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An Improved Exponential Method of Estimation for Current Population Mean in Two-Occasion Successive Sampling

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Abstract: The present article proposes an improved exponential method of estimation for current population mean in two-occasion successive sampling when the information on an auxiliary variable is readily available on both the occasions. The behavior of the proposed estimator has been examined and its optimal replacement strategy is formulated. Empirical studies are carried out to show the dominance of the proposed estimation procedure. Results are interpreted and suitable recommendations are made to the survey practitioners.

Keywords: Successive sampling, auxiliary variables, bias, mean square error, optimum replacement strategy.

1 Introduction

There are many real life problems of practical interest in various fields of applied sciences, where the study characters of a finite population is subject to change over time and a survey carried out on a single occasion will provide information about the characteristics of the surveyed population only for the given occasion and cannot give information related to the nature or rate of change over different occasions or the estimates of the population parameters over all occasions or on the most recent occasion. For such cases, the use of successive (rotation) sampling as advocated by [1] is the most appropriate sampling procedure to generate the reliable (in terms of cost and precision) estimates of population parameters on various desired occasions. The theory of successive sampling was further extended by [2],[3],[4],[5], and [6] among others. [7],[8] applied this theory to design the estimators of the population parameters on the current occasion by using information on two or more auxiliary variables which were readily available on the previous occasion.

In many practical situations, information on an auxiliary variable may be readily available on both occasions in two-occasion successive sampling. In follow up of this argument, [9], [10],[11],[12],[13], [14], [15],[16] among others have proposed several estimators of population mean on current (second) occasion in two occasion successive sampling. Motivated with the above cited works, in this paper we have proposed an improved exponential type estimator of current population mean in two-occasion successive sampling by utilizing the information on an auxiliary variable which is readily available on both occasions. Theoretical properties of the proposed estimator have been discussed and empirical studies are carried out to show the dominance over other estimators. Results have been interpreted and suitable recommendations are made to the survey practitioners.

2 Formulation of the estimator

Let $U = (U_1, U_2, ..., U_N)$ be the finite population of N units, which has been sampled over two occasions. The character under study is denoted by $x(y)$ on the first (second) occasion respectively. It is assumed that information on an auxiliary variable $z_h\,(h = 1, 2)$ is readily available on hth occasion whose population mean is known and has positive

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correlation with x and y on the first and the second occasions respectively. A simple random sample of n units is drawn on the first occasion using without replacement scheme. A random sub sample of $m(=n\lambda)$ units is matched (retained) from the sample on the first occasion for its use on the second occasion, while afresh random sample $u=(n-m)=n\mu$ of units is drawn without replacement on the second occasion so that the sample size on the second occasion is also n. Here λ and μ $(\lambda+\mu=1)$ are the fractions of matched and fresh samples respectively on the current occasion. The values of λ or μ would be chosen optimally.
The following notations have been considered for further use.

$\bar{X},\bar{Y}$: Population means of the study variables x and y respectively.
$\bar{Z}_1$ $(\bar{Z}_2)$: Population mean of the auxiliary variable z_h on the hth $(h=1,2)$ occasion.
$\bar{x}_n,\bar{x}_m,\bar{y}_u,\bar{y}_m,\bar{z}_{ju},\bar{z}_{jn},\bar{z}_{jm}(j=1,2)$: Sample means of the respective variables based on the sample sizes shown in suffices.
$\rho_{yx},\rho_{xz_1},\rho_{xz_2},\rho_{yz_1},\rho_{yz_2},\rho_{z_1z_2}$: Population correlation coefficients between the variables shown in suffices.
$S_x^2,S_y^2,S_{z_1}^2,S_{z_2}^2$: Population variances of the variables x,y,z_1,z_2 respectively.
C_y,C_x,C_{z_1},C_{z_2}: Population coefficients of variation of the variables y,x,z_1,z_2 respectively.

Utilizing the information on auxiliary variables z_1 and z_2 on first and second occasions respectively, we formulate two estimators of current population mean which are based on fresh and matched samples. The estimator based on fresh sample of size u is of exponential structure and formulated as

$$T_u=\bar{y}_u\left(\frac{\bar{Z}_2}{\bar{z}_{2u}}\right)\exp\left(\frac{\bar{Z}_2-\bar{z}_{2u}}{\bar{Z}_2+\bar{z}_{2u}}\right) \tag{1}$$

The second estimator based on matched sample of size m is also have an exponential structure and formulated as

$$T_m=\bar{y}_m\left(\frac{\bar{x}_n}{\bar{x}_m}\right)\exp\left(\frac{\bar{Z}_1-\bar{z}_{1n}}{\bar{Z}_1+\bar{z}_{1n}}\right)\exp\left(\frac{\bar{Z}_2-\bar{z}_{2m}}{\bar{Z}_2+\bar{z}_{2m}}\right) \tag{2}$$

To estimate the current population mean $\bar{Y}$, the estimator T_u is most appropriate and for estimating change over both occasions the estimator T_m is suitable while to deal with both the problems simultaneously the contribution of T_u and T_m are highly desirable. Motivated with these arguments, the final estimator of current population mean $\bar{Y}$ is proposed as

$$T=\varphi T_u+(1-\varphi)T_m \tag{3}$$

where $\varphi(0\le\varphi\le1)$ is an unknown constant (scalar) to be determined under certain criterion such that the mean square error (MSE) of the estimator T attains its minimum.

3 Properties of the proposed estimator T

Since the estimators T_u and T_m have the exponential structures, therefore, they are biased estimators of the current population mean $\bar{Y}$, the final estimator T is a convex linear combination of the estimators T_u and T_m, hence it is also a biased estimator. The bias $B(.)$ and mean square error $M(.)$ of the estimator T is derived up to first-order of approximations under large sample assumption using the following transformations:

$\bar{y}_u=\bar{Y}(1+e_1),\bar{y}_m=\bar{Y}(1+e_2),\bar{x}_m=\bar{X}(1+e_3),\bar{x}_n=\bar{X}(1+e_4),\bar{z}_{2u}=\bar{Z}(1+e_5)\,\bar{z}_{1n}=\bar{Z}(1+e_6),\bar{z}_{2m}=\bar{Z}(1+e_7)$
Such that $E(e_i)=0$ *and* $|e_i|\le1,\forall\, i=1,2,...,7$
Under the above transformations, the estimators T_u and T_m take the following forms:

$$T_u=\left[\bar{Y}(1+e_1)(1+e_5)^{-1}\exp\left\{-\frac{1}{2}e_5\left(1+\frac{1}{2}e_5\right)^{-1}\right\}\right] \tag{4}$$

$$T_m=\left[\bar{Y}(1+e_2)(1+e_4)(1+e_3)^{-1}\exp\left\{-\frac{1}{2}e_6\left(1+\frac{1}{2}e_6\right)^{-1}\right\}\exp\left\{-\frac{1}{2}e_7\left(1+\frac{1}{2}e_7\right)^{-1}\right\}\right] \tag{5}$$

Thus, we have the following theorems.
Theorem 1: The bias of the estimator up to the first order approximations is derived as:

$$B(T)=\varphi B(T_u)+(1-\varphi)B(T_m) \tag{6}$$

where

$$B(T_u) = \bar{Y}\left(\frac{1}{u} - \frac{1}{N}\right)\left[\frac{15}{8}C_{0002} - \frac{3}{2}C_{0101}\right] \tag{7}$$

$$B(T_{2m}) = \bar{Y}\begin{bmatrix} \left(\frac{1}{m} - \frac{1}{N}\right)C_{2000} + \left(\frac{1}{n} - \frac{1}{m}\right)C_{1100} \\ +\left\{\frac{1}{2}\left(\frac{1}{m} - \frac{1}{n}\right)C_{1001} - \left(\frac{1}{m} - \frac{1}{N}\right)C_{0101} - \left(\frac{1}{n} - \frac{1}{N}\right)C_{0110} + \frac{1}{2}\left(\frac{1}{n} - \frac{1}{N}\right)C_{0011}\right\} \\ +\frac{3}{8}\left\{\left(\frac{1}{m} - \frac{1}{N}\right)C_{0002} + \left(\frac{1}{n} - \frac{1}{N}\right)C_{0020}\right\} \end{bmatrix} \tag{8}$$

where

$C_{pqrs} = E\left[(x_i - \bar{X})^p (y_i - \bar{Y})^q (z_{i1} - \bar{Z})^r (z_{i2} - \bar{Z})^s\right]$; $(p,q,r,s) \geq 0$ are integers.

Proof: The bias of the estimator T is given by

$$B(T) = E(T - \bar{Y}) = \varphi E(T_u - \bar{Y}) + (1-\varphi)E(T_m - \bar{Y})$$

$$= \varphi B(T_u) + (1-\varphi)B(T_m) \tag{9}$$

where

$B(T_u) = E(T_u - \bar{Y})$ and $B(T_m) = E(T_m - \bar{Y})$
From equations (4) and (5), substituting the expressions of T_u and T_m into equation (9), expanding exponentially and binomially, taking expectations and retaining the term up to first order of approximations, we have the expression of the bias of the estimator T as given in equation (6).

Theorem 2: The mean square error of the estimator T up to the first degree of approximations is obtained as

$$M(T) = \varphi^2 M(T_u) + (1-\varphi)^2 M(T_m) + 2\varphi(1-\varphi)C(T_u, T_m) \tag{10}$$

where

$$M(T_u) = \left(\frac{1}{u} - \frac{1}{N}\right)\left[\frac{13}{4} - 3\rho_{yz_2}\right]S_y^2. \tag{11}$$

$$M(T_m) = S_y^2\begin{bmatrix} \left(\frac{9}{4m} - \frac{3}{4n} - \frac{6}{4N}\right) + 2\left(\frac{1}{n} - \frac{1}{m}\right)\rho_{yx} + \left(\frac{1}{m} - \frac{1}{n}\right)\rho_{xz_2} \\ +\left(\frac{1}{n} - \frac{1}{N}\right)\left(\rho_{z_1z_2} - \rho_{yz_1}\right) - \left(\frac{1}{m} - \frac{1}{N}\right)\rho_{yz_2} \end{bmatrix} \tag{12}$$

$$C(T_u, T_m) = -\frac{1}{N}\left[\frac{7}{4} - \frac{1}{2}\left(\rho_{yz_2} + \rho_{yz_1}\right) + \frac{3}{4}\left(\rho_{z_1z_2} - \rho_{yz_2}\right)\right]S_y^2 \tag{13}$$

Proof: The mean square error of the estimator T is given by

$$M(T) = E[T - \bar{Y}]^2 = E[\varphi(T_u - \bar{Y}) + (1-\varphi)(T_m - \bar{Y})]^2$$

$$= \varphi^2 M(T_u) + (1-\varphi)^2 M(T_m) + 2\varphi(1-\varphi)C(T_u, T_m) \tag{14}$$

where

$M(T_u) = E[T_u - \bar{Y}]^2$, $M(T_m) = E[T_m - \bar{Y}]^2$ and $C(T_u, T_m) = E\left[(T_u - \bar{Y})(T_m - \bar{Y})\right]$

From equations (4) and (5), substituting the expressions of T_u and T_m into equation (14), expanding exponentially and binomially, taking expectations and retaining the term up to first order of approximations, we have the expression of the mean square error of the estimator T as given in equation (10).

Remark: The bias and mean square error of the estimator T shown in equations (6) and (10) respectively are derived under the assumption that the coefficients of variation of the variables x, y,z_1 and z_2 are approximately equal, which is an intuitive assumption and also considered by [9].

3.1 Minimum mean square error of the estimator T

Since the mean square error of the estimator T in equation (10) is a function of the unknown constant φ, therefore, it is minimized with respect to φ and subsequently the optimum value of φ is obtained as

$$\varphi_{opt} = \frac{M(T_m) - C(T_u, T_m)}{M(T_u) + M(T_m) - 2C(T_u, T_m)} \tag{15}$$

From equation (15), substituting the value of φ_{opt} in equation (10) we get the optimum mean square error of the estimator T as

$$M(T)_{opt} = \frac{M(T_u)M(T_m) - [C(T_u, T_m)]^2}{M(T_u) + M(T_m) - 2C(T_u, T_m)} \tag{16}$$

Now substituting the values from equations (11) - (13) in equations (15) and (16), the simplified values of φ_{opt} and $M(T)_{opt}$ is obtained as

$$\varphi_{opt} = \frac{\mu[\mu A_6 + fA_7 + A_8]}{[\mu^2 A_9 + \mu A_6 + A_{11}]} \tag{17}$$

$$M(T)_{opt} = \frac{[A_{13}\mu^2 + A_{14}\mu + A_{15}]}{[A_9\mu^2 + A_1\mu + A_{11}]}\frac{S_y^2}{n} \tag{18}$$

where

$$A_1 = 2\rho_{yx} - \rho_{xz_2}, A_2 = \rho_{z_1z_2} - \rho_{yz_1}, A_3 = \rho_{yz_2}, A_4 = \rho_{z_1z_2} - 2\rho_{yz_2}, A_5 = \rho_{yz_2} + \rho_{yz_1}$$

$$A_6 = \left(\frac{3}{4} - \frac{1}{4}f - A_1 - A_2 + fA_2 - fA_3 - \frac{3}{4}fA_4 + \frac{1}{2}fA_5\right), A_7 = \left(\frac{1}{4} - A_2 + A_3 + \frac{3}{4}A_4 - \frac{1}{2}A_5\right)$$

$$A_8 = A_2 - A_3,\ A_9 = \frac{3}{4} + \frac{5}{4}f - 3fA_3 - A_1 - A_2 + fA_2 - fA_3 - \frac{3}{2}fA_4 + fA_5$$

$$A_{10} = \left[-\frac{7}{4} - f\left(\frac{5}{4} - 6A_3 - \frac{3}{2}A_4 + A_5\right)\right], A_{11} = \left(\frac{13}{4} - 3A_3\right)$$

$$A_{12} = \left[\frac{49}{16} + \frac{1}{4}\left(\frac{9}{4}A_4^2 + A_5^2 + \frac{21}{2}A_4 - \frac{3}{4}A_4A_5 - 7A_5\right)\right]$$

$$A_{13} = \left[f^2(A_{12} + A_3A_{11} - A_2A_{11}) + f\left(A_1A_{11} + A_2A_{11} - \frac{3}{4}A_{11}\right)\right]$$

$$A_{14} = f^2\left(\frac{6}{4}A_{11} + A_2A_{11} - A_3A_{11} - A_{12}\right) + \frac{9}{4}fA_{11} - \frac{3}{2}A_{11} - A_1A_{11} - A_2A_{11}$$

$$A_{15} = (1-f)\left(\frac{6}{4}A_{11} + A_2A_{11} - A_3A_{11}\right)$$

$\mu = \frac{u}{n}$ is the fraction of the fresh sample drawn on the current (second) occasion.

4 Optimum replacement strategy

To determine the optimum value of μ (fraction of sample to be drawn afresh on the current occasion) so that $\bar{Y}$ be estimated with maximum precision and minimum cost, we minimize $M(T)_{opt}$ in equation (18) with respect to μ, which results in a quadratic equation in μ, which is given as

$$T_1\mu^2 + 2T_2\mu + T_3 = 0 \tag{19}$$

Solving the equation (19), the solutions of μ (say $\hat{\mu}$) are given as

$$\hat{\mu} = \frac{-T_2 \pm \sqrt{T_2^2 - T_1T_3}}{T_1} \tag{20}$$

where

$$T_1 = A_{10}A_{13} - A_9A_{14}, T_2 = A_{11}A_{13} - A_9A_{15}, T_3 = A_{11}A_{14} - A_{10}A_{15}$$

The real value of $\hat{\mu}$ exists, if $T_2^2 - T_1T_3 \geq 0$ for any combination of $\rho_{yx}, \rho_{xz_1}, \rho_{xz_2}, \rho_{yz_1}, \rho_{yz_2}, \rho_{z_1z_2}$ which satisfy the condition of real solution, two real values of $\hat{\mu}$ are possible. Hence while choosing the value of $\hat{\mu}$, it should be remembered that $0 \leq \hat{\mu} \leq 1$; all other values of $\hat{\mu}$ are inadmissible. If both the value of $\hat{\mu}$ are admissible, the lowest one is the best choice as it reduces the cost of the survey. From equation (20), substituting the admissible value of $\hat{\mu}$ (say μ_0) in equation (18), we have the optimum value of mean square error of the estimator T which is shown below

$$M(T)^{(0)}{}_{opt} = \frac{\left[A_{13}\mu_0{}^2 + A_{14}\mu_0 + A_{15}\right]}{\left[A_9\mu_0{}^2 + A_1\mu_0 + A_{11}\right]} \frac{S_y^2}{n} \tag{21}$$

5 Efficiency comparison

For evaluating the efficiency of the proposed estimator T, we compare the proposed estimator with sample mean estimator $\bar{y}_n$ (when there is no matching) and with natural successive sampling estimator $\hat{\bar{Y}} = \varphi \bar{y}_u + (1-\varphi)\bar{y}'_m$ (when no auxiliary information is used on any occasion),where $\bar{y}'_m = \bar{y}_m + \beta_{yx}(\bar{X}_n - \bar{X}_m)$ and T_1, where T_1 is the [16] estimator. The percent relative efficiencies of the estimator T have been obtained for different choices of $\rho_{yx}, \rho_{xz_1}, \rho_{xz_2}, \rho_{yz_1}, \rho_{yz_2}, \rho_{z_1z_2}$. Since $\bar{y}_n$ and $\hat{\bar{Y}}$ are unbiased estimator of $\bar{Y}$, following [17] the variance of $\bar{y}_n$ and optimum variance of $\hat{\bar{Y}}$ and the optimum mean square error of [16] estimator are as follows

$$V(\bar{y}_n) = \left(\frac{1}{n} - \frac{1}{N}\right) S_y^2$$

$$V(\hat{\bar{Y}})_{opt} = \left[1 + \sqrt{1-\rho_{yx}^2}\right] \frac{S_y^2}{2n} - \frac{S_y^2}{N}$$

$$M(T_1)^*_{opt} = \frac{\left[A_5 + \mu_1^{(0)}A_6 + \mu_1^{(0)2}A_7\right]}{\left[A_1 + \mu_1^{(0)2}A_3\right]} \frac{S_y^2}{n}$$

where A_i $(i = 1, 2...., 17)$ are same as given by [16].

For N=5000, n=500 and different choices of correlations $\rho_{yx}, \rho_{xz_1}, \rho_{xz_2}, \rho_{yz_1}, \rho_{yz_2}, \rho_{z_1z_2}$, Table 1 presents the optimum values of μ and percent relative efficiencies E_1 and E_2 of the estimator T with respect to the estimators $\bar{y}_n$ and $\hat{\bar{Y}}$ respectively and Table 2 presents the optimum values of μ and percent relative efficiencies E of the estimator T with respect to T_1 are as follows:

$E_1 = \frac{V(\bar{y}_n)}{M(T)^{(0)}{}_{opt}} \times 100$, $E_2 = \frac{M\left(\hat{\bar{Y}}_{opt}\right)}{M(T)^{(0)}{}_{opt}} \times 100$ and $E = \frac{M(T_1)^*_{opt}}{M(T)^{(0)}_{opt}} \times 100$

Note: * indicate μ do not exist and ** indicate no gain.

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6 Interpretations of empirical results

1.The following interpretations may be read out from Table 1.

(i)For fixed values of $\rho_{yx}, \rho_{xz2}, \rho_{yz_1}, \rho_{z_1z_2}$ the values of μ_0 decrease and the values of E_1 and E_2 increase with the increasing values of ρ_{yz_2}.

(ii)For fixed values of $\rho_{yx}, \rho_{xz_2}, \rho_{yz_1}, \rho_{yz_2}$,, the values of μ_0 and the values of E_1 and E_2 decrease with the increasing values of $\rho_{z_1z_2}$.

(iii)For fixed values of $\rho_{yz_2}, \rho_{yz_1}, \rho_{z_1z_2}, \rho_{xz_2}$, the values of μ_0 and E_1 increase but the values of E_2 do not follow any pattern with the increasing values of ρ_{yx}.

Table 1: Optimum values of μ and percent relative efficiencies of T with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$ (for ρ_{yz_2}=0.5)

	ρ_{yx}	ρ_{yz_1}	ρ_{yz_2}	0.5	0.6	0.7	0.8	0.9
0.4	0.7	0.8	μ_0	0.6849	0.6471	0.5946	0.517	0.3903
			E_1	109.29	108.64	107.7	106.28	103.93
			E_2	104.23	103.6	102.71	101.35	**
		0.9	μ_0	0.5213	0.4592	0.3729	0.2452	0.0365
			E_1	149.56	146.43	142.05	135.53	124.89
			E_2	142.62	139.64	135.46	129.25	119.1
	0.9	0.8	μ_0	0.7388	0.7166	0.6881	0.6502	0.5973
			E_1	110.38	110.03	109.57	108.93	108
			E_2	105.26	104.93	104.48	103.87	102.99
		0.9	μ_0	0.6071	0.5703	0.5229	0.4599	0.3719
			E_1	154.25	152.47	150.16	147.03	142.61
			E_2	147.09	145.4	143.19	140.21	135.99
0.5	0.7	0.8	μ_0	0.7784	0.7439	0.6884	0.5834	0.4111
			E_1	113.98	113.68	113.16	112.12	109.32
			E_2	105.49	105.22	104.74	103.77	101.18
		0.9	μ_0	0.5785	0.5004	0.3746	0.1386	0.4665
			E_1	159.1	156.02	151	141.59	125.16
			E_2	147.26	144.41	139.76	131.06	109.36
	0.9	0.8	μ_0	0.8233	0.8068	0.7839	0.75	0.6946
			E_1	114.46	114.38	114.21	113.95	113.46
			E_2	105.94	105.86	105.72	105.47	105.02
		0.9	μ_0	0.6736	0.6348	0.5811	0.5017	0.3723
			E_1	163.21	161.79	159.76	156.69	151.63
			E_2	151.06	149.75	147.87	145.03	140.34
0.6	0.7	0.8	μ_0	0.9567	0.9672	0.9936	0.9952	0.8172
			E_1	117.54	117.6	117.65	117.44	118.07
			E_2	104.48	104.54	104.57	104.54	104.95
		0.9	μ_0	0.6772	0.5731	0.386	*	*
			E_1	170.74	168.29	162.11	**	**
			E_2	151.77	149.59	144.09	**	**
	0.9	0.8	μ_0	0.9553	0.9607	0.9694	0.9861	*
			E_1	117.46	117.53	117.59	117.64	**
			E_2	104.41	104.47	104.53	104.57	**
		0.9	μ_0	0.7743	0.7394	0.6832	0.5772	0.3832
			E_1	173.28	172.61	171.42	169.05	162.76
			E_2	154.02	153.43	152.37	150.27	144.67

(iv)For fixed values of $\rho_{yx}, \rho_{yz2}, \rho_{z_1z_2}, \rho_{xz_2}$, the values of μ_0, E_1 and E_2 do not follow any pattern with the increasing values of ρ_{yz_1}.

(v)The minimum value of μ_0 is $0.0365\,(\cong 0.04)$, which shows that the fraction to be replaced on the current occasion is as low as about 4 percent of the total sample size leading to a reduction of the considerable amount of the cost of the survey.

2.The following interpretations may be observed from Table 2.

(i)For fixed values of ρ_{yx} and ρ_{yz}, the values of μ_0 increase but the values of E decrease with the increasing values of $\rho_{z_1z_2}$.

(ii)For fixed values of ρ_{yx} and $\rho_{z_1z_2}$, the values of E decrease and there is no change in the values of μ_0 with the increasing values of ρ_{yz}.

Table 2: For fixed (g=0.6, ρ_{xz_2}=0.2, ρ_{yz_1}=0.9, ρ_{yz_2}=0.6, ρ_{xz}=0.9), the optimum values of μ and percent relative efficiencies of T with respect to [16] estimator.

ρ_{yx}	ρ_{yz}	$\rho_{z_1z_2}$	0.8	0.85	0.9
0.7	0.5	μ_0	*	0.0887	0.1994
		E	**	137.67	131.33
	0.55	μ_0	*	0.0887	0.1994
		E	**	125.18	119.41
	0.6	μ_0	*	0.0887	0.1994
		E	**	110.61	105.51
0.8	0.5	μ_0	0.3693	0.4233	0.4683
		E	153.04	147.33	142.54
	0.55	μ_0	0.3693	0.4233	0.4683
		E	139.64	134.43	130.06
	0.6	μ_0	0.3693	0.4233	0.4683
		E	123.98	119.35	115.48
0.9	0.5	μ_0	0.7172	0.7343	0.7494
		E	187.26	181.18	175.83
	0.55	μ_0	0.7172	0.7343	0.7494
		E	171.69	166.11	161.2
	0.6	μ_0	0.7172	0.7343	0.7494
		E	153.69	148.7	144.31

(iii)For fixed values of ρ_{yz} and $\rho_{z_1z_2}$, the values of μ_0 and E increase with the increasing values of ρ_{yx}.

(iv)The minimum value of μ_0 is 0.0887, which indicates that the fraction of fresh sample to be replaced on the current occasion is as low as about 9 percent of the total sample size, which is highly helpful in reducing the cost of the survey.

It is visible that from Table 1-2 almost all the values of E_1,E_2 and E are more than 100 which indicate that the proposed estimator is uniformly dominating over $\bar{y}_n$, $\hat{\bar{Y}}$ and [16] estimators.

7 Conclusions and Recommendations

From the above numerical study, it may be concluded that the proposed estimator T is significantly better than $\bar{y}_n$,$\hat{\bar{Y}}$ and [16] estimators. It may also be concluded that proposed estimator is highly rewarding in terms of precision as well as in reducing the cost of the survey to the considerable amount. Hence the proposed estimator represents the nice behavior and may be recommended to survey statisticians for their real life applications.

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