Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/060308

Estimation of Population mean using Exponential Dual to Ratio Type Compromised Imputation for Missing data in Survey Sampling

B. K. Singh and Upasana Gogoi[*]

Department of Mathematics, North Eastern Regional Institute of Science and Technology, Nirjuli, Arunachal Pradesh - 791109, India

Received: 22 Jun. 2017, Revised: 20 Aug. 2017, Accepted: 25 Aug. 2017
Published online: 1 Nov. 2017

Abstract: In this paper, we have proposed a class of exponential dual to ratio type compromised imputation technique and corresponding point estimator. Its mean square error expression is compared with the sample mean, ratio and compromised methods of imputation in the case of missing data. Further, numerical illustrations are provided with the help of some natural population data sets to compare their efficiencies for different non-response rate.

Keywords: Missing data, Imputation methods, Exponential Dual to ratio, Bias, Mean Square Error, Non-response, Simple Random Sampling, Efficiency.

1 Introduction

In sample survey it is commonly experienced that complete data from the sampling units or respondents are not obtainable for various reasons, for example in an opinion survey, the selected family might have shifted to some other places, and selected person might have died. In mailed questionnaire many respondents do not sent their replies. Such a problem of incomplete sample data due to non availability of information from the respondents is known as the problem of non-response.

A common technique for handling non-response is imputation. The term imputation refers to the process of assigning one or more values to an item when there is no reported value for that item and where the missing values are filled into create a complete data set that can be analysed with traditional analysis methods. There are two principal uses of imputation. These are(1) Imputation for item non-response only and (2) Imputation for item non-response as well as for the unit non-response. Most of the currently used imputation methods involve the substitution of an imperfect predicted value. Some of the important imputation methods of this kind are mean imputation, ratio and product method of imputation, multiple imputation, hot deck imputation, cold deck imputation, distance function matching, regression imputation, etc. to improve the estimation of population mean with non-response. In recent past, a number of efficient compromised imputation strategies have been proposed by several survey statisticians.

In addition to the obvious advantage of allowing complete-data methods of analysis, imputation by the data collector (e.g. the Census Bureau) also has the important advantage of being able to utilize information available to the data collector but not available to an external data analyst such as a university social scientist analyzing a public-use file. This information may involve detailed knowledge of interviewing procedures and reasons for nonresponse that are too cumbersome to place in public-use files, or may be facts, such as street adresses of dwelling units, that cannot be placed on public-use files because of confidentiality constraints. This kind of information, even though inaccessible to the user of a public-use file, can often narrow the possible range of imputed values.

[8] suggested the first attempt in the estimation procedure for population mean in presence of non-response by mail questionnaire and the second attempt by a personal interview. [10] and [14] suggested imputation methods that make an incomplete data set structurally complete and its analysis. [11] used the information on an auxiliary variable for the purpose of imputation. [13] addressed two key concepts: missing completely at random, when the response indicator to

[*] Corresponding author e-mail: 21upasanagogoi@gmail.com

the survey is independent of all other variables in the survey; missing at random, when the response indicator depends only on some characteristics observed in the survey and available also for non-respondents. [23] and [19] have suggested the class of estimators in simple random sampling. [21] introduced a compromised method of imputation. [20] suggested a new power transformation estimator of population mean. Other authors such as [17], [9], [1], [6], [12], [2], [18], [16], [7], [4], [5], [15] has studied the problem of imputation methods under single and double sampling scheme.

For improving the precisions in estimating the unknown mean $\overline{Y}$ of a finite population by using the auxiliary variable, let for a finite (survey) population U i.e; $U = (U_1, U_2...U_N)$ be the finite population of size N. To each unit $U_i (i = 1, 2...N)$ in the population, paired values (y_i, x_i) corresponding to study variable y and auxiliary variable x correlated with y are attached. Now, define the population means of the study variable y and auxiliary variable x as

$$\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i, \overline{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

A simple random sample without replacement (SRSWOR), s, of size n is drawn from the population. Let r be the number of responding units out of n sampled units. Let the set of responding units be denoted by R and that of non-responding units be denoted by R^C. For every $i \in R$, the value of y_i is observed. However, for the units $i \in R^c$, the y_i values are missing and imputed values are derived using different methods. The imputation is carried out with the aid of an auxiliary variable x, such that x_i is the value of auxiliary variable x for unit i is known and positive for every $i \in s$ i.e; the data $x_s = \{x_i : i \in s\}$ are known.

2 Notations:

The following notations have been used

Y : Study variable.

X : Auxiliary variable.

$\overline{X}, \overline{Y}$: The population mean of the variates X and Y respectively.

$\overline{x}_n$: The sample mean of X for the sample of size n.

$\overline{y}_r$: The mean of the variable Y for the set R.

ρ_{YX} : The correlation coefficient between the variates Y and X.

S_X^2, S_Y^2 : The population mean squares of X and Y respectively.

$S_X^2 = \frac{\sum_{i=1}^{N}(x_i - \overline{X})^2}{N-1}, S_Y^2 = \frac{\sum_{i=1}^{N}(y_i - \overline{Y})^2}{N-1}$

C_X, C_Y : The coefficient of variation of X and Y respectively.

$C_X = \frac{S_X}{\overline{X}}, C_Y = \frac{S_Y}{\overline{Y}}$

$g = \frac{n}{N-n}$,

$\phi_{YX} = \frac{\rho_{YX} C_Y}{C_X}$.

3 Some Available Methods of Imputation:

Mean method of imputation: The mean method is to replace each missing datum with the mean of the observed value. The data after imputation becomes

$$y_{.i} = \begin{cases} y_i & i \in R \\ \overline{y}_r & i \in R^C \end{cases}$$

Under this method of imputation, the point estimator of population mean given by,

$\overline{y}_s = \frac{1}{n}\sum_{i\in s} y_{.i}$

$= \frac{1}{n}\left\{\sum_{i\in R} y_i + \sum_{i\in R^C} \overline{y}_r\right\}$

$= \frac{1}{n}\left\{r\overline{y}_r + (n-r)\overline{y}_r\right\}$

$= \frac{1}{n}\left\{r\overline{y}_r + n\overline{y}_r - r\overline{y}_r\right\}$

$= \frac{1}{n} n\overline{y}_r$

$= \overline{y}_r$

Lemma 1.

$$B(\overline{y}_r) = 0 \tag{1}$$

$$V(\overline{y}_r) = \overline{Y}^2\left(\frac{1}{r} - \frac{1}{N}\right)C_Y^2 \tag{2}$$

Ratio method of imputation: Following the notations of [11], in the case of single imputation method, if the i^{th} unit requires imputation, the value $\hat{b}x_i$ is imputed, where $\hat{b} = \frac{\sum_{i\in R} y_i}{\sum_{i\in R} x_i}$

$$y_{.i} = \begin{cases} y_i & i \in R \\ \hat{b}x_i & i \in R^C \end{cases}$$

This method of imputation is called the ratio method of imputation. Under this method of imputation, the point estimator of the population mean $\overline{Y}$ is given by

$$\overline{y}_{RAT} = \overline{y}_r \frac{\overline{x}_n}{\overline{x}_r} \tag{3}$$

where $\overline{y}_r = \frac{1}{r}\sum_{i\in R} y_{.i}$, $\overline{x}_r = \frac{1}{r}\sum_{i\in R} x_i$, $\overline{x}_n = \frac{1}{n}\sum_{i\in s} x_i$

Lemma 2.

$$B(\overline{y}_{RAT}) = \overline{Y}\left(\frac{1}{r} - \frac{1}{n}\right)(1-\phi_{YX})C_X^2 \tag{4}$$

$$MSE(\overline{y}_{RAT}) = \overline{Y}^2\left\{\left(\frac{1}{r} - \frac{1}{N}\right)C_Y^2 + \left(\frac{1}{r} - \frac{1}{n}\right)(1-2\phi_{YX})C_X^2\right\} \tag{5}$$

Compromised method of imputation: [21] suggested a compromised method of imputation. It based on using information from imputed values for the responding units in addition to non-responding units. In case of compromised imputation procedures, the data take the form

$$y_{.i} = \begin{cases} \alpha\frac{n}{r}y_i + (1-\alpha)\hat{b}x_i & i \in R \\ (1-\alpha)\hat{b}x_i & i \in R^C \end{cases}$$

where α is a suitably chosen constant, such that the variance of the resultant estimator is minimum.

The point estimator of the population mean under compromised method of imputation method becomes

$$\overline{y}_{COMP} = \alpha\overline{y}_r + (1-\alpha)\overline{y}_r\frac{\overline{x}_n}{\overline{x}_r} \tag{6}$$

Lemma 3.

$$B(\overline{y}_{COMP}) = \overline{Y}\left(\frac{1}{r} - \frac{1}{n}\right)(1-\alpha)(1-\phi_{YX})C_X^2 \tag{7}$$

$$MSE(\overline{y}_{COMP}) = \overline{Y}^2\left[\left(\frac{1}{r} - \frac{1}{N}\right)C_Y^2 + \left(\frac{1}{r} - \frac{1}{n}\right)\left\{(1-\alpha)^2 - 2(1-\alpha)\phi_{YX}\right\}C_X^2\right] \tag{8}$$

$$\alpha_{opt} = 1 - \phi_{YX}$$

$$MSE(\overline{y}_{COMP})_{opt} = \overline{Y}^2\left\{\left(\frac{1}{r} - \frac{1}{N}\right)C_Y^2 - \left(\frac{1}{r} - \frac{1}{n}\right)\rho_{YX}^2 C_Y^2\right\} \tag{9}$$

4 Properties of the Imputed Estimators:

To obtain the bias and MSE of imputed estimator, we write

$e_1 = \frac{\bar{y}_r}{\bar{Y}} - 1$, $e_2 = \frac{\bar{x}_r}{\bar{X}} - 1$ and

$e_3 = \frac{\bar{x}_n}{\bar{X}} - 1$

such that

$|e_i| < 1 \ \forall \ i = 1,2,3$

Hence, we have

$E(e_1) = E(e_2) = E(e_3) = 0$ and $E(e_1^2) = \left(\frac{1}{r} - \frac{1}{N}\right) C_Y^2$,

$E(e_2^2) = \left(\frac{1}{r} - \frac{1}{N}\right) C_X^2, E(e_3^2) = \left(\frac{1}{n} - \frac{1}{N}\right) C_X^2$,

$E(e_1 e_2) = \left(\frac{1}{r} - \frac{1}{N}\right) \rho_{YX} C_Y C_X, E(e_2 e_3) = \left(\frac{1}{n} - \frac{1}{N}\right) C_X^2$,

$E(e_1 e_3) = \left(\frac{1}{n} - \frac{1}{N}\right) \rho_{YX} C_Y C_X$.

where
$\rho_{YX} = S_{XY}/S_X S_Y$
and S_{XY} is the covariance between variables Y and X.

5 The Proposed estimator:

Motivated with [22] and [3], we here propose the following exponential dual to ratio type compromised method of imputation

$$y_{.i} = \begin{cases} k\frac{n}{r} y_i + (1-k)\bar{y}_r \exp\left(\frac{\Psi - \bar{X}}{\Psi + \bar{X}}\right) & i \in R \\ (1-k)\bar{y}_r \exp\left(\frac{\Psi - \bar{X}}{\Psi + \bar{X}}\right) & i \in R^C \end{cases}$$

where

$$\Psi = \frac{N\bar{X} - n\bar{x}_r}{N - n}$$

The point estimator of the population mean $\bar{Y}$ under proposed method of imputation is

$$\bar{y}_{EDR} = k\bar{y}_r + (1-k)\bar{y}_r \exp\left(\frac{\Psi - \bar{X}}{\Psi + \bar{X}}\right) \tag{10}$$

where k is a suitably chosen constant to be determined under certain conditions.
Under large sample approximations the estimator takes the form

$$\bar{y}_{EDR} = \bar{Y}\left[k(1+e_1) + (1-k)(1+e_1)\exp\left\{\frac{-ge_2}{2}\left(1 - \frac{ge_2}{2}\right)^{-1}\right\}\right] \tag{11}$$

Theorem 1. *The bias of $\bar{y}_{EDR}$ is given by*

$$B(\bar{y}_{EDR}) = \bar{Y}\left(\frac{1}{r} - \frac{1}{N}\right)(k-1)\frac{g}{2}\left(\frac{g}{4} + \phi_{YX}\right)C_X^2 \tag{12}$$

*Proof.*The estimator $\overline{y}_{EDR}$ in terms of e_1 and e_2 can be written as

$$\overline{y}_{EDR} = \overline{Y}\left\{1 + e_1 + \frac{ge_2}{2}(k-1) + \frac{g^2e_2^2}{4}(k-1) - \frac{g^2e_2^2}{8}(k-1) + \frac{ge_1e_2}{2}(k-1)\right\} \tag{13}$$

Taking expectations in equation 13 and using the results of $E\left(e_2^2\right)$ and $E\left(e_1e_2\right)$, we get the bias of the estimator

$$B(\overline{y}_{EDR}) = E\left(\overline{y}_{EDR} - \overline{Y}\right) = \overline{Y}(k-1)\frac{g}{2}E\left(\frac{ge_2^2}{4} + e_1e_2\right) \tag{14}$$

given as in equation 12

Theorem 2.*The mean square error of* $\overline{y}_{EDR}$ *is given by*

$$MSE\left(\overline{y}_{EDR}\right) = \overline{Y}^2\left(\frac{1}{r} - \frac{1}{N}\right)\left\{C_Y^2 + g\left(\frac{gk^2}{4} + \frac{g}{4} + k\phi_{YX} - \frac{gk}{2} - \phi_{YX}\right)C_X^2\right\} \tag{15}$$

*Proof.*Squaring and taking expectations on both the sides of 13 and neglecting the second and higher order terms, we get the MSE of $\overline{y}_{EDR}$ to the first degree of approximation as

$$MSE\left(\overline{y}_{EDR}\right) = E\left(\overline{y}_{EDR} - \overline{Y}\right)^2 = \overline{Y}^2E\left(e_1^2 + \frac{g^2k^2e_2^2}{4} + \frac{g^2e_2^2}{4} + gke_1e_2 - \frac{g^2ke_2^2}{2} - ge_1e_2\right) \tag{16}$$

Putting the results of $E\left(e_1^2\right)$, $E\left(e_2^2\right)$ and $E\left(e_1e_2\right)$ in equation 16, we get the mean square error (MSE)of the estimator 10 that poves the theorem 2

Theorem 3.*The minimum mean square error of* $\overline{y}_{EDR}$ *is given by*

$$MSE\left(\overline{y}_{EDR}\right)_{min} = \overline{Y}^2\left(\frac{1}{r} - \frac{1}{N}\right)\left(1 - \rho_{YX}^2\right)C_Y^2 \tag{17}$$

for the optimum value of k which is given by

$$k_{opt} = 1 - \frac{2\phi_{YX}}{g} \tag{18}$$

*Proof.*Differentiating equation 15 with respect to k and equating it to zero, we get optimum value of k as

$$k_{opt} = 1 - \frac{2\phi_{YX}}{g}$$

Putting the value of k in equation 15, we get the minimum mean square error (MSE) of the proposed estimator $\overline{y}_{EDR}$ given as in equation 17

*Remark.*1. When $k = 1$, the proposed class of estimators reduces to mean method of imputation
$\overline{y}_{EDR} = \overline{y}_r$
The bias and MSE of $\overline{y}_r$ can be obtained by putting $k = 1$ in 12 and 15 respectively as

$$B(\overline{y}_r) = 0 \tag{19}$$

$$MSE\left(\overline{y}_r\right) = \overline{Y}^2\left(\frac{1}{r} - \frac{1}{N}\right)C_Y^2 \tag{20}$$

2. When $k = 0$,
the proposed class of estimators reduces to imputed exponential dual to ratio estimator
$\overline{y}_{DR} = \overline{y}_r exp\left(\frac{\Psi - \overline{X}}{\Psi + \overline{X}}\right)$
The bias and MSE of $\overline{y}_{DR}$ can be obtained by putting $k = 0$ in 12 and 15 respectively as

$$B(\overline{y}_{DR}) = -\overline{Y}\left(\frac{1}{r} - \frac{1}{N}\right)\frac{g}{2}\left(\frac{g}{4} + \phi_{YX}\right)C_X^2 \tag{21}$$

$$MSE\left(\overline{y}_{DR}\right) = \overline{Y}^2\left(\frac{1}{r} - \frac{1}{N}\right)\left\{C_Y^2 + g\left(\frac{g}{4} - \phi_{YX}\right)C_X^2\right\} \tag{22}$$

6 Efficiency comparison of the estimator $(\bar{y}_{EDR})_{opt}$:

1.Comparison of the estimator $(\bar{y}_{EDR})_{opt}$ and the estimator $\bar{y}_r$

$$V(\bar{y}_r) - MSE(\bar{y}_{EDR})_{opt} = \bar{Y}^2\left(\frac{1}{r}-\frac{1}{N}\right)\rho_{YX}^2 C_Y^2 > 0 \tag{23}$$

which is always true, hence the estimator $(\bar{y}_{EDR})_{opt}$ is always better than the estimator $\bar{y}_r$ under the optimality condition 18.

2.Comparison of the estimator $(\bar{y}_{EDR})_{opt}$ and the estimator $\bar{y}_{RAT}$

$$MSE(\bar{y}_{RAT}) - MSE(\bar{y}_{EDR})_{opt} = \bar{Y}^2\left\{\left(\frac{1}{n}-\frac{1}{N}\right)\rho_{YX}^2 C_Y^2 + \left(\frac{1}{r}-\frac{1}{n}\right)(C_X - \rho_{YX}C_Y)^2\right\} > 0 \tag{24}$$

which is always true, hence the estimator $(\bar{y}_{EDR})_{opt}$ is always better than the estimator $\bar{y}_{RAT}$ under the optimality condition 18.

3.Comparison of the estimator $(\bar{y}_{EDR})_{opt}$ and the estimator $(\bar{y}_{COMP})_{opt}$

$$MSE(\bar{y}_{COMP})_{opt} - MSE(\bar{y}_{EDR})_{opt} = \bar{Y}^2\left\{\left(\frac{1}{n}-\frac{1}{N}\right)\rho_{YX}^2 C_Y^2\right\} > 0 \tag{25}$$

which is always true, hence it can be concluded that the proposed estimator $(\bar{y}_{EDR})_{opt}$ is always preferable over the estimator $(\bar{y}_{COMP})_{opt}$.

4.Comparison of the estimator $(\bar{y}_{EDR})_{opt}$ and the estimator $\bar{y}_{DR}$

$$MSE(\bar{y}_{DR}) - MSE(\bar{y}_{EDR})_{opt} = \bar{Y}^2\left(\frac{1}{r}-\frac{1}{N}\right)\left(\frac{g}{2}-\phi_{YX}\right)^2 C_X^2 > 0 \tag{26}$$

which is always true, hence it can be concluded that the proposed estimator $(\bar{y}_{EDR})_{opt}$ is always better than the estimator $(\bar{y}_{DR})_{opt}$.

7 Emperical study:

To illustrate the findings, we consider the parameters of four different population data sets, which are given in Table 1.
We have also computed the percent relative efficiencies (PRE) of different estimators which are given in Table 2.

Table 1: Population Parameters of four different populations

parameters	Population A Kadilar and Cingi(2008)	Population B Singh (2009)	Population C Diana and Perri(2010)	Population D Mukhopadhyaya(2000)
N	19	3055	8011	20
$\bar{Y}$	575.00	308582.4	28229.43	41.5
$\bar{X}$	13537.68	56.5	1.69	441.95
S_Y	858.36	425312.8	22216.56	9.784518

parameters	Population A Kadilar and Cingi(2008)	Population B Singh (2009)	Population C Diana and Perri(2010)	Population D Mukhopadhyaya(2000)
[0.5 ex] S_X	12945.38	72.3	0.78	101.0703
ρ	0.88	0.677	0.46	0.6521
C_X	0.953712	1.279646018	0.461538	0.2286
C_Y	1.4928	1.378279513	0.787	0.2358
n	10	611	400	7
r	8	520	250	5

Table 2: PRE of the considered estimator under four different populations

Estimator	Population A $V(\bar{y}_r) = 41290.80821$	Population B $V(\bar{y}_r) = 288655816$	Population C $V(\bar{y}_r) = 1309431$	Population D $V(\bar{y}_r) = 14.36399$
$\bar{y}_r$	100	100	100	100
$\bar{y}_{RAT}$	132.88	107.63	108.19	114.11
$\bar{y}_{COMP}$	136.52	108.96	108.92	119.33
$\bar{y}_{DR}$	199.48	116.78	101.41	137.42
$\bar{y}_{EDR}$	443.26	184.61	126.84	173.98

8 conclusion

In this paper, the PRE of the suggested estimator $\bar{y}_{EDR}$ has been compared with several other estimators, viz; $\bar{y}_r$ $\bar{y}_{RAT}$, $\bar{y}_{COMP}$, and $\bar{y}_{DR}$. From table 2, it is observed that the proposed estimator $\bar{y}_{EDR}$ in its optimality is more efficient than the other estimators taken for comparisons under considerations.

References

[1] M. S. Ahmed, O. Al-Titi, Z. Al-Rawi, and W. Abu-Dayyeh. Estimation of a population mean using different imputation methods. *Statistics in transition*, 7:1247–1264, 2006.

[2] R. Arnab and S. Singh. A new method for estimating variance from data imputed with ratio method of imputation. *Statistics and Probability lettersl*, 76:513–519, 2006.

[3] S. Bahl and R. K. Tuteja. Ratio and product type exponential estimators. *Journal of Information and Optimization Sciences*, 12:159–164.

[4] Amanda N Baraldi and Craig K Enders. An introduction to modern missing data analyses. *journal of school psychology*, 48(1):5–37, 2010.

[5] Carlos N Bouza. Estimation of the population mean with missing observations using product type estimators. *Revista Investigación Operacional*, 29(3):207–223, 2008.

[6] G. Diana and P. F. Perri. Improved estimators of the populatiom mean for missing data. *Communications in Statistics-Theory and Methods*, 39:3245–3251.http://dx.doi.org/10.1080/03610920903009400, 2010.

[7] S González, M Rueda, and A Arcos. An improved estimator to analyse missing data. *Statistical Papers*, 49(4):791–796, 2008.

[8] M.N. Hansen and W.N. Hurwitz. The problem of non-response in sample surveys. *Journal of the American Statistical Association*, 41:517–529, 1946.

[9] C. Kadilar and H. Cingi. Estimators for the population mean in the case of missing data. *Communications in Statistics-Theory and Methods*, 37:2226–2236.http://dx.doi.org/10.1080/03610920701855020, 2008.
[10] D. Kalton, G. Kasprzyk and R. Santos. Issues of non-response and imputation in the survey of income and program participation. *Current Topics in Survey Sampling,(D. Krewski, R. Platek and J. N. K. Rao, eds.)*, pages 455–480, 1981.
[11] E. Lee, H. Rancourt and C.E. Sarndal. Experiments with variance estimation from survey data with imputed values. *StatisticsJournal of Official*, 10:231–243, 1994.
[12] J. N. K Rao and Sitter R. R. Variance estimation under two-phase sampling with application to imputation for missing data. *Biometrika*, 82:453–460, 1995.
[13] D. B. Rubin. Inference and missing data. *Biometrika*, 63:581–593, 1976.
[14] I. G. Sande. A personal view of hot-deck imputation procedures. *Survey Methodology,Statistics Canada*, pages 238–247, 1979.
[15] D. Shukla and N. S. Thakur. Estimation of mean with imputation of missing data using factor type estimator. *Statistics in transition*, 9:33–48, 2008.
[16] A.K Singh, R.K Singh, and Thaker M.K. Improved estimators of population mean for missing data in simple random sampling. *Imperial Journal of Interdisciplinary Research*, 2(8), 2016.
[17] P. Singh, A.K. Singh and V.K. Singh. Exponential - type compromised imputation in survey sampling. *Journal of Statistics Applications and Probability*, 3.
[18] P. Jong-Min K. Singh, G. N. Kumari and S. Singh. Estimation of population mean using imputation techniques in sample surveys. *Journal of the Korean Statistical Society*, 39:67–74.
[19] P. Sawan N Singh, R. Chauhan and F. Smarandache. Ratio estimators in simple random sampling using information on auxiliary attribute. *Pakistan Journal of Statistics and Operational Research*, 4(1):46–56, 2009.
[20] S. Singh and B. Deo. Imputation by power transformation. *Statistical Papers*, 44:555–579, 2003.
[21] S. Singh and S. Horn. Compromised imputation in survey sampling. *Metrika*, 51:266–276, 2000.
[22] T. Srivenkataramana and D. S. Tracy. An alternative to ratio method in sample surveys. *Annals of the Institute of Statistical Mathematics*, 32:111–120, 1980.
[23] L. N. Upadhyaya and H. P. Singh. Use of transformed auxiliary variable in estimating the finite population mean. *Biometrical Journal*, 41:627–636, 2009.

B. K. Singh received his PhD degree in Statistics from Banaras Hindu University, India. Currently, he is Professor of Statistics, in the Dept. of Mathematics, North Eastern Regional Institute of Science and Technology, India. His research interests are in the areas of Sampling Techniques. He has published research articles in reputed international journals of mathematical and statistical sciences. He has published forty research papers in the field of Sampling Techniques. Till now four students have been awarded Ph.D. degree under his supervision. He has participated in many international conferences of Mathematics and Statistics in India and abroad.

Upasana Gogoi is Ph.D. scholar under the supervision of Prof. B.K. Singh, Dept. of Mathematics, North Eastern Regional Institute of Science and Technology, India.