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Discrete Burr Type XII Minimax Distribution: A New Discrete Model

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Abstract: In this article, we attempt to introduce a new count data model which is obtained by compounding discrete Burr type XII distribution with Minimax distribution. Several distributional properties of the model have been discussed. Finally, real data set is analyzed to investigate the suitability of the proposed distribution in modeling count.

Keywords: Discrete Burr type XII Distribution, Minimax distribution, compound distribution, count data, reliability.

1 Introduction

Researchers employ different techniques like discretization [18,19,20], transmutation [21], compounding [17] etc. to generate new probability distributions to handle complex data. Compound distribution arises when all or some parameters of a distribution known as parent distribution vary according to some probability distribution called the compounding distribution, for instance negative binomial distribution can be obtained from Poisson distribution when its parameter λ follows gamma distribution. If the parent distribution is discrete then resultant compound distribution will also be discrete and if the parent distribution is continuous then resultant compound distribution will also be continuous i,e. the support of the original (parent) distribution determines the support of compound distributions. In several research papers it has been found that compound distributions are very flexible and can be used efficiently to model different types of data sets. With this in mind many compound probability distributions have been constructed. In the early 1970s, Dubey [10] derived a compound gamma, Minimax and F distribution by compounding a gamma distribution with another gamma distribution and reduced it to the beta Ist and 2nd kind and to the F distribution by suitable transformations. Sankaran [1] introduced a compound of Poisson distribution with that of Lindley distribution for modeling count data. Gerstenkorn [11,12] proposed several compound distributions, he obtained compound of gamma distribution with exponential distribution by treating the parameter of gamma distribution as an exponential variate and also obtained compound of polya with beta distribution. Ghitany, Al-Mutairi and Nadarajah [2,3] introduced zero-truncated Poisson-Lindley distribution, who used the distribution for modeling count data in the case where the distribution has to be adjusted for the count of missing zeros. Zamani and Ismail [4] constructed a new compound distribution by compounding negative binomial with one parameter Lindley distribution that provides good fit for count data where the probability at zero has a large value.

In this paper we propose a new count data model by compounding two parameter discrete Burr type XII distribution with Minimax distribution, as there is a need to find more plausible discrete probability models or survival models in medical science and other fields, to fit to various discrete data sets. It is well known in general that a compound model is more flexible than the ordinary model and it is preferred by many data analysts in analyzing statistical data. Moreover, it presents beautiful mathematical exercises and broadened the scope of the concerned model being compounded.

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2 Material and Methods

A discrete analogue of the continuous Burr type XII distribution was introduced by Krishna and Punder [6], and is defined by the probability mass function (pmf):

$$f_1(x;q,\gamma)=q^{\log(1+x^{\gamma})}-q^{\log(1+(x+1)^{\gamma})},\quad x=0,1,2,...... \tag{1}$$

where $\gamma>0$ and $0<q<1$ are its parameter. The first and the second moments of the discrete Burr type XII random variable X are given by

$$E(X)=\sum_{x=1}^{\infty}q^{\log(1+x^{\gamma})}$$

$$E(X^2)=2\sum_{x=1}^{\infty}xq^{\log(1+x^{\gamma})}+E(X) \tag{2}$$

There are various types of life time models such as exponential, Pareto and Gamma that are used in reliability and life testing. Jones [15] studied two-parameter distribution on (0,1) which he has called the Minimax distribution, Minimax (α,β) ,where its two shape parameters α and β are positive. It has many of the same properties as the beta distribution but has some advantages in terms of tractability. The probability density function of Minimax distribution is given by

$$f(x;\gamma,\alpha,\beta)=\alpha\beta x^{\alpha-1}(1-x^{\alpha})^{\beta-1};\qquad 0<x<1;\alpha,\beta>0 \tag{3}$$

If we put $\alpha=\beta=1$, the equation (3) reduces to Uniform distribution.

Where $\alpha,\beta>0$ are shape parameters. The raw moments of Minimax distribution (MD) are given by

$$E(X^r)=\int_0^1 x^r f_2(X;\alpha,\beta)dx$$

$$E(X^r)=\frac{\beta\Gamma(1+\frac{1}{\alpha})\Gamma(\beta)}{\Gamma\left(1+\frac{1}{\alpha}+\beta\right)} \tag{4}$$

Minimax distribution is not very popular among statisticians because researchers have not analyzed and investigated it systematically in much detail. Minimax distribution is similar to the Beta distributionbut unlike beta distribution it has a closed form of cumulative distribution function which makes it very simple to deal with. For more detailed properties one can see references [15,16]

Usually the parameter q in DBXIID is fixed constant but here we have considered a problem in which the probability parameter q is itself a random variable following Minimax with pdf (3).

3 Definition of proposed model

If $X|q\sim$ DBXII(q,γ) where q is itself a random variable following Minimax distribution Minimax(α,β), then determining the distribution that results from marginalizing over q will be known as a compound of discrete Burr type XII distribution with that of Minimax distribution, which is denoted by DBXIIMD(α,β,γ). It may be noted that proposed model will be a discrete since the parent distribution DBXIID is discrete.

Theorem 3.1: The probability mass function of a compound of DBXII(q,γ) with Minimax(α,β) is given by

$$f_{DBXIIMD}(X;\alpha,\beta,\gamma)=\beta[B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1],$$

where $x=0,1,2,......$ and $\alpha,\beta,\gamma>0$

Proof: Using the definition (3), the pmf of a compound of DBXIID (q,γ) with Minimax (α,β) can be obtained as

$$f_{DBXIIMD}(X;\alpha,\beta,\gamma)=\int_0^1 f_1(x|q)\,f_2(q)\,dq$$

$$f_{DBXIIMD}(X;\alpha,\beta,\gamma)=\int_0^1 (q^{\log(1+x^{\gamma})}-q^{\log(1+(x+1)^{\gamma})})\,q^{\alpha-1}(1-q^{\alpha})^{\beta-1}\,dq$$

$$f_{DBXIIMD}(X;\alpha,\beta,\gamma)=\beta[B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1)]$$

$$f_{DBXIIMD}(X;\alpha,\beta,\gamma)=\left[\frac{\Gamma(\beta)\,\Gamma\left(\frac{\log(1+x^{\gamma})}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{\log(1+x^{\gamma})}{\alpha}+1\right)}-\frac{\Gamma(\beta)\,\Gamma\left(\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1\right)}\right] \tag{5}$$

where $x=0,1,2,...$ and $\alpha,\beta,\gamma>0$. From here a random variable X following a compound of DBXIID with Minimax will be symbolized by DBXIIMD (α,β,γ).

Fig.1 provides a pmf plot of the proposed model DBXIIM $(x;\alpha,\beta,\gamma)$ for different values of parameters. It is evident that the proposed model is right skewed with unimodel behavior.

The Cumulative distribution function of the DBXIIMD (α,β,γ) is given by

$$F(x)=1-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1)\,; \qquad x=0,1,2,...\text{ and } (\alpha>0,\beta>0,\gamma>0).$$

Where $B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)=\frac{\Gamma(\beta)\,\Gamma\left(\frac{\log(1+x^{\gamma})}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{\log(1+x^{\gamma})}{\alpha}+1\right)}$

Fig.2 provides a CDF plot of the proposed model DBXIIMD (α,β,γ) for different values of parameters. The initial rise of the CDF plot decreases as α increases but as β increases, initial rise of the CDF plot also increases.

3.1 Random Data Generation from Discrete Burr Type XII Minimax distribution

For simulating random sample data of size n, $x_1,x_2,....,x_n$ of the discrete Burr Type XII Minimax random variable X with pmf $p(X=x_i)=p_i,\sum_{i=0}^{k}p_i=1$ and a cdf F(x), where k may be finite or infinite can be described as

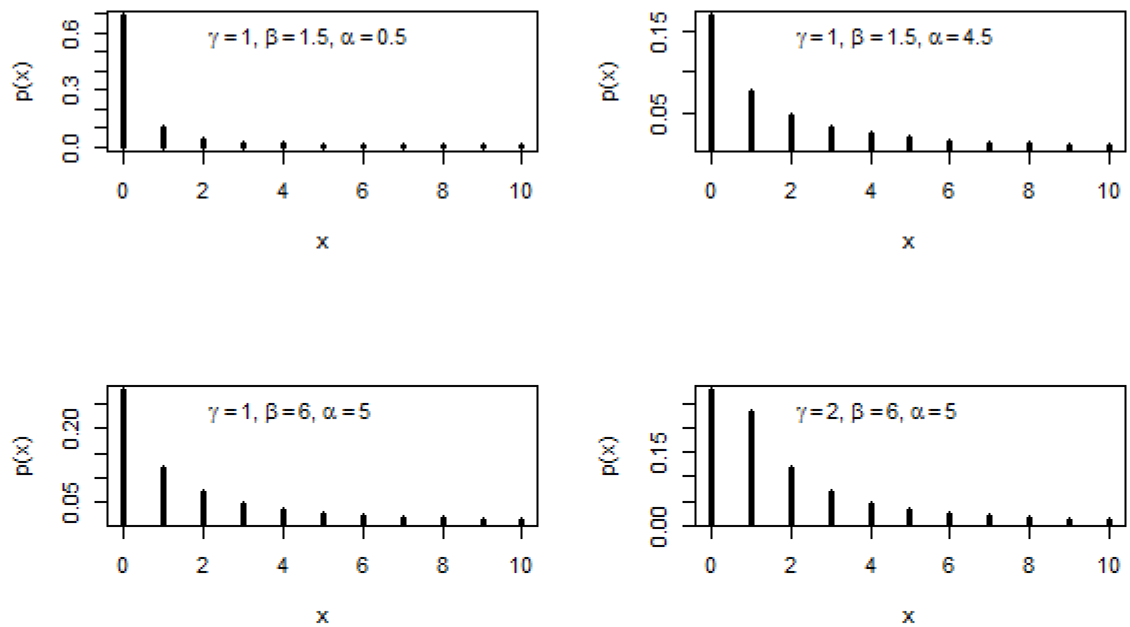

Fig1 . Pmf plot of Discrete Burr type XII Minimax distribution

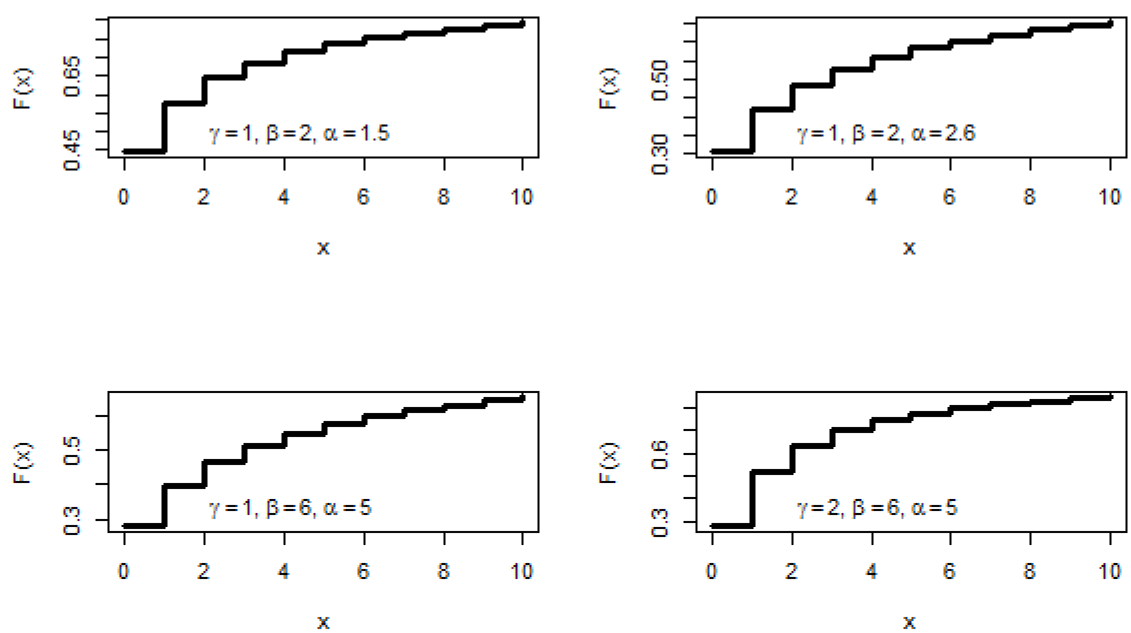

Fig 2. CDF plot of Discrete Burr type XII Minimax distribution

Step1: Generate a random number u from uniform distribution U(0,1).

Step2: Generate random number x_i based on

$$
\begin{aligned}
&if\ u \le p_0 = F(x_0)\ then\ X = x_0 \\
&if\ p_0 < u \le p_0 + p_1 = F(x_1)\ then\ X = x_1 \\
&. \\
&. \\
&.if\ \sum_{j=0}^{k-1} p_j < u \le \sum_{j=0}^{k} p_j = F(x_k)\ then\ X = x_k
\end{aligned}
$$

In order to generate n random numbers from Discrete Burr Type XII Minimax distribution, $x_1, x_2,...., x_n$, repeat step 1 to step 2 n times.

4 Nested Distributions

Discrete Burr XII and Minimax distributions have six special combinations. For Minimax distribution, we have the following cases.

Case 1: When $\alpha = \beta = 1$, then Minimax distribution (1.3) reduces to UD (Uniform distribution) with probability density function as $f(x) \;\; = \; 1; \quad 0 < x < 1$

Case 2: When $\beta = 1$, then Minimax distribution (1.3) reduces to Power distribution (PD) with probability density function as $f(x;\alpha) = \alpha \; x^{\alpha-1}; \quad 0 < x < 1 \quad ,\alpha > 0$

Case 3: When $\alpha = 1$, then Minimax distribution (1.3) reduces to one parameter Minimax distribution with probability density function as $f(x;\beta) = \beta \; (1-x)^{\beta-1}; \quad 0 < x < 1 \quad ,\beta > 0$

In case of discrete Burr type XII distribution, when $\gamma = 1$, discrete Burr type XII reduces to Pareto distribution.

In this particular section we show that the proposed model can be nested to different models under specific parameter setting.

Proposition 4.1: If $X \sim DBXIIMD(\alpha,\beta,\gamma)$ then by setting $\alpha = \beta = 1$, we obtain a compound of DBXII distribution with uniform distribution.

Proposition 4.2: If then by setting $\alpha = \beta = \gamma = 1$, we obtain a compound of discrete Pareto distribution with uniform distribution. $X \sim DBXIIMD(\alpha,\beta,\gamma)$

Proposition 4.3: If $X \sim DBXIIMD(\alpha,\beta,\gamma)$ then by setting $\alpha = \beta = \gamma = 1$, we obtain a compound of discrete Pareto distribution with uniform distribution.

Proposition 4.4: If $X \sim DBXIIMD(\alpha,\beta,\gamma)$ then by setting $\beta = \gamma = 1$, we obtain a compound of discrete Pareto distribution with Power distribution.

Proposition 4.5: If $X \sim DBXIIMD(\alpha,\beta,\gamma)$ then by setting $\beta = 1$, we obtain a compound of discrete Burr type XII distribution with Power distribution.

Proposition 4.6: If $X \sim DBXIIMD(\alpha, \beta, \gamma)$ then by setting $\alpha = 1$, we obtain a compound of discrete Burr type XII distribution with standard Minimax distribution.

5 Reliability Measures of Compound Discrete Burr type XII Minimax Distribution

If $X \sim DBXIIMD(X; \alpha, \beta, \gamma)$, then the various reliability measures of a random variable X are given by

(I) Survival Function.

$$s(x) = \beta\, B(\beta, \log(1+x^{\gamma})/\alpha + 1),\ x = 0,1,2,... \text{ and } (\alpha > 0, \beta > 0, \gamma > 0), \tag{6}$$

where $B(\beta, \log(1+x^{\gamma})/\alpha + 1) = \dfrac{\Gamma(\beta)\,\Gamma(\log(1+x^{\gamma})/\alpha + 1)}{\Gamma(\beta + \log(1+x^{\gamma})/\alpha + 1)}$

(II) Rate of Failure Function.

$$r(x) = \frac{p(x)}{s(x)} = 1 - \frac{B(\beta, \frac{\log(1+(x+1)^{\gamma})}{\alpha} + 1)}{B(\beta, \log(1+x^{\gamma})/\alpha + 1)},\ x = 0,1,2... \text{ and } \alpha > 0, \beta > 0, \gamma > 0 \tag{7}$$

(III) Second Rate of Failure Function.

$$h(x) = \log\left(\frac{s(x)}{s(x+1)}\right) = \log\left(\frac{B(\beta, \log(1+x^{\gamma})/\alpha + 1)}{B(\beta, \log(1+(x+1)^{\gamma})/\alpha + 1)}\right),\quad x = 0,1,2,... \text{ and } \alpha > 0, \beta > 0, \gamma > 0 \tag{8}$$

where, B(.) refers to the beta function defined by $B(a,b) = \dfrac{\Gamma a\, \Gamma b}{\Gamma(a+b)}$

Fig 3 provides the first rate of failure plot for DBXIIM distribution for different values of parameters.

6 MGF and PGF Of discrete Burr Type XII Minimax Distribution

(a) The MGF (moment generating function) of the Compound discrete Burr type XII Minimax distribution is

$$M_x(t) = \sum_{x=0}^{\infty} e^{tx} p(x)$$

$$M_x(t) = \sum_{x=0}^{\infty} e^{tx}\left[\frac{1}{B(\alpha,\beta)}[B(\beta, \frac{\log(1+x^{\gamma})}{\alpha} + 1) - B(\beta, \frac{\log(1+(x+1)^{\gamma})}{\alpha} + 1)]\right]$$

$$M_x(t) = \sum_{x=0}^{\infty} e^{tx}[\psi(x; \beta, \alpha, \gamma) - \psi(x+1; \beta, \alpha, \gamma)],$$

where $\psi(x; \beta, \alpha, \gamma) = \beta\, B(\beta, \dfrac{\log(1+x^{\gamma})}{\alpha} + 1)$

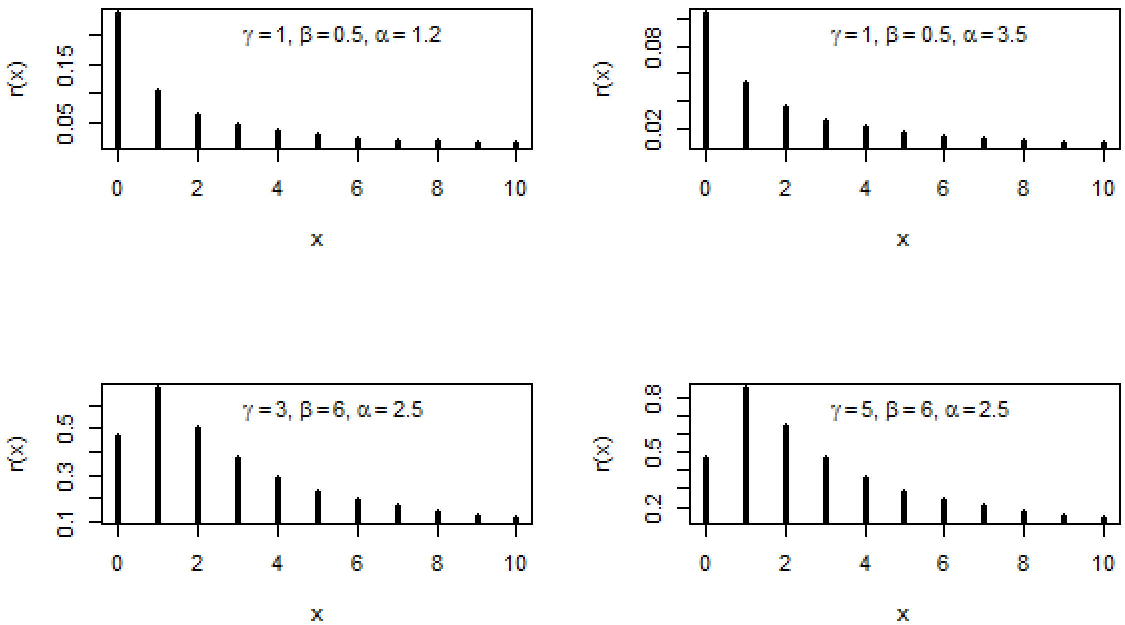

Fig 3. r(X) plot of Discrete Burr type XII Minimax distribution

$$M_x(t)=\begin{pmatrix}\psi(0;\beta,\alpha,\gamma)+e^{t}\psi(1;\beta,\alpha,\gamma)+e^{2t}\psi(2;\beta,\alpha,\gamma)+e^{3t}\psi(3;\beta,\alpha,\gamma)+....-\{\psi(1;\beta,\alpha,\gamma)\\ +e^{t}\psi(2;\beta,\alpha,\gamma)+e^{2t}\psi(3;\beta,\alpha,\gamma)+e^{3t}\psi(4;\beta,\alpha,\gamma)+......\}\end{pmatrix}$$

$$M_x(t)=\psi(0;\gamma,\beta,\alpha)+(e^{t}-1)\psi(1;\gamma,\beta,\alpha)+(e^{2t}-e^{t})\psi(2;\gamma,\beta,\alpha)+(e^{3t}-e^{2t})\psi(3;\gamma,\beta,\alpha)+....$$

$$M_x(t)=1+\sum_{x=1}^{\infty}(e^{xt}-e^{(x-1)t})\psi(x;\beta,\alpha,\gamma)$$

Differentiating $M_x(t)$ r times with respect to t

$$M_x^{(r)}(t)=\sum_{x=1}^{\infty}(x^{r}e^{xt}-(x-1)^{r}e^{(x-1)t})\psi(x;\beta,\alpha,\gamma)$$

First four moments of the proposed model are given by

$$\mu_1'=\sum_{x=1}^{\infty}\psi(x;\beta,\alpha,\gamma)$$

$$\mu_2'=\sum_{x=1}^{\infty}(2x-1)\psi(x;\beta,\alpha,\gamma)$$

$$\mu_3'=\sum_{x=1}^{\infty}(3x^{2}-3x+1)\psi(x;\beta,\alpha,\gamma)$$

$$\mu_4' = \sum_{x=1}^{\infty}(4x^3 - 6x^2 + 4x - 1)\psi(x;\beta,\alpha,\gamma)$$

(b) Probability generating function of the Compound discrete Burr type XII Minimax distribution is

$$G_{[x]}(t) = \sum_{x=0}^{\infty} t^x p(x)$$

$$G_{[x]}(t) = \sum_{x=0}^{\infty} t^x \left[\beta[B(\beta, \frac{\log(1+x^\gamma)}{\alpha}+1) - B(\beta, \frac{\log(1+(x+1)^\gamma)}{\alpha}+1)]\right]$$

$$G_{[x]}(t) = \sum_{x=0}^{\infty} t^x [\psi(x;\beta,\alpha,\gamma) - \psi(x+1;\beta,\alpha,\gamma)]$$

Where $\psi(x;\beta,\alpha) = \beta\left\{B(\beta, \frac{\log(1+x^\gamma)}{\alpha}+1)\right\}$

$$G_{[x]}(t) = \psi(0;\gamma,\beta,\alpha) + (t-1)\psi(1;\gamma,\beta,\alpha) + t(t-1)\psi(2;\gamma,\beta,\alpha) + t^2(t-1)\psi(3;\gamma,\beta,\alpha) + ...$$

$$G_{[x]}(t) = 1 + (t-1)\sum_{x=1}^{\infty} t^{x-1}\psi(x;\beta,\alpha,\gamma)$$

Differentiating $G_{[x]}(t)$ with respect to t

$$G_{[x]}'(t) = \sum_{x=1}^{\infty}((t-1)(x-1)t^{x-2} + t^{x-1})\psi(x;\beta,\alpha,\gamma)$$

$$G_{[x]}'(t) = \sum_{x=1}^{\infty}(t^{x-2}(xt - x + 1))\psi(x;\beta,\alpha,\gamma)$$

$$G_{[x]}''(t) = \sum_{x=1}^{\infty}(x-1)t^{x-3}\{(t-1)(x-2) + 2t\}\psi(x;\beta,\alpha,\gamma)$$

At t=1, $G_{[x]}'(t)$, $G_{[x]}''(t)$ gives first and second factorial moments

$$E(x) = G_{[x]}'(1) = \sum_{x=1}^{\infty} \psi(x;\beta,\alpha,\gamma)$$

$$E(x^2) = G_{[x]}'(1) + G_{[x]}''(1) = \beta\sum_{x=1}^{\infty}(2x-1)\psi(x;\beta,\alpha,\gamma)$$

7 Parameter Estimation

In this section the estimation of parameters of $DBXIIMD(x;\alpha,\beta,\gamma)$ model will be discussed through method of moments and maximum likelihood estimation.

7.1 Moments Method of Estimation

In order estimate three unknown parameters of $DBXIIMD(x;\alpha,\beta,\gamma)$ model by the method of moments, we need to equate first three sample moments with their corresponding population moments.

$$m_1=\gamma_1;\ m_2=\gamma_2\ ;\ m_3=\gamma_3.$$

Where γ_i is the ith sample moment and m_i is the ith corresponding population moment and the solution for $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ may be obtained by solving above equations simultaneously through numerical methods.

7.2 Maximum Likelihood Method of Estimation

The estimation of parameters of $DBXIIMD(x;\alpha,\beta,\gamma)$ model via maximum likelihood estimation method requires the log likelihood function of $DBXIIMD(x;\alpha,\beta,\gamma)$

$$£(X;\alpha,\beta,\gamma)=\log L(X;\alpha,\beta,\gamma)=\sum_{i=1}^{n}\log(B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1))+n\log(\beta) \tag{9}$$

The maximum likelihood estimate of $\Theta=(\hat{\alpha},\hat{\beta},\hat{\gamma})^T$ can be obtained by differentiating (9) with respect unknown parameters α and β respectively and then equating them to zero.

$$\frac{\partial}{\partial\beta}£(X;\alpha,\beta,\gamma)=\sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\beta}\left(B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1))\right)}{\left(B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1)\right)}\right)+\frac{n}{\beta} \tag{10}$$

$$\frac{\partial}{\partial\alpha}£(X;\gamma,\alpha,\beta)=\sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\alpha}\left(B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1)\right)}{\left(B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1)\right)}\right) \tag{11}$$

$$\frac{\partial}{\partial\gamma}£(X;\gamma,\alpha,\beta)=\sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\gamma}\left(B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1)\right)}{\left(B(\beta,\frac{\log(1+x^{\gamma})}{\alpha}+1)-B(\beta,\frac{\log(1+(x+1)^{\gamma})}{\alpha}+1)\right)}\right) \tag{12}$$

These two derivative equations cannot be solved analytically, therefore $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ will be obtained by maximizing the log likelihood function numerically using Newton-Raphson method which is a powerful technique for solving equations iteratively and numerically.

8 Application of Discrete Burr type XII Minimax distribution

In this section, we present a real data set to examine the fit of the proposed model. MLE based on the likelihood Equations (10), (11) and (12) was used to obtain the parameter estimates of the proposed distribution, although it is also possible to perform a direct search of the maximum likelihood function to obtain the maximum likelihood estimators. This can be done using appropriate software such as R Studio statistical software.

In this section an attempt has been made to fit to data relating to automobile claims as given in table 1 (Automobile claims frequencies data in Willmot[1]), using discrete Burr type XII Minimax distribution (DBXIIMD) in comparison with some compound discrete models like, Poisson Akasha distribution (PAD) [9], Poisson Lindley distribution (PLD) [8] , Poisson Sujatha distribution (PSD) [5] and other classical discrete models.

Table 1. Automobile claim data studied by Willmot [7]

Count	0	1	2	3	4	5
Observed	3719	212	38	7	3	1

The ML estimates provided by the fitdistr procedure in R studio are given in the table 2.

Table 2. Estimated parameters by ML method for fitted distributions for Counts of Automobile claim data studied by Willmot [7]

Distribution	parameter Estimates	Model function
Discrete Burr type XII Minimax	$\beta = 35.01$ $\alpha = 0.94$ $\gamma = 1.25$	$p(x) = \beta[B(\beta, \frac{\log(1+x^{\gamma})}{\alpha}+1) - B(\beta, \frac{\log(1+(x+1)^{\gamma})}{\alpha}+1)]$ $x = 0,1,2,...$ for $\beta > 0, \alpha > 0, \gamma > 0$
Poisson	$\lambda = 0.08$,	$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$ $\lambda > 0;\ x = 0,1,2,.....$
Poisson Akasha	$\theta = 12.48$,	$p(x) = \frac{\theta^{3}(x^{2}+3x+(\theta^{2}+2\theta+2))}{(\theta^{2}+2)(\theta+1)^{x+3}}$ $x = 0,1,2,....\ \theta > 0$
Poisson Lindley	$\theta = 13.08$,	$p(x) = \frac{\theta^{2}(x+\theta+2)}{(\theta+1)^{x+3}}$, $x = 0,1,2,....\ \theta > 0$
Poisson Sujatha	$\theta = 13.30$,	$p(x) = \frac{\theta^{3}}{\theta^{2}+\theta+2}\frac{x^{2}+(\theta+4)x+(\theta^{2}+3\theta+4)}{(\theta+1)^{x+3}}$, $x = 0,1,2,....\ \theta > 0$
Discrete Rayleigh	$q = 0.12$,	$p(x) = q^{x^{2}} - q^{(x+1)^{2}}$, $0 < q < 1; x = 0,1,2,.....$

The p-value of Pearson's Chi-square statistic is 0.2210 for discrete Burr type XII Minimax distribution and <0.01for Poisson, discrete Rayleigh, Poisson Lindley, Poisson Akasha, and Poisson Sujatha distributions, respectively (see Table 3). This reveals that Poisson, discrete Rayleigh, PLD, PAD and PSD distributions are not good fit at all, whereas discrete Burr type XII Minimax model being the significant model for automobile claim data. The null hypothesis that data come from discrete inverse Burr type XII Minimax distribution is accepted

Table 3. Table for goodness of fit for Counts of Automobile claim data (Willmot [7]).

Observed	DBXIIBD	Poison	DRayleigh	PLD	PAD	PSD
3719	3725.62	3667.00	3509.20	3678.71	3678.13	3678.71
212	202.06	300.36	470.03	278.44	278.75	278.44
38	35.09	12.30	0.78	21.11	21.34	21.11
7	9.91	0.34	0.00	1.60	1.65	1.60
3	3.67	0.01	0.00	0.12	0.13	0.12
1	3.66	0.00	0.00	0.01	0.01	0.01
P-values	0.2210	<0.01	<0.01	0.0003	0.000001	0.000071

Table 4.AIC, BIC and loglikelihood values for fitted distributions

Criterion	DBXIIBD	Poisson	DRayleigh	PLD	PAD	PSD
Loglikelihood Value	-1128.03	-1194.9	-1535.85	-1154.92	-1154.13	-1154.61
AIC	2260.05	2391.802	3073.705	2311.842	2310.258	2311.217
BIC	2272.63	2398.091	3079.994	2318.131	2316.547	2317.506

We have compared discrete Burr type XII Minimax distribution with Poisson, discrete Rayleigh, Poisson Lindley, Poisson Akasha, and Poisson Sujatha distributions using the Akaike information criterion (AIC), given by Akaike [13] and the Bayesian information criterion (BIC), given by Schwarz [14] . Generic function calculating Akaike's 'An Information Criterion' for one or several fitted model objects for which a log-likelihood value can be obtained, according to the formula -2*log-likelihood + k*npar, where npar represents the number of parameters in the fitted model, and k = 2 for the usual AIC, or k = log(n) (n being the number of observations) for the so called BIC or SBC (Schwarz's Bayesian criterion). From table 4, Comparing the fits using AIC and BIC criterion, it is obvious that AIC and BIC criterion favors discrete Burr type XII Minimax distribution in comparison with the Poisson, discrete Rayleigh, Poisson Lindley, Poisson Akasha, and Poisson Sujatha distributions, in the case of Counts of Automobile claim data.

Conclusion

In this paper, a new discrete model is proposed by compounding discrete Burr type XII distribution (DBXII) with Minimax distribution and it has been shown that proposed model can be nested to different compound distributions. Some important probabilistic properties and the problem of estimation of its parameters are studied. The proposed model is well competitive of some well known compound and classical discrete distributions.

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