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A Simple Algorithm for Complete Factorization of an N-Partite Pure Quantum State

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Abstract: We present a simple algorithm to completely factorize an arbitrary N-partite pure quantum state. This complete factorization of such a pure state also specifies its complete entanglement status : whether the given N-partite pure quantum state is completely separable (N factors), or completely entangled (no factors), or partially entangled having entangled factors of different sizes which cannot be factored further. The problem of deciding entanglement status of a bipartite pure quantum state is one of the initial problems encountered in quantum information research and this problem is usually tackled using the well known Schmidt decomposition procedure. One obtains Schmidt number of the state which decides the entanglement status of the state. In this paper we first develop a simple criterion which when fulfilled enables us to factorize given N-partite pure quantum state as tensor product of an m-partite pure quantum state and an n-partite pure quantum state where $m+n=N$. This criterion gives rise to an effective mechanical procedure in terms of an easy algorithm to perform complete factorization of given N-partite pure quantum state and thus provides an easy method to determine complete entanglement status of the state. In this paper we carry out our discussion for the case of N-qubit pure quantum state instead of N-qudit case for the sake of simplicity of presentation. The extension to the case of N-qudit pure quantum state is straightforward and follows by proceeding along similar lines. We just mention this extension to avoid repetition and only briefly demonstrate it with the help of one of the examples discussed at the end of the paper.

Keywords: Multipartite pure quantum state, criterion for entanglement and separability, complete factorization

1 Introduction

One of the central issues in quantum information theory is whether a given multipartite pure quantum state is separable or entangled [1,2,3,4,5]. This important question of deciding whether a given multipartite pure quantum state is separable or entangled is completely solved in this paper. In this paper, we present an algorithm to completely factorize an arbitrary N-partite pure quantum state, that is, it is factorized until no further factorization is possible. An N-partite pure quantum state may be completely separable, that is, it is a tensor product of N states, each pertaining to one of the individual parts, or it may be a product of M $(M < N)$ states, each belonging to one of the M subsystems, some of them containing two or more parts. If the completely factorized N-partite state has such a structure, then the states of the subsystems containing more than one part appearing in this factorization are necessarily entangled, otherwise, they would have factorized further. In this paper we carry out our discussion for the case of N-qubit pure quantum state instead of N-qudit case for the sake of simplicity of presentation. The extension to the case of N-qudit pure quantum state is straightforward and follows by proceeding along similar lines. We just mention this extension to avoid repetition and only briefly demonstrate it with the help of one of the examples discussed at the end of the paper.

2 The criterion for factorization

Notation: Let $|\psi\rangle$ be an N-qubit pure state :

$$|\psi\rangle = \sum_{s=1}^{2^N} a_{r_s} |r_s\rangle \tag{1}$$

expressed in terms of the computational basis. Here the basis vectors $|r_s\rangle$ are ordered lexicographically. That is, the corresponding binary sequences are ordered lexicographically: $r_1 = 00\cdots00$, $r_2 = 00\cdots01$, $\ldots$,

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$r_{2^N} = 11\cdots11$, so that $|r_1\rangle = |00\cdots00\rangle$, $|r_2\rangle = |00\cdots01\rangle$, $\ldots$, $r_{2^N} = |11\cdots11\rangle$. Let m,n be any integers such that $1 \leq m,n < N$ and $m+n = N$. Let the corresponding two sets of computational basis vectors ordered lexicographically be $|i_1\rangle,\ldots,|i_{2^m}\rangle$ (each of length m) and $|j_1\rangle,\ldots,|j_{2^n}\rangle$ (each of length n). Rewrite $|\psi\rangle$ thus :

$$|\psi\rangle = \sum_{u=1}^{2^m}\sum_{v=1}^{2^n} a_{i_u j_v}|i_u\rangle \otimes |j_v\rangle. \qquad (2)$$

Here in the symbol $a_{i_u j_v}$, the suffix $i_u j_v$ is the juxtaposition of the binary sequences i_u and j_v in that order. Thus we get a $2^m \times 2^n$ matrix $A = [a_{i_u j_v}]$ which will be called the $2^m \times 2^n$ *matrix associated to* $|\psi\rangle$.

Lemma 1: The state $|\psi\rangle$ given by (1) can be factored as the product, $|\psi_1\rangle \otimes |\psi_2\rangle$, of an $m-$qubit state $|\psi_1\rangle$ and an $n-$qubit state $|\psi_2\rangle$ if and only if the $2^m \times 2^n$ matrix A associated to $|\psi\rangle$ can be expressed as $B^T C$ where B is a 1×2^m matrix, C is a 1×2^n matrix and B^T is the transpose of B.

Proof: With the above notation, let

$$|\psi_1\rangle = \sum_{u=1}^{2^m} b_{i_u}|i_u\rangle,$$
$$\text{and}\quad |\psi_2\rangle = \sum_{v=1}^{2^n} c_{j_v}|j_v\rangle.$$

Then the product $|\psi_1\rangle \otimes |\psi_2\rangle$ is

$$|\psi_1\rangle \otimes |\psi_2\rangle = \sum_{u=1}^{2^m}\sum_{v=1}^{2^n} b_{i_u} c_{j_v}|i_u\rangle \otimes |j_v\rangle. \qquad (3)$$

Comparing (2) and (3) we see that $|\psi\rangle$ can be factored as $|\psi_1\rangle \otimes |\psi_2\rangle$ if and only if

$$a_{i_u j_v} = b_{i_u} c_{j_v}, \quad \text{for} \quad u = 1,\ldots,2^m \quad \text{and} \quad v = 1\ldots,2^n$$

i.e. if and only if $A = B^T C$ where $B = [b_{i_u}]$ and $C = [c_{j_v}]$. $\square$

We also need the following standard result:

Lemma 2: An $a \times b$ non-zero matrix A over complex numbers can be expressed as $B^T C$ for some $1 \times a$ matrix B and $1 \times b$ matrix C if and only if rank$(A) = 1$.

Now we can prove the

Theorem: The state $|\psi\rangle$ given by (1) can be factored as the product, $|\psi_1\rangle \otimes |\psi_2\rangle$, of an $m-$qubit state $|\psi_1\rangle$ and an $n-$qubit state $|\psi_2\rangle$ if and only if the $2^m \times 2^n$ matrix A associated to $|\psi\rangle$ is of rank 1.

Proof: With the above notation, let $|r_w\rangle$ be the *first* basic vector such that $a_{r_w} \neq 0$. Choose integers m,n such that $1 \leq m,n < N$ and $m+n = N$. Let the corresponding two sets of computational basis vectors ordered lexicographically be $|i_1\rangle,\ldots,|i_{2^m}\rangle$ (each of length m) and $|j_1\rangle,\ldots,|j_{2^n}\rangle$ (each of length n). Then we can write

$$|\psi\rangle = \sum_{u=1}^{2^m}\left[|i_u\rangle \otimes \sum_{v=1}^{2^n} a_{i_u j_v}|j_v\rangle\right]. \qquad (4)$$

Consider the associated $2^m \times 2^n$ matrix $A = [a_{i_u j_v}]$. Suppose $|r_w\rangle = |i_p\rangle|j_q\rangle$ so that the *first* non-zero element of A is the qth element in the pth row, namely $a_{i_p j_q}$. Thus the pth row of A is non-zero. Now, suppose rank$(A) = 1$. Then there exist numbers $k_1,\ldots,k_{2^m}$ such that $k_p = 1$ and row$_u$ $=$ $k_u$row$_p$ $(u = 1,\ldots,2^m)$ i.e. $a_{i_u j_v} = k_u a_{i_p j_v}$, $(u = 1,\ldots,2^m,\ v = 1,\ldots,2^n)$. Hence (4) can be written as

$$|\psi\rangle = \sum_{u=1}^{2^m}\left[|i_u\rangle \otimes \sum_{v=1}^{2^n} k_u a_{i_p j_v}|j_v\rangle\right]$$
$$= \sum_{u=1}^{2^m} k_u|i_u\rangle \otimes \sum_{v=1}^{2^n} a_{i_p j_v}|j_v\rangle$$
$$= |\psi_1\rangle \otimes |\psi_2\rangle,$$
$$\text{where} \quad |\psi_1\rangle = \sum_{u=1}^{2^m} k_u|i_u\rangle \quad \text{and} \quad |\psi_2\rangle = \sum_{v=1}^{2^n} a_{i_p j_v}|j_v\rangle.$$

Thus $|\psi\rangle$ factors as stated. Conversely, suppose $|\psi\rangle$ given by (1) can be factored as the product of an $m-$qubit state and an $n-$qubit state, in that order. Then by lemma 1, the $2^m \times 2^n$ non-zero matrix A associated to $|\psi\rangle$ can be expressed as $B^T C$ where B is a 1×2^m matrix and C is a 1×2^n matrix. Hence by lemma 2, rank$(A) = 1$.

This proves the theorem. $\square$

3 Algorithm

We now proceed to present our algorithm, based on the above theorem, for complete factorization of an arbitrary N-qubit pure quantum state. Later we will indicate how this algorithm could be modified to cater to an arbitrary N-partite pure quantum state. We use the above notation.The steps of the algorithm are as follows.

(i) We express the given N-qubit pure state $|\psi\rangle$ in terms of the computational basis as

$$|\psi\rangle = \sum_{s=1}^{2^N} a_{r_s}|r_s\rangle$$

where the basis vectors $|r_s\rangle$ are ordered lexicographically as before.

(ii) Now our aim is to check as first step (using the Theorem just proved above) whether given $|\psi\rangle$ has a linear (1-qubit) factor and an $(N-1)$-qubit factor. In order to find the corresponding $2^1 \times 2^{N-1}$ matrix A associated to $|\psi\rangle$, we rewrite this state as

$$|\psi\rangle = \sum_{u=1}^{2}|i_u\rangle \otimes \left[\sum_{v=1}^{2^{N-1}} a_{i_u i_v}|j_v\rangle\right].$$

Here the basis vectors ordered lexicographically are $|i_1\rangle = |0\rangle, |i_2\rangle = |1\rangle$ (each of length 1) and $|j_1\rangle, \dots, |j_{2^{(N-1)}}\rangle$ (each of length $N-1$). Hence the associated matrix is

$$A = \begin{bmatrix} a_{00\cdots 0} & a_{00\cdots 01} & a_{00\cdots 010} & \cdots & a_{01\cdots 11} \\ a_{10\cdots 0} & a_{10\cdots 01} & a_{10\cdots 010} & \cdots & a_{11\cdots 11} \end{bmatrix}.$$

(iii) Now there are two cases.
Case I

If $\mathrm{rank}(A) = 1$, then by above theorem there exist numbers k_1, k_2 such that

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle,$$
$$\text{where} \quad |\psi_1\rangle = \sum_{u=1}^{2} k_u |i_u\rangle \quad \text{and} \quad |\psi_2\rangle = \sum_{v=1}^{2^{N-1}} a_{i_p j_v} |j_v\rangle,$$

with $k_p = 1$ where the pth row of A is the *first* non-zero row of A ($p = 1$ or 2). Thus in this case the state has a factor $|\psi_1\rangle$. In this case we go back to step (i) with $|\psi\rangle = |\psi_2\rangle$.
Case II

If $\mathrm{rank}(A) \neq 1$, then by above theorem we do not get a factor like $|\psi_1\rangle = k_1|i_1\rangle + k_2|i_2\rangle$ with $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$. In this case our aim is to check as next step whether given $|\psi\rangle$ has a 2-qubit factor and an $(N-2)$-qubit factor. For this we proceed with the originally given state $|\psi\rangle$ as given in the next step (iv).
(iv) In order to find the corresponding $2^2 \times 2^{N-2}$ matrix A associated to $|\psi\rangle$, we rewrite this state as

$$|\psi\rangle = \sum_{u=1}^{2^2} |i_u\rangle \otimes \left[\sum_{v=1}^{2^{N-2}} a_{i_u i_v} |j_v\rangle \right].$$

Here the basis vectors ordered lexicographically are $|i_1\rangle = |00\rangle$, $|i_2\rangle = 01$, $|i_3 = |10\rangle$, $|i_4\rangle = |11\rangle$ (each of length 2) and $|j_1\rangle, \dots, |j_{2^{(N-2)}}\rangle$ (each of length $N-2$). Thus

$$A = \begin{bmatrix} a_{000\cdots 0} & a_{000\cdots 01} & a_{000\cdots 010} & \cdots & a_{001\cdots 11} \\ a_{010\cdots 0} & a_{010\cdots 01} & a_{010\cdots 010} & \cdots & a_{011\cdots 11} \\ a_{100\cdots 0} & a_{100\cdots 01} & a_{100\cdots 010} & \cdots & a_{101\cdots 11} \\ a_{110\cdots 0} & a_{110\cdots 01} & a_{110\cdots 010} & \cdots & a_{111\cdots 11} \end{bmatrix}.$$

Again there are two cases.
Case I

If $\mathrm{rank}(A) = 1$, then by above theorem there exist numbers $k_1, \dots, k_4$ such that

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle,$$
$$\text{where} \quad |\psi_1\rangle = \sum_{u=1}^{4} k_u |i_u\rangle \quad \text{and} \quad |\psi_2\rangle = \sum_{v=1}^{2^{N-2}} a_{i_p j_v} |j_v\rangle,$$

with $k_p = 1$ where the pth row of A is the *first* non-zero row of A. Thus in this case the state has a factor $|\psi_1\rangle$. In this case we go back to step (i) with $|\psi\rangle = |\psi_2\rangle$.
Case II

If $\mathrm{rank}(A) \neq 1$, then by above theorem we do not get a factor like $|\psi_1\rangle = k_1|i_1\rangle + \cdots + k_4|i_4\rangle$ with $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$. In this case our aim is to check as next step whether given $|\psi\rangle$ has a 3-qubit factor and an $(N-3)$-qubit factor. In order to find the corresponding $2^3 \times 2^{N-3}$ matrix A associated to $|\psi\rangle$, we rewrite this state as

$$|\psi\rangle = \sum_{u=1}^{2^3} |i_u\rangle \otimes \left[\sum_{v=1}^{2^{N-3}} a_{i_u i_v} |j_v\rangle \right].$$

Here the basis vectors ordered lexicographically are $|i_1\rangle = |000\rangle, \dots, |i_8\rangle = |111\rangle$ (each of length 3) and $|j_1\rangle, \dots, |j_{2^{(N-3)}}\rangle$ (each of length $N-3$). We again construct associated matrix matrix $A = [a_{i_u j_v}]$. This, again, leads to two separate cases, namely, whether the rank of the associated matrix A is equal to unity or not and so on. Thus, as above the algorithm continues until $|\psi\rangle$ is completely factored.

4 Generalization to N-qudit case

For this remark we use the usual notation. A general N-qudit state with dimensions $d_1, d_2, \dots, d_N$ can be written as

$$|\psi\rangle = \sum_{i_1, i_2, \dots i_N} a_{i_1 i_2 \dots i_N} |i_1 i_2 \dots i_N\rangle$$

where $i_k \in \{0, 1, \dots, d_k - 1\}$; $k = 1, 2, \dots, N$. To check whether $|\psi\rangle$ has a linear factor (on left), we re-write this state as

$$|\psi\rangle = |0\rangle \otimes \sum_{i_2, \dots, i_N} a_{0 i_2 \dots i_N} |i_2 \dots i_N\rangle + |1\rangle \otimes \sum_{i_2, \dots, i_N} a_{1 i_2 \dots i_N} |i_2 \dots i_N\rangle$$
$$+ \cdots + |d_1 - 1\rangle \otimes \sum_{i_2, \dots, i_N} a_{(d_1 - 1) i_2 \dots i_N} |i_2 \dots i_N\rangle.$$

This allows us to write down the d_1 by $(d_2 \times d_3 \times \cdots \times d_N)$ matrix A associated to $|\psi\rangle$ and $|\psi\rangle$ facotrs as

$$|\psi\rangle = \left(\sum_{i=0}^{d_1 - 1} k_i |i\rangle \right) \otimes \left(\sum_{i_2, \dots, i_N} a_{p i_2 \dots i_N} |i_2 \dots i_N\rangle \right),$$

if and only if $\mathrm{rank}(A) = 1$. Here $k_p = 1$ and row_p is the *first* non-zero row of A. If $\mathrm{rank}(A) \neq 1$, we check whether a two partite state factors out and so on. From this point, the algorithm proceeds exactly as in the N-qubit case. Please see Example (iv) below.

5 Examples

(i) Consider following example ([6], page 423)

$$|\psi\rangle = \frac{1}{\sqrt{2}} |01\rangle - \frac{1}{\sqrt{2}} |10\rangle.$$

We proceed as per our algorithm and first check whether $|\psi\rangle$ has a linear factor (on left). For this we rewrite $|\psi\rangle$ thus:

$$|\psi\rangle = 0|00\rangle + \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle + 0|11\rangle.$$
$$= |0\rangle \otimes \left[0|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right] + |1\rangle \otimes \left[-\frac{1}{\sqrt{2}}|0\rangle + 0|1\rangle\right]$$

Therefore the associated matrix A is

$$A = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

Clearly, $\text{rank}(A) = 2 > 1$, so that $|\psi\rangle$ has no linear factor and therefore the state $|\psi\rangle$ is entangled.
(ii) Consider the following two qubit state

$$|\psi\rangle = \frac{1}{\sqrt{3}}|00\rangle - \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{6}}|10\rangle - \frac{1}{\sqrt{6}}|11\rangle.$$

To check whether $|\psi\rangle$ has a linear factor (on left), we rewrite $|\psi\rangle$ thus:

$$|\psi\rangle = |0\rangle \otimes \left[\frac{1}{\sqrt{3}}|0\rangle - \frac{1}{\sqrt{3}}|1\rangle\right] + |1\rangle \otimes \left[\frac{1}{\sqrt{6}}|0\rangle - \frac{1}{\sqrt{6}}|1\rangle\right]$$

Therefore the associated matrix A is

$$A = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{bmatrix}$$

Clearly, $\text{row}_2 = 1/(\sqrt{2})\ \text{row}_1$ so that $\text{rank}(A) = 1$ and

$$|\psi\rangle = |0\rangle \otimes \left[\frac{1}{\sqrt{3}}|0\rangle - \frac{1}{\sqrt{3}}|1\rangle\right] + |1\rangle \otimes \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{3}}|0\rangle - \frac{1}{\sqrt{3}}|1\rangle\right].$$

Hence with $k_1 = 1, k_2 = 1/(\sqrt{2})$, $|\psi\rangle$ factors into two linear factors as follows:

$$|\psi\rangle = \left(|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{3}}|0\rangle - \frac{1}{\sqrt{3}}|1\rangle\right).$$

Hence the state $|\psi\rangle$ is separable.
(iii) Consider the following four qubit state,

$$|\psi\rangle = \frac{1}{2}\left[|0001\rangle + |0010\rangle + |1101\rangle + |1110\rangle\right]$$

First, to check whether there exists a linear factor we construct the associated matrix A :

$$A = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Clearly, $\text{rank}(A)$ is greater than 1, therefore, clearly $|\psi\rangle$ has no linear factor and therefore it is entangled. We now continue as per algorithm to check whether there exists a two qubit factor to given $|\psi\rangle$. For this we construct the following associated matrix A :

$$A = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Clearly, $\text{rank}(A) = 1$ and $|\psi\rangle$ factors into two 2-qubit factors:

$$|\psi\rangle = \left(\frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]\right) \otimes \left(\frac{1}{\sqrt{2}}\left[|01\rangle + |10\rangle\right]\right).$$

By applying our method we can readily see that these two factor states of $|\psi\rangle$ are both entangled states.
(iv) Consider the following 3-partite state comprising a qubit, a qutrit and, a qudit with $d = 4$.

$$|\psi\rangle = \frac{5}{4}\sqrt{\frac{35}{241}}i|001\rangle + \sqrt{\frac{35}{241}}|003\rangle + \frac{3}{2}\sqrt{\frac{7}{241}}i|022\rangle$$
$$-\frac{15}{4}\sqrt{\frac{5}{241}}|101\rangle + 3\sqrt{\frac{5}{241}}i|103\rangle - \frac{9}{2\sqrt{241}}|122\rangle.$$

As per the algorithm to check whether there exists a linear factor, we rewrite $|\psi\rangle$ thus:

$$|\psi\rangle = \frac{1}{4\sqrt{241}}\left[|0\rangle \otimes \left(5\sqrt{35}i|01\rangle + 4\sqrt{35}|03\rangle + 6\sqrt{7}i|22\rangle\right)\right]$$
$$+ \frac{1}{4\sqrt{241}}\left[|1\rangle \otimes \left(-15\sqrt{5}|01\rangle + 12\sqrt{5}i|03\rangle - 18|22\rangle\right)\right]$$

Therefore the associated 2 by (3×4) matrix A is

$$A = \begin{bmatrix} 0 & \frac{5}{4}\sqrt{\frac{35}{241}}i & 0 & \sqrt{\frac{35}{241}} & 0 \cdots 0 & \frac{3}{2}\sqrt{\frac{7}{241}}i & 0 \\ 0 & \frac{-15}{4}\sqrt{\frac{5}{241}} & 0 & 3\sqrt{\frac{5}{241}}i & 0 \cdots 0 & \frac{-9}{2\sqrt{241}} & 0 \end{bmatrix},$$

where columns six to nine consist of zeros.
By applying the algorithm, $|\psi\rangle$ factors thus:

$$|\psi\rangle = \left(|0\rangle + \frac{3i}{\sqrt{7}}|1\rangle\right) \otimes \left(\frac{5\sqrt{35}}{4\sqrt{241}}i|01\rangle + \frac{4\sqrt{35}}{\sqrt{241}}|03\rangle + \frac{6\sqrt{7}}{4\sqrt{241}}i|22\rangle\right).$$

To check the second factor, say $|\psi_2\rangle$, for a linear factor, we rewrite it thus:

$$|\psi_2\rangle = \frac{1}{4\sqrt{241}}\left[|0\rangle \otimes \left(0|0\rangle + 5\sqrt{35}i|1\rangle + 0|2\rangle + 4\sqrt{35}|3\rangle\right)\right.$$
$$+ |1\rangle \otimes \left(0|0\rangle + 0|1\rangle + 0|2\rangle + 0|3\rangle\right)$$
$$\left.+ |2\rangle \otimes \left(0|0\rangle + 0|1\rangle + 6\sqrt{7}i|2\rangle + 0|3\rangle\right)\right]$$

Therefore the associated 3 by 4 matrix A is

$$A = \frac{1}{4\sqrt{241}}\begin{bmatrix} 0 & 5\sqrt{35}i & 0 & 4\sqrt{35} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 6\sqrt{7}i & 0 \end{bmatrix},$$

Here rank$(A) = 2 > 1$, so that $|\psi_2\rangle$ is entangled. Thus the state $|\psi\rangle$ is entangled and it has one linear factor and one bipartite entangled factor.

Conclusion:

We believe that our algorithm is a very useful tool to understand the structure of a multipartite pure quantum state vis-a-vis entanglement.

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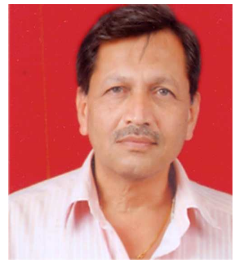

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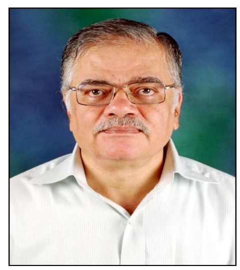

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