}$	4.244	9.7711	17239.7	4.096	9.1349	16654.2
$\hat{Y}_{p1}$	2.522	3.4523	11828.3	2.285	2.8440	11269.7
$\hat{Y}_{p2}$	1.822	1.8003	10413.5	1.7138	1.5993	10203.8
$\hat{Y}_{p3}$	2.412	3.1557	11574.3	2.1244	2.4578	10939.1
$\hat{Y}_{p4}$	2.392	3.1051	11531.1	0.4578	0.1141	8931.92
$\hat{Y}_{p5}$	1.693	1.5550	10203.5	0.2746	0.0411	8869.33
$\hat{Y}_{p6}$	2.280	2.8201	11286.9	0.3980	0.0863	8908.03

Table 3: PRE of the Proposed Estimators with the Estimators in Literature for population I.

	$\hat{Y}_{p1}$	$\hat{Y}_{p2}$	$\hat{Y}_{p3}$	$\hat{Y}_{p4}$	$\hat{Y}_{p5}$	$\hat{Y}_{p6}$
$\hat{Y}_{1}$	147.4236	167.4528	150.6588	151.2232	170.8992	154.4950
$\hat{Y}_{2}$	146.8791	166.8344	150.1024	150.6647	170.2680	153.9245
$\hat{Y}_{3}$	146.6703	166.5972	149.8890	150.4505	170.0260	153.7056
$\hat{Y}_{4}$	146.9019	166.8603	150.1257	150.6881	170.2945	153.9484
$\hat{Y}_{5}$	146.4268	166.3206	149.6402	150.2008	169.7437	153.4505
$\hat{Y}_{6}$	147.1006	167.0860	150.3287	150.8919	170.5248	154.1566
$\hat{Y}_{7}$	146.9958	166.9669	150.2216	150.7844	170.4033	154.0467
$\hat{Y}_{8}$	146.2104	166.0748	149.4190	149.9788	169.4928	153.2236
$\hat{Y}_{9}$	147.1141	167.1014	150.3426	150.9058	170.5405	154.1708
$\hat{Y}_{10}$	145.7496	165.5514	148.9481	149.5061	168.9587	152.7408

Table 4: PRE of the Proposed Estimators with the Estimators in Literature for population II.

	$\hat{Y}_{p1}$	$\hat{Y}_{p2}$	$\hat{Y}_{p3}$	$\hat{Y}_{p4}$	$\hat{Y}_{p5}$	$\hat{Y}_{p6}$
$\hat{Y}_1$	147.9498	163.4048	152.4211	186.6732	187.9905	187.1738
$\hat{Y}_2$	147.4715	162.8766	151.9284	186.0697	187.3828	186.5687
$\hat{Y}_3$	147.8841	163.3323	152.3535	186.5903	187.9071	187.0907
$\hat{Y}_4$	143.2744	158.2410	147.6045	180.7741	182.0498	181.2589
$\hat{Y}_5$	147.8593	163.3048	152.3279	186.5590	187.8755	187.0593
$\hat{Y}_6$	147.6517	163.0755	152.1140	186.2970	187.6117	186.7966
$\hat{Y}_7$	147.5363	162.9481	151.9951	186.1515	187.4651	186.6507
$\hat{Y}_8$	146.8930	162.2376	151.3324	185.3398	186.6477	185.8368
$\hat{Y}_9$	144.9790	160.1237	149.3606	182.9248	184.2157	183.4154
$\hat{Y}_{10}$	147.7786	163.2157	152.2447	186.4571	187.7729	186.9572

6 Conclusions

So in the present study using the auxiliary information of coefficient of kurtosis, coefficient of skewness and non-conventional location parameters, we conclude that our proposed estimators are more efficient than the existing estimators as there mean square error and bias is much lower than the existing estimators. Hence we strongly recommend that our proposed estimators preferred over existing estimators for practical applications.

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