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Optimum Stratification Using Mathematical Programming Approach: A Review

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Abstract: The stratification technique which results in minimum possible variance is called optimum stratification. The main objective of stratification is to give a better cross-section of the population so as to gain a higher degree of relative precision. The strata may be constructed either on the basis of study variable itself or by using some other variable(s) closely related with study variable known as auxiliary variable. Over the past many years several computational methods have been developed to obtain stratification points. This paper reviews the contribution toward obtaining optimum strata boundaries using mathematical programming. For several methods the comparative study has been made.

Keywords: Mathematical programming approach, Optimum strata boundaries, Dynamic programming, Multistage decision problem, Bellman's principal of optimality.

1 Introduction

Optimal stratification is usually separated into three optimisation problems: the number of strata to construct, the placement of stratum boundaries, and the number of observations to be selected from each stratum. Notable contributions to these problems include Cochran [1] on the number of strata, Dalenius [2], Dalenius& Hodges [3], Ekman [4] , and Lavallee &Hidiroglou [5] on the placement of stratum boundaries, and Neyman [6] on the number of observations to be selected from each stratum. The first ever mathematical programming problem (MPP) was perhaps the problem of optimum allocation of limited resources recognized by economists in early 1930s. After World War II in 1947 the United States Air Force team SCOOP (Scientific Computation of Optimum Programs) started intensive research on some optimum resource allocation problem which led to the development of the famous simplex method by George B. Dantzig for solving a linear programming problem. Kuhn and Tucker [7] developed the necessary conditions (which become sufficient also under special circumstances) to be satisfied by an optimal solution of an MPP. These conditions, known as K-T conditions, laid the foundation for great deal of later research and development in nonlinear programming techniques.

Till date no single technique is available which can provide an optimal solution to every nonlinear programming problem (NLPP) like simplex method for linear programming problem (LPP). However different methods are available for some special types of NLPPs. In [8] Beal gave a method for solving convex quadratic programming problem (CQPP). One of the most powerful techniques for solving an NLPP is to transform it, by some means, into a form which permits the use of simplex method of LPP. Using K-T conditions Wolfe in [9] transformed the CQPP into an equivalent LPP to which simplex method could be applied with some additional restrictions on the vectors entering the basis at various iterations. A similar method is developed by Panne and Whinston ([10], [11], [12]) . Some other techniques for solving quadratic programming problems are due to Lemke [13], Graves [14] , Kelley [15], Aggarwal ([16] , [17]), Finkbeiner and Kail [18] , Arshad et al. [19], Khan et al. [20] , Todd [21] , Fukushima [22] , Powell and Yuan [23] etc. Some recent work is due to Du et al.[24] , Kalantari and Bagchi [25] , Yuan [26], Wei [27] , Benzi [28] , Fletcher [29] , Botnze and Danninger ([30] , [31]), Anstreicher et al. [32].In this paper ,we have made an attempt to review and summarise the literature relation to optimum stratification using mathematical programming approach.

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2 The problem of Obtaining Stratification Points

Among other NLPP methods there are Gradient Methods and Gradient Projection Methods. Like simplex method of LPP these are iterative procedures in which at each step we move from one feasible solution to another in such a way that the value of the objective function is improved. Rosen([33] , [34]), Kelley [15], Goldfarb [35] , Du et al. [24] , Lai et al.[36] etc. gave gradient projection methods for nonlinear programming with linear and nonlinear constraints. Des Raj in [37] determines the optimum probabilities of selection in sampling without replacement. The methods for the approximate solution to the separable programming problems are found in the works of Charnes and Cooper [38], Markowitz and Manne [39], Dantzig et al.[40], Millier [41], Fleury [42] and Megiddo and Tamir [43] etc. The principle of optimality enunciated by Bellman [44] paved the way for the development of the dynamic programming technique which has been applied successfully for solving certain special types of MPPs. The dynamic programming became rich and more applicable by the contributions of several authors like Li [45], Li and Haimes [46], Wang ([47] , [48]), Wang and Xing [49], Chen, Hearn and Lee [50], Lin [51], Odanaka [52], etc. Since dynamic programming technique is used as the main tool in this manuscript for solving nonlinear programming problems arising in sampling, in the next section it is discussed in some detail. The problems in which decisions are to be made sequentially at different stages of solutions are called multistage decision problems. Many multistage decision problems can be formulated as a MPP. The dynamic programming technique is a computational procedure which is well suited for solving MPPs that may be treated as a multistage decision problem. The basic idea which led to the development of the dynamic programming technique was given by Richard Bellman in early 1950s.This is the approach used by various authors for obtaining optimum strata boundaries. The problem of optimum stratification was studied through dynamic programming by Buhler and Deutler in [53].They found that by a suitable transformation a global optimal solution could be obtained through dynamic programming. The results are illustrated for discrete and continuous variables by numerical results. A comparative study of solution procedures as regards the determination of optimum stratification was also made by Buhler and Deutler in [53].He considered two groups of solution methods including procedure of operations research. In both the groups it has been discussed which procedure are capable of finding the global optimum.

Khan et al. [54] proposed a technique for determining the optimum number of strata. The problem of determining the optimum strata boundaries, when the main study variable is used as a stratification variable and a stratified sample, using Neyman allocation is to be selected to estimate the population mean, has been formulated as a MPP has been discussed by Khan et al. [55].It has been shown that with some modification that MPP may be converted into a multistage decision problem that could be solved using dynamic programming technique. In other study, Khan et al. [56] examined the problem of determining an optimum compromise allocation in multivariate stratified random sampling, when the population means of several characteristics are to be estimated. Formulating the problem of allocation as an all integer nonlinear programming problem, they developed a solution procedure using a dynamic programming technique. The compromise allocation discussed is optimal in the sense that it minimizes a weighted sum of the sampling variances of the estimates of the population means of various characteristics under study. Once again, Khan et al. [57] studied the problem of optimum stratification and formulated as a MPP assuming exponential frequency distribution of the main study variable. The stratum boundaries are optimum in the sense that they minimize the sampling variance of the stratified sample mean under Neyman allocation. The formulated MPP found to be separable with respect to the decision variables and is treated as multistage decision problem. A solution procedure has also been developed using dynamic programming. The problem of finding OSB has been considered as the problem of determining OSW by Khan et al. [58].The problem has been formulated as MPP, which minimizes the variance of the estimated population parameter under Neyman allocation subject to the restriction that the sum of the widths of all the strata is equal to the total range of the distribution. The distribution of the study variable has been considered as continuous with triangular and standard normal density functions. The formulated MPPs which turn out to be multistage decision problems, can then be solved using dynamic programming technique proposed by Buhler and Deutler [53].Comparative study reveals that proposed technique is better than Dalenius and Hodges [59] method. Khan et al.[58] found that in order to make the strata internally homogenous, the strata should be constructed in such a way that the strata variance for the characteristic under study be as small as possible. This could be achieved effectively by having the known distribution of the main study variable and create strata by cutting the range of the distribution at suitable points. If the frequency distribution of the study variable is unknown, it may be approximated from the past experience or some prior knowledge obtained at a recent study. They considered the problem of finding optimum strata boundaries as the problem of determining optimum strata width. Khan et al. [60] proposed the method of choosing the best boundaries that make strata internally homogenous given some sample allocation, known as optimum allocation. In this paper the problem of OSB has been discussed when strata are formed based on a single auxiliary variable with a varying measurement cost per unit strata the auxiliary variable considered in the problem is a size variable that holds a common model for the population. The OSB were achieved effectively by assuming a suitable distribution of the auxiliary variable and creating strata by cutting the range of the distribution at suitable points. The problem of finding the OSB, which minimizes the variance of the estimated population mean under a weighted stratified balanced sampling, has been formulated as a MPP.

One of the other advanced step for obtaining OSB has been taken by Sebnem [61] that having stratified many populations with different characteristics reveals that Kozak's random search method and Keskinturk and Er's genetic algorithm gives more or less the same results both from the point of coefficient of variations and boundaries. The results obtained there shows that either genetic algorithm or Kozak's random search method could be efficiently applied in order to obtain near optimum boundaries in stratified sampling. Khan et al.[62] developed a method of choosing the boundaries that have high level of precision. They took the problem of finding the OSB for a skewed population with standard log-normal distribution. The problem is then redefined as the problem of determining optimum strata width and is formulated as MPP that seeks minimization of the variance of the estimated population parameter under Neyman allocation subject to the constraint that sum of the widths of all the strata is equal to the total range of the distribution. Rao et al. [63] proposed a technique of determining the multivariate calibrated estimator to improve the survey estimates when more than one auxiliary variable is available. The problem of determining optimum calibrated weights is formulated as an MPP, which is solved using Lagrange multiplier technique. The numerical illustration presented in the paper for computation details reveals that the proposed estimator performs better than the usual estimator. Fonolahi and Khan [64] proposed a technique to determine the optimum strata boundaries when the measurement cot per unit varies across the strata. The problem has been formulated as a MPP and solved to obtain the strata width, which is then used to obtain the OSB. The study variable is assumed to be exponentially distributed. Rao et al. [65] developed a technique of solving a combined problem of determining OSB and optimum strata size of each stratum, when the population under study is skewed and the study variable has a pareto frequency distribution. The problem is formulated as an MPP, which has been solved by using a dynamic programming technique. The problem of construction optimum stratification for two study variables based on auxiliary variable that follow respectively a uniform and a right-triangular distribution has been discussed by Khan et al. [66].The problem of determining the OSB are formulated as NLPP ,which turn out to be multistage decision problems and were solved using dynamic programming techniques. The comparative study has been done on simulated data of two sets and the results were obtained by cum√f ,Dalenius and Hodges [58], geometric method by Gunning and Horgan [67],the generalized method of Lavallee and Hidiriglous [5] and proposed method and it was found that construction of strata using auxiliary variable for the populations with uniform and right-triangular distributions, leads to substantial gains in precision of the estimates while using the proposed technique.

Khan et al. [68] proposed a technique by using auxiliary information for determining the optimum strata boundaries of the population that has uniformly distributed auxiliary variable. The problem has been formulated as NLPP that seek minimization of the variance of the estimated population parameter under Neyman allocation. The NLPP is then solved by developing a solution procedure using a dynamic programming technique. In a stratified sapling design for economic surveys based on auxiliary information has been developed by Khan et al.[69], which can be used for constructing optimum stratification and determining optimum sample allocation to maximize the precision in estimate. They made comparison of the proposed method with Dalenius and Hodges [59] ,Gunning and Horgan [67] and generalized [5] method with Kozak's algorithm and concluded that the construction of strata and determination of sample allocation using auxiliary variable of the populations with Gamma distribution ,leads to substantial gains in the precision of the estimates while using proposed technique than others taken in comparison. The problem of finding the OSB and the optimum sample sizes within the stratum for a skewed population with log-normal distribution has been studied by Khan et al. [70]. The problem of determining the OSB has been redefined as the problem of determining optimum strata width (OSW) and is formulated as a NLPP that seeks minimization of the variance of the estimated population mean under Neyman allocation subject to the constraint that the sum of the widths of all the strata is equal to the range of the distribution. The formulated NLPP turns out to be a multistage decision problem that can be solved by dynamic programming technique. Furthermore, a comparison studied is carried out using 10 artificial populations to compare efficiency of the proposed method with the cum√f , geometric and L-H methods. The results in the study reveal that the proposed method and the L-H method are more efficient than cum√f and geometric methods in minimizing the variance of the estimate of the population mean.

One of the advanced distribution has been discussed by Para and Jan [71]. Danish et al.[72] discussed the problem of determining the OSB of a study variable based on auxiliary variable under proportional allocation. The problem has been formulated as a MPP which minimizes the objective function subject to the constraint that the sum of the widths of all the strata is equal to the whole range of the distribution. The distributions of the auxiliary variable were considered continuous with uniform and exponential density functions separately. The formulated MPPs, which turn out to be multistage decision problems, were then be solved using dynamic programming approach. The theoretical results have been illustrated empirically for both the distributions which show that there is substantial gain in efficiency with the increase in the number of strata.

3 Conclusion

Having stratified many populations with different characteristics reveals that the method used at any situation depends about the nature of the problem. It can be also concluded from the whole paper that mathematical programming approach has an increasing trend rather than classical methods proposed by Dalenius, Dalenius and Gurney, Ekman etc. Bellman's principal of optimality is one of the main tools used to solve the nonlinear programming problems to obtain the optimum strata boundaries. One of the main advantages of the proposed methods is that they don't require any initial approximate solution.

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