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# Characterization for Generalized Power Function Distribution Using Recurrence Relations Based on General Progressively Type-II Right Censored Order Statistics

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**Abstract:** In this article, we establish recurrence relations for single and product moments for the generalized power function distribution. Moreover we use the relation between the probability density function and distribution function and recurrence relations to characterize the generalized power function distribution based on general progressively Type-II right censored order statistics.

**Keywords:** Characterization; General Progressively Type-II Right Censored Order Statistics; Generalized Power Function Distribution; Recurrence Relations.

## 1 Introduction

In failure data analysis, it is common that some individuals cannot be observed for the full failure times. general progressively Type-II right censored order statistics (GPTIIRCOS) is a useful and more general scheme in which a specific fraction of individuals at risk may be removed from the study at each of several ordered failure times. Progressively censored samples have been considered, among others, by Davis and Feldstein (1979), Balakrishnan et al. (2001), and Guilbaud (2001). This scheme of censoring was generalized by Balakrishnan and Sandhu (1996) as follows: at time $X_0 \equiv 0$, $n$ units are placed on test; the first $k$ failure times, $X_1,\ldots,X_k$, are not observed; at time $X_i+0$, where $X_i$ is the $i^{th}$ ordered failure time $(i=k+1,\ldots,m-1)$, $R_i$ units are removed from the test randomly, so prior to the $(i+1)^{th}$ failure there are $n_i = n -- i-\sum_{j=k+1}^{i} R_j$ units on test; finally, at the time of the $m^{th}$ failure, $X_m$, the experiment is terminated, i.e., the remaining $R_m$ units are removed from the test. The $R_i$'s, $m$ and $r$ are prespecified integers which must satisfy the conditions: $0 \le k < m \le n$, $0 \le R_i \le n_{i-1}$ $for$ $i=k+1,\ldots,m-1$ with $n_k = n-k$ and $R_m = n_{m-1}-1$.
If the failure times are based on an absolutely continuous distribution function (cdf) $F$ with probability density function (pdf) $f$, the joint probability density function of the general progressively Type II censored failure times $X_{k+1:m:,n},\ldots,X_{m:m:,n}$ , is given by

$$f_{X_{k+1:m:,n},\ldots,X_{m:m:,n}}(x_{k+1},\ldots,x_m) = K_{(n,m-1)}[F(x_{k+1},\theta)]^k \prod_{i=k+1}^{m} f(x_i,\theta)[1-F(x_i,\theta)]^{R_i}, \quad x_{k+1}<x_{k+2}<\cdots<x_m, \tag{1}$$

where,

$$K_{(n,m-1)} = \frac{n!}{k!\,(n-k-1)!}\left(\prod_{j=k+2}^{m} n - \sum_{i=k+1}^{j-1} R_i - j + 1\right).$$

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Aggrawala and Balakrishnan (1996) derived recurrence relations for single and product moments of progressively Type-II right censored order statistics from exponential, Pareto and power function distributions and their truncated forms. Imtiyaz et al. (2015) derived translation, contraction and dilation of dual generalized order statistics. Mohie El-Din et al. (2017a,b) derived characterization for Gompertz and linear failure rate distributions using recurrence relations of single and product moments based on general progressively Type-II right censored order statistics.
In this paper, we shall introduce recurrence relations among single and product moments based on GPTIIRCOS. Characterization for generalized power function distribution (GPFD) using recurrence relations are obtained.
Let $X_{k+1:m:n}^{(R_{k+1},R_{k+2},\dots,R_m)} < \cdots < X_{m:m:n}^{(R_{k+1},R_{k+2},\dots,R_m)}$ be the $m-k$ ordered observed failure times in a sample of size $(n-k)$ under general progressively Type-II right censoring scheme from the GPFD with pdf is given by (see Imtiyaz et al. (2015))

$$f(x) = \alpha x^{\alpha-1}[1-(m+1)x^{\alpha}]^{\frac{-m}{m+1}}, \qquad m>-1, \quad \alpha>0\,, \quad 0\leq x<(m+1)^{\frac{1}{\alpha}}, \tag{2}$$

and cdf is given by

$$F(x) = 1-[1-(m+1)x^{\alpha}]^{\frac{1}{m+1}}. \tag{3}$$

It may be noted that from (2) and (3) the relation between pdf and cdf is given by,

$$[1-(m+1)x^{\alpha}]\, f(x) = \alpha x^{\alpha-1}[1-F(x)]\,. \tag{4}$$

For any continuous distribution, we shall denote the $i^{th}$ single moment based on GPTIIRCOS in view of Eq. (1) as

$$\begin{aligned} \mu_{q:m:n}^{(R_{k+1},R_{k+2},\dots,R_m)^{(i)}} &= E\left[X_{q:m:n}^{(R_{k+1},R_{k+2},\dots,R_m)}\right]^i \\ &= K_{(n,m-1)}\iint\cdots\int_{0<x_{k+1}<\cdots<x_m<\infty} x_q^i[F(x_{k+1})]^k f(x_{k+1})[1-F(x_{k+1})]^{R_{k+1}}\times \\ &\qquad f(x_{k+2})[1-F(x_{k+2})]^{R_{k+2}}\dots f(x_m)[1-F(x_m)]^{R_m}dx_{k+1}\dots dx_m, \end{aligned} \tag{5}$$

and the $i^{th}$ and $j^{th}$ product moments as

$$\begin{aligned} \mu_{q,s:m:n}^{(R_{k+1},R_{k+2},\dots,R_m)^{(i,j)}} =& K_{(n,m-1)}\iint\cdots\int_{0<x_{k+1}<\cdots<x_m<\infty} x_q^i x_s^j[F(x_{k+1})]^k f(x_{k+1})\times \\ &[1-F(x_{k+1})]^{R_{k+1}} f(x_{k+2})[1-F(x_{k+2})]^{R_{k+2}}\dots f(x_m)[1-F(x_m)]^{R_m}dx_{k+1}\dots dx_m. \end{aligned} \tag{6}$$

## 2 Recurrence Relations of Single and Product Moments

In this section, we introduce the recurrence relations for single and product moments based on GPTIIRCOS.
In the next theorem we introduce the recurrence relation for single moments based on GPTIIRCOS.

**Theorem 1.** *For* $k+2\leq r\leq m-1,\ m\leq n$ *and* $i\geq 0,$

$$\begin{aligned} \mu_{r:m:n}^{(R_{k+1},R_{k+2},\dots,R_m)^{(i)}} &- (m+1)\,\mu_{r:m:n}^{(R_{k+1},\dots,R_m)^{(i+\alpha)}} = \alpha\left(\frac{R_r+1}{i+\alpha}\right)\mu_{r:m:n}^{(R_{k+1},\dots,R_m)^{(i+\alpha)}} \\ &-(n-R_{k+1}-\cdots-R_{r-1}-r+1)\left[\frac{\alpha}{(i+\alpha)}\mu_{r-1:m-1:n}^{\left(R_{k+1},R_{k+2},\dots,R_{r-2},(R_{r-1}+R_r+1),R_{r+1},\dots,R_m\right)^{(i+\alpha)}}\right] \\ &+(n-R_{k+1}-\cdots-R_r-r)\left[\frac{\alpha}{(i+\alpha)}\mu_{r:m-1:n}^{(R_{k+1},R_{k+2},\dots,R_{r-1},(R_r+R_{r+1}+1),R_{r+2},\dots,R_m)^{(i+\alpha)}}\right]. \end{aligned} \tag{7}$$

*Proof.* From Eq. (4) and Eq. (5), we get

$$\begin{aligned} \mu_{r:m:n}^{(R_{k+1},R_{k+2},\dots,R_m)^{(i)}} &- (m+1)\,\mu_{r:m:n}^{(R_{k+1},\dots,R_m)^{(i+\alpha)}} = \\ &\alpha\, K_{(n,m-1)}\iint\cdots\int_{0<x_{k+1}<\cdots<x_{r-1}<x_{r+1}<\cdots<x_m<\infty}[F(x_{k+1})]^k Y_1(x_{r-1},x_{r+1})\ \cdots\times \\ &f(x_{k+1})[1-F(x_{k+1})]^{R_{k+1}}\dots f(x_{r-1})[1-F(x_{r-1})]^{R_{r-1}} f(x_{r+1})[1-F(x_{r+1})]^{R_{r+1}}\dots\times \\ &f(x_m)[1-F(x_m)]^{R_m}dx_{k+1}dx_{k+2}\dots dx_{r-1}dx_{r+1}\dots dx_m, \end{aligned} \tag{8}$$

where

$$Y_1(x_{r-1},x_{r+1})=\int_{x_{r-1}}^{x_{r+1}} x_r^{i+\alpha-1}[1-F(x_r)]^{R_r+1}\,dx_r. \tag{9}$$

Now, integrating by parts gives

$$Y_1(x_{r-1},x_{r+1})=\frac{x_{r+1}^{i+\alpha}[1-F(x_{r+1})]^{R_r+1}-x_{r-1}^{i+\alpha}[1-F(x_{r-1})]^{R_r+1}}{i+\alpha}+\left(\frac{R_r+1}{i+\alpha}\right)\int_{x_{r-1}}^{x_{r+1}} x_r^{i+\alpha} f(x_r)[1-F(x_r)]^{R_r}dx_r \tag{10}$$

Now substituting for the resultant expression of $Y_1(x_{r-1},x_{r+1})$ from Eq. (10) in Eq. (8) and simplifying, yields Eq. (7).

This completes the proof.

In the next two theorems, we shall introduce recurrence relations for product moments based on GPTIIRCOS.

**Theorem 2.** *For* $k+1\le r<s\le m-1,\quad m\le n$ *and* $i,j\ge 0$,

$$\begin{aligned}\mu_{r,s:m:n}^{(R_{k+1},\dots,R_m)^{(i,j)}}-(m+1)\,\mu_{r,s:m:n}^{(R_{k+1},\dots,R_m)^{(i+\alpha,j)}}&=\alpha\left(\frac{R_r+1}{i+\alpha}\right)\mu_{r,s:m:n}^{(R_{k+1},\dots,R_m)^{(i+\alpha,j)}}\\&-(n-R_{k+1}-\dots-R_{r-1}-r+1)\left[\frac{\alpha}{(i+\alpha)}\mu_{r-1,s-1:m-1:n}^{\left(R_{k+1},\dots,R_{r-2},(R_{r-1}+R_r+1),R_{r+1},\dots,R_m\right)^{(i+\alpha,j)}}\right]\\&+(n-R_{k+1}-\dots-R_r-r)\left[\frac{\alpha}{(i+\alpha)}\mu_{r,s-1:m-1:n}^{(R_{k+1},\dots,R_{r-1},(R_r+R_{r+1}+1),R_{r+2},\dots,R_m)^{(i+\alpha,j)}}\right].\end{aligned} \tag{11}$$

*Proof.* Similarly as proved in theorem 1.

**Theorem 3.** *For* $k+1\le r<s\le m-1,\quad m\le n$ *and* $i,j\ge 0$,

$$\begin{aligned}\mu_{r,s:m:n}^{(R_{k+1},\dots,R_m)^{(i,j)}}-(m+1)\,\mu_{r,s:m:n}^{(R_{k+1},\dots,R_m)^{(i,j+\alpha)}}&=\alpha\left(\frac{R_s+1}{j+\alpha}\right)\mu_{r,s:m:n}^{(R_{k+1},\dots,R_m)^{(i,j+\alpha)}}\\&-(n-R_{k+1}-\dots-R_{s-1}-s+1)\left[\frac{\alpha}{(j+\alpha)}\mu_{r,s-1:m-1:n}^{\left(R_{k+1},\dots,R_{s-2},(R_{s-1}+R_s+1),R_{s+1},\dots,R_m\right)^{(i,j+\alpha)}}\right]\\&+(n-R_{k+1}-\dots-R_s-s)\left[\frac{\alpha}{(j+\alpha)}\mu_{r,s-1:m-1:n}^{(R_{k+1},\dots,R_{s-1},(R_s+R_{s+1}+1),R_{s+2},\dots,R_m)^{(i,j+\alpha)}}\right].\end{aligned} \tag{12}$$

*Proof.* Similarly as proved in theorem 1.

## 3 The Characterization

In this section we introduce the characterization of GPFD using the relation between pdf and cdf and using recurrence relations for single and product moments based on GPTIIRCOS.

In the next theorem we introduce the characterization of the GPFD using relation between pdf and cdf.

**Theorem 4.** *Let* $X$ *be a continuous random variable with pdf* $f(\bullet)$, *cdf* $F(\bullet)$ *and survival function* $[1-F(\bullet)]$. *Then* $X$ *GPFD iff*

$$[1-(m+1)x^{\alpha}]\,f(x)=\alpha x^{\alpha-1}[1-F(x)]. \tag{13}$$

*Proof.* **Necessity:** From Eq. (2) and Eq. (3) we can easily obtain Eq. (13).

**Sufficiency:** Suppose that $X$ is a continuous random variable with pdf $f(\bullet)$ and cdf $F(\bullet)$. Suppose, also, that Eq. (13) is true. Then we have:

$$\frac{-d[1-F(x)]}{1-F(x)}=\frac{\alpha x^{\alpha-1}}{1-(m+1)x^{\alpha}}dx.$$

On integrating, we get

$$(m+1)\ln|1-F(x)|=\ln|1-(m+1)x^{\alpha}|+C,$$

where $C$ is an arbitrary constant.

Now, since $[1-F(0)]=1$, then putting $x=0$ in this equation, we get $C=0$.
Therefore,

$$F(x)=1-\left[1-(m+1)x^{\alpha}\right]^{\frac{1}{m+1}}.$$

That is the distribution function of GPFD. This completes the proof.

In the next theorem, we introduce the characterization of the GPFD using recurrence relation for single moments based on GPTIIRCOS.

**Theorem 5.***Let $X$ be a continuous random variable with pdf $f(\bullet)$, cdf $F(\bullet)$ and survival function $[1-F(\bullet)]$. Let $X_{k+1:n} \leq \ldots \leq X_{n:n}$ be the order statistics of a random sample of size $(n-k)$. Then $X$ has GPFD iff, for $k+2 \leq r \leq m-1,\ m \leq n$ and $i \geq 0$,*

$$\begin{aligned}\mu_{r:m:n}^{(R_{k+1},R_{k+2},\ldots,R_m)^{(i)}}-(m+1)\mu_{r:m:n}^{(R_{k+1},\ldots,R_m)^{(i+\alpha)}}&=\alpha\left(\frac{R_r+1}{i+\alpha}\right)\mu_{r:m:n}^{(R_{k+1},\ldots,R_m)^{(i+\alpha)}}\\&-(n-R_{k+1}-\cdots-R_{r-1}-r+1)\left[\frac{\alpha}{(i+\alpha)}\mu_{r-1:m-1:n}^{(R_{k+1},R_{k+2},\ldots,R_{r-2},(R_{r-1}+R_r+1),R_{r+1},\ldots,R_m)^{(i+\alpha)}}\right]\\&+(n-R_{k+1}-\cdots-R_r-r)\left[\frac{\alpha}{(i+\alpha)}\mu_{r:m-1:n}^{(R_{k+1},R_{k+2},\ldots,R_{r-1},(R_r+R_{r+1}+1),R_{r+2},\ldots,R_m)^{(i+\alpha)}}\right].\end{aligned}\tag{14}$$

*Proof.* **Necessity:** Theorem 1 proved the necessary part of this theorem.

**Sufficiency:** Suppose that $X$ is a continuous random variable with pdf $f(\bullet)$ and cdf $F(\bullet)$. Assuming that equation (14) holds and from Eq. (5), we get then we have:

$$\begin{aligned}\mu_{r:m:n}^{(R_{k+1},\ldots,R_m)^{(i+\alpha)}}=&K_{(n,m-1)}\iint\cdots\int_{0<x_{k+1}<\cdots<x_{r-1}<x_{r+1}<\cdots<x_m<\infty}[F(x_{k+1})]^k Y_2(x_{r-1},x_{r+1})\times\\&f(x_{k+1})[1-F(x_{k+1})]^{R_{k+1}}\ldots f(x_{r-1})[1-F(x_{r-1})]^{R_{r-1}}f(x_{r+1})[1-F(x_{r+1})]^{R_{r+1}}\cdots\times\\&f(x_m)[1-F(x_m)]^{R_m}dx_{k+1}\ldots dx_{r-1}dx_{r+1}\ldots dx_m,\end{aligned}\tag{15}$$

where

$$Y_2(x_{r-1},x_{r+1})=\int_{x_{r-1}}^{x_{r+1}}x_r^{i+\alpha}f(x_r)[1-F(x_r)]^{R_r}dx_r.\tag{16}$$

Using, integrating by parts of $Y_2(x_{r-1},x_{r+1})$ and by substituting in Eq. (15), we get

$$\begin{aligned}\mu_{r:m:n}^{(R_{k+1},R_{k+2},\ldots,R_m)^{(i)}}-(m+1)\mu_{r:m:n}^{(R_{k+1},\ldots,R_m)^{(i+\alpha)}}=&\alpha K_{(n,m-1)}\iint\cdots\int_{0<x_{k+1}<\cdots<x_m<\infty}x_r^i\left(x_r^{\alpha-1}\right)\\&[F(x_{k+1})]^k[1-F(x_r)]^{R_r+1}f(x_{k+1})[1-F(x_{k+1})]^{R_{k+1}}\ldots f(x_{r-1})\times\\&[1-F(x_{r-1})]^{R_{r-1}}f(x_{r+1})[1-F(x_{r+1})]^{R_{r+1}}\ldots f(x_m)[1-F(x_m)]^{R_m}dx_{k+1}\ldots dx_m.\end{aligned}\tag{17}$$

We get

$$\begin{aligned}K_{(n,m-1)}\iint\cdots\int_{0<x_{k+1}<\cdots<x_m<\infty}&x_r^i[1-F(x_r)]^{R_r}[F(x_{k+1})]^k f(x_{k+1})[1-F(x_{k+1})]^{R_{k+1}}\cdots\times\\&f(x_{r-1})[1-F(x_{r-1})]^{R_{r-1}}f(x_{r+1})[1-F(x_{r+1})]^{R_{r+1}}\ldots f(x_m)[1-F(x_m)]^{R_m}\times\\&\left[\left[1-(m+1)x_r^{\alpha}\right]f(x_r)-\alpha x_r^{\alpha-1}[1-F(x_r)]\right]dx_{k+1}\ldots dx_m=0.\end{aligned}$$

Using Muntz-Szasz theorem, [See, Hwang and Lin [6]], we get

$$\left[1-(m+1)x_r^{\alpha}\right]f(x_r)=\alpha x_r^{\alpha-1}[1-F(x_r)].$$

Using Theorem 4, we get the distribution function

$$F(x)=1-\left[1-(m+1)x^{\alpha}\right]^{\frac{1}{m+1}}.$$

This completes the proof.

In the next two theorems, we introduce the characterization of the GPFD using recurrence relation for product moments based on GPTIIRCOS.

**Theorem 6.** *Let $X$ be a continuous random variable with pdf $f(\bullet)$, cdf $F(\bullet)$ and survival function $[1-F(\bullet)]$. Let $X_{k+1:n} \leq \ldots \leq X_{n:n}$ be the order statistics of a random sample of size $(n-k)$. Then $X$ has GPFD iff, for $k+1 \leq r < s \leq m-1$, $m \leq n$ and $i, j \geq 0$,*

$$\mu_{r,s:m:n}^{(R_{k+1},\ldots,R_m)^{(i,j)}} - (m+1)\,\mu_{r,s:m:n}^{(R_{k+1},\ldots,R_m)^{(i+\alpha,j)}} = \alpha\left(\frac{R_r+1}{i+\alpha}\right)\mu_{r,s:m:n}^{(R_{k+1},\ldots,R_m)^{(i+\alpha,j)}} - (n-R_{k+1}-\cdots-R_{r-1}-r+1)\left[\frac{\alpha}{(i+\alpha)}\mu_{r-1,s-1:m-1:n}^{\left(R_{k+1},\ldots,R_{r-2},(R_{r-1}+R_r+1),R_{r+1},\ldots,R_m\right)^{(i+\alpha,j)}}\right] + (n-R_{k+1}-\cdots-R_r-r)\left[\frac{\alpha}{(i+\alpha)}\mu_{r,s-1:m-1:n}^{(R_{k+1},\ldots,R_{r-1},(R_r+R_{r+1}+1),R_{r+2},\ldots,R_m)^{(i+\alpha,j)}}\right]. \tag{18}$$

*Proof.* Similarly as proved in theorem 5.

**Theorem 7.** *Let $X$ be a continuous random variable with pdf $f(\bullet)$, cdf $F(\bullet)$ and survival function $[1-F(\bullet)]$. Let $X_{k+1:n} \leq \ldots \leq X_{n:n}$ be the order statistics of a random sample of size $(n-k)$. Then $X$ has GPFD iff, for $k+1 \leq r < s \leq m-1$, $m \leq n$ and $i, j \geq 0$,*

$$\mu_{r,s:m:n}^{(R_{k+1},\ldots,R_m)^{(i,j)}} - (m+1)\,\mu_{r,s:m:n}^{(R_{k+1},\ldots,R_m)^{(i,j+\alpha)}} = \alpha\left(\frac{R_s+1}{j+\alpha}\right)\mu_{r,s:m:n}^{(R_{k+1},\ldots,R_m)^{(i,j+\alpha)}} - (n-R_{k+1}-\cdots-R_{s-1}-s+1)\left[\frac{\alpha}{(j+\alpha)}\mu_{r,s-1:m-1:n}^{\left(R_{k+1},\ldots,R_{s-2},(R_{s-1}+R_s+1),R_{s+1},\ldots,R_m\right)^{(i,j+\alpha)}}\right] + (n-R_{k+1}-\cdots-R_s-s)\left[\frac{\alpha}{(j+\alpha)}\mu_{r,s-1:m-1:n}^{(R_{k+1},\ldots,R_{s-1},(R_s+R_{s+1}+1),R_{s+2},\ldots,R_m)^{(i,\alpha+j)}}\right]. \tag{19}$$

*Proof.* Similarly as proved in theorem 5.

## 4 Conclusions

Above investigations demonstrated that using the above relations, we can obtain all the single and product moments of all GPTIIRCOS for all sample sizes and all censoring schemes in a simple recursive way. Since recurrence relations reduce the amount of direct computation and hence the time, cost and labour. Therefore the relations under consideration may be useful in computing the moments of any order of GPTIIRCOS from GPFD.

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