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Instability of Collision Electron-Ion Warm Nonmagnetized Plasma

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Abstract: The instability of collisional electron and ion in warm plasma is investigated. However, the dispersion relation of collisional electron at equilibrium ion and the dispersion relation of collisional ions at equilibrium electron are studied. It is found that the collision term is more effective in the instability. Also, the separation of variables method is used to find the transverse electrostatic wave potential in cylindrical coordinates, of electrons and ions plasma. It is found that this wave potential is independent of the collision parameter. On the other hand, the longitudinal electrostatic wave potential is investigated and found to be strongly depending on the collision parameter.

Keywords: electron-ion plasmas, collisional plasmas, warm plasma, unmagnetized plasma, plasma in cylindrical coordinates

1 Introduction

The number of theoretical publications devoted to various collective processes in electron–ion plasmas has increased enormously in recent years. This interest is primarily due to the fact that such plasma is typical rather than exceptional in astrophysical conditions. For example, it is assumed that such plasmas exist in the inner regions of accretion disks near black holes [1], in magnetospheres of neutron stars [2,3], in active galactic nuclei [4], and even in solar flares [5]. The theory of collective processes in quantum plasma is one of the most actively developing fields in plasma physics [6,7,8,9] Ion-acoustic waves (IAWs) in plasma have been attracting attention of physicists for decades. The interest in IAWs remains alive because not all problems have been solved in this field [10]. Ion-acoustic waves (IAWs) in quantum electron-ion plasma with degenerate components are theoretically investigated using a system of quantum equations of gas dynamics by Dubinov and Kitayev, 2015 [11]. Models of ionization equilibrium of thermal plasmas with multicharged ions are exactly solved [12]. Collisionless unmagnetized *e–p–i* plasma consisting of quantum degenerate gases of ions, electrons, and positrons at nonzero temperatures was considered. The dispersion equation for isothermal ionic sound waves was derived and analyzed [13]. The collisionless unmagnetized degenerate plasma with zero temperature components was considered and the exact barometric formulas for the electron and ion degenerate gases and the exact expressions for the electron and ion Debye radii were derived [14].

Binary Coulomb collisions between charged particles, characterized by a cubic dependence of the collision rates on the relative particle velocity, are one of the distinguishing features of plasma physics. It plays a crucial role in a variety of transport, relaxation and dissipative phenomena in magnetically confined plasmas [15,16]. The effects of electron-electron and electron-ion Coulomb collisions on the electron distribution function are studied by Hagelaar, 2015 [17]. A difficult problem that often arises when trying to derive theoretical predictions for the behavior of waves in plasma is how to include effects of particle collisions in the theory [19,18]. When collision effects are considered, three regimes of sheath behavior are evident. There is a collisionally dominated (i.e., mobility limited) regime, a collisionless regime, and a transition regime that separates them [20].In this paper we present the study of the warm plasma of electron and ion with take into consideration the collision between the particles of electron-ion, and the effect of this collision on the potential function.

The layout of this paper is as follows; In Sec. 2, we derive the basic electron ion plasma differential equations

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describing the model. In Sec.3 the separation of variable method is used to find the complete solution of these equations. In Sec.4 is assigned for numerical results and conclusions.

2 Basic Equations

We consider a warm unmagnetized collisional plasma system consists of ions and electrons. This system is governed by the continuity equation, the momentum equation, and Poisson‘s equation respectively [21,22,23, 24]

$$\frac{\partial n_{e,i}}{\partial t}+\nabla.(n_{e,i}V_{e,i})=0, \quad (1)$$

$$m_{e,i}n_{e,i}(\frac{\partial V_{e,i}}{\partial t}+(V_{e,i}.\nabla)V_{e,i}) = \pm e n_{e,i}E-\nabla P_{e,i}-m_{e,i}n_{e,i}\Gamma_{e,i}V_{e,i}, \quad (2)$$

$$\nabla.E=\frac{e}{\varepsilon_0}(n_i-n_e), \quad (3)$$

Where$E$ is the electric field.$n_{e,i}$, $V_{e,i}$, $m_{e,i}$, and $\Gamma_{e,i}$, are the density, velocity, mass and collision term of electrons and ions respectively. The positive (negative) sign in Eq.(2) is assigned for ions (electrons).The pressure term is $P_{e,i}=\gamma n_{e,i}K_BT_{e,i}$, where $\gamma=\frac{2+N}{N}$, is the ratio of specific heats of substrate. The degree of freedom, $N=1$, and hence, $\gamma=3$ so $P_{e,i=}3n_{ei}K_BT_{e,i}$, while K_B, $T_{e,i}$ are Boltzmann constant and electron and ion temperatures respectively.

2.1 Collision of Electron in warm plasma at Equilibrium Ion

At first, assuming a small perturbation around the equilibrium number density of electrons and a constant number density of ions, the plasma parameters appearing in Eqs.(1)–(3) can be expanded as

$$\left\{\begin{array}{cc} n_e=n_0+n_{e1}, & n_{e1}\ll n_0 \\ V_e=V_0+V_{e1}, & V_{e1}\ll V_0 \\ E=E_0+E_1, & E_1\ll E_0 \\ P_e=P_0+P_{e1}, & P_{e1}\ll P_0 \end{array}\right\}, \quad (4)$$

At plasma equilibrium state we have

$$E=E_0,\ n_i=n_e=n_0,\ \rho=e(n_i-n_e)=0, \quad (5)$$

Applying Eqs.(4) and (5) in Eqs. (1),(2) and (3):

$$\frac{\partial(n_0+n_{e1})}{\partial t}+\nabla.(n_0+n_{e1})(V_0+V_{e1})=0, \quad (6)$$

$$m_e(n_0+n_{e1})(\frac{\partial(V_0+V_{e1})}{\partial t}+((V_0+V_{e1}).\nabla)(V_0+V_{e1}) = -e(n_0+n_{e1})(E_0+E_1)-\nabla(P_0+P_{e1})-m_e(n_0+n_{e1})\Gamma_e(V_0+V_{e1})=0, \quad (7)$$

$$\nabla.(E_0+E_{e1})=\frac{e}{\varepsilon_0}(n_0-(n_0+n_{e1})), \quad (8)$$

In this perturbation approximation, we assume that $n_i=n_0$ is a constant andV_0=$E_0=0$.In addition we set$P_0=3n_0K_BT_e$, $P_{e1=}3n_{e1}K_BT_e$and $E_1=\nabla\emptyset_{e1,i1}$, where$\emptyset_{e1,i1}$ is the perturbed part of the electric potential affected on the electrons and ions. For first order perturbation, Eqs.(6),(7) and (8), with some mathematical manipulation, give the 4^{th} order differential equation;

$$(\nabla^2+K_e^2)\nabla^2\emptyset_{e1}=0, \quad (9)$$

Where the electron wave vector, K_e, satisfy the dispersion relation:

$$K_e^2=\frac{1}{3\lambda_{De}^2}[\frac{\omega(\omega+i\Gamma_e)}{\omega_{pe}^2}-1]. \quad (10)$$

Hence, the electron plasma frequency $\omega_{pe}^2=\frac{n_0e^2}{m_e\varepsilon_0}$, and the electron Debye length $\lambda_{De}^2=\frac{\varepsilon_0K_BT_e}{n_0e^2}$.

2.2 Collision of Ion in Warm Plasma at Equilibrium Electron

In this case, we assume a small perturbation around the equilibrium number density of ions and a constant number density of electrons. The plasma parameters appearing in Eqs.(1)–(3) can be expanded, in a similar way, as

$$\left\{\begin{array}{cc} n_i=n_0+n_{i1}, & n_{i1}\ll n_0 \\ V_i=V_0+V_{i1}, & V_{i1}\ll V_0 \\ E=E_0+E_1, & E_1\ll E_0 \\ P_i=P_0+P_{i1}, & P_{i1}\ll P_0 \end{array}\right\}, \quad (11)$$

Substitute from Eqs.(5) and (11) in Eqs.(1), (2) and (3):

$$\frac{\partial(n_0+n_{i1})}{\partial t}+\nabla.(n_0+n_{i1})(V_0+V_{i1})=0, \quad (12)$$

$$m_e(n_0+n_{i1})(\frac{\partial(V_0+V_{i1})}{\partial t}+((V_0+V_{i1}).\nabla)(V_0+V_{i1}) = e(n_0+n_{i1})(E_0+E_1)-\nabla(P_0+P_{i1})-m_i(n_0+n_{i1})\Gamma_i(V_0+V_{i1})=0, \quad (13)$$

$$\nabla.(E_0+E_1)=\frac{e}{\varepsilon_0}(n_0-(n_0+n_{i1})), \quad (14)$$

Assuming $n_e=n_0$, for first order perturbation, we can obtain a similar 4^{th} order differential equation for ions as

$$(\nabla^2+K_i^2)\nabla^2\emptyset_{i1}=0, \quad (15)$$

Where the ion wave vector, K_i, satisfy the following dispersion relation:

$$K_i^2=\frac{1}{3\lambda_{Di}^2}[\frac{\omega(\omega+i\Gamma_i)}{\omega_{pi}^2}-1]. \quad (16)$$

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Hence, the ion plasma frequency $\omega_{pi}^2 = \frac{n_0 e^2}{m_i \varepsilon_0}$, and the electron Debye length $\lambda_{Di}^2 = \frac{\varepsilon_0 K_B T_i}{n_0 e^2}$.
Eqs. (10) and (16) are the dispersion relations for electron and ion acoustic waves which can be rewritten as a quadratic equation in ω as

$$\omega^2 + i\Gamma_{e,i}\omega - \omega_{pe,i}^2(1 + 3\lambda_{De,i}^2 K_{e,i}^2) = 0 \qquad (17)$$

The roots of equation (17) in this case are

$$\omega = -\frac{i\,\Gamma_{e,i}}{2} \pm \sqrt{\frac{-\Gamma_{e,i}^2}{4} + \omega_{pe,i}^2(1 + 3\lambda_{De,i}^2 K_{e,i}^2)} \qquad (18)$$

It is clear that, the square root in Eq. (18) may be real or imaginary. If it is real we assume that $\omega = \omega_{r+} i\omega_i$ to obtain

$$\omega_r = \pm\sqrt{\frac{\Gamma_{e,i}^2}{4} - \omega_{pe,i}^2(1 + 3\lambda_{De,i}^2 K_{e,i}^2)} \qquad (19)$$

and

$$\omega_i = -\frac{i\,\Gamma_{e,i}}{2} \qquad (20)$$

If ω is imaginary we get

$$\omega_r = 0 \text{ And } \omega_i = -\frac{\Gamma_{e,i}}{2} \pm \sqrt{\frac{\Gamma_{e,i}^2}{4} - \omega_{pe,i}^2(1 + 3\lambda_{De,i}^2 K_{e,i}^2)}$$

From Eq.(20), it is clear that the damping rate increases linearly with the collision rate, $\Gamma_{e,i}$,and it does not depend on the wave number, $K_{e,i}$. However, from Eq.(19), the damping rate depends on $\Gamma_{e,i}$, and the wave number, $K_{e,i}$. In addition, the instability occurs when $\Gamma_{e,i}^2 > 4\omega_{pe,i}^2(1 + 3\lambda_{De,i}^2 K_{e,i}^2)$.

3 The acoustic wave solution of Eqs. (9) and (15)

We will seek the solution for electron and ion wave propagating in three variables cylindrical geometry. The equations describing this acoustic wave are Eqs. (9) and (15). These equations can be separated to two 2^{nd} order differential equations as;

$$\nabla^2 \emptyset_{e,\,i1} = 0, \qquad (21)$$

And

$$(\nabla^2 + K_{e,\,i}^2)\emptyset_{e,\,i1} = 0. \qquad (22)$$

Equation (21) describes a finite wavelength transverse wave of electrons and ions plasma. It is clear that this wave is independent of the collision parameter, $\Gamma_{e,i}$. On the other hand, equation (22) describes the longitudinal electron and ion plasma waves which strongly depend on the collision parameter. In three dimensional cylindrical coordinates Equation (21) takes the form

$$\nabla^2 \emptyset_{e,\,i1} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \emptyset_{e,\,i1}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \emptyset_{e,\,i1}}{\partial \theta^2} + \frac{\partial^2 \emptyset_{e,\,i1}}{\partial z^2} = 0. \qquad (23)$$

Let the function; $\emptyset_{e,\,i1}(r,\ \vartheta,\ z)$is separated in the form; $\emptyset_{e,\,i1}(r,\ \vartheta,\ z) = R(r)\vartheta(\theta)Z(z)$, then equation (23) can be written as

$$\left(\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr}\right)\frac{1}{R} + \frac{1}{\vartheta r^2}\frac{\partial^2\vartheta}{\partial\theta^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = 0. \qquad (24)$$

To separate the dependence of Z, equation (24) can be rewritten as

$$\left(\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr}\right)\frac{1}{R} + \frac{1}{\vartheta r^2}\frac{\partial^2\vartheta}{\partial\theta^2} = -\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -a^2, \quad (25)$$

Where$a$ is a constant. Hence,

$$Z = Z_0\, e^{-az} \qquad (26)$$

Similarly, to separate the dependence of $R(r)$and $\vartheta(\theta)$ in equation (25) we can write

$$\left(\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr}\right)\frac{r^2}{R} + a^2r^2 = -\frac{1}{\vartheta}\frac{\partial^2\vartheta}{\partial\theta^2} = n^2, \qquad (27)$$

This gives;

$$\vartheta = \vartheta_0 e^{in\vartheta} \qquad (28)$$

And

$$\left(r^2\frac{d^2R}{dr^2} + r\frac{dR}{dr}\right) + (a^2r^2 - n^2)R = 0 \qquad (29)$$

Let $(S = ar)$ so by chain rule, equation (29) takes the form of Bessel equation as;

$$\left(S^2\frac{d^2R}{dS^2} + S\frac{dR}{dS}\right) + (S^2 - n^2)R = 0 \qquad (30)$$

Eq.(30) has the following solution

$$J_n(S) = \sum_{m=0}^{\infty}\frac{(-1)^n}{m!\,(m+n+1)\,2^{2m}}\left(\frac{S}{2}\right)^{2m+n}. \qquad (31)$$

where n is an integer and it is the order of Bessel function, $J_n(S)$.In view of Eqs.(26), (28) and (31), the complete solution of Eq.(24) is

$$\emptyset_{e,i1}(r,\ \vartheta,\ z) = C_{a,n}\, J_n(ar)\, e^{-az}e^{in\vartheta}, a \le 0, n = 0, \pm 1, \pm 2, \ldots, \qquad (32)$$

Similarly, Eq. (22) in three dimensional cylindrical coordinates takes the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \emptyset_{e,i1}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \emptyset_{e,i1}}{\partial\theta^2} + \frac{\partial^2 \emptyset_{e,i1}}{\partial z^2} + K_{e,\,i}^2\emptyset_{e,\,i1} = 0, \qquad (33)$$

By the same way of separation of previous solution, equation (33) solved to take the form

$$\emptyset_{e,i1}(r,\ \vartheta,\ z) = C_{a,n}\, J_n(ar)\, e^{ik_{e,i}z}e^{in\vartheta}, a \le 0, n = 0, \pm 1, \pm 2, \ldots, \qquad (34)$$

Equation (32) describes a potential function as a solution of equation (21) for finite wavelength transverse wave of electrons and ions plasma. On the other hand, equation (34) describes the potential function of longitudinal electron and ion plasma waves which strongly depend on the collision parameter.

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4 Numerical Results

For numerical calculations purpose we can rescale the wave frequency, ω, and the wave vector, $K_{e,i}$, by the plasma frequency,$\omega_{pe,i}$, and the reciprocal of Debye length,$\frac{1}{\lambda_{De,i}}$,respectively. In this case Eq.(20) becomes

$$\overline{\omega}_i = -\frac{\overline{\Gamma}_{e,i}}{2} \pm \sqrt{\frac{\overline{\Gamma}_{e,i}^2}{4} - (1+3\overline{K}_{e,i}^2)}, \qquad (35)$$

Where$\overline{\Gamma}_{e,i} = \frac{\Gamma_{e,i}}{\omega_{pe,i}}$.

From Eq. (35), we notice that the damping rate depends on the collision parameter $\overline{\Gamma}_{e,i}$ and the wave number, $K_{e,i}$. In Fig. 1, we plotted this equation for electrons, where the collision for electron plasma is small compared with ion plasma and it is clear that the damping rate increases with $\overline{\Gamma}_e$ and it decreases with K_e.Fig. 2.shows the same behavior for the damping rate for ions where the collision is high.

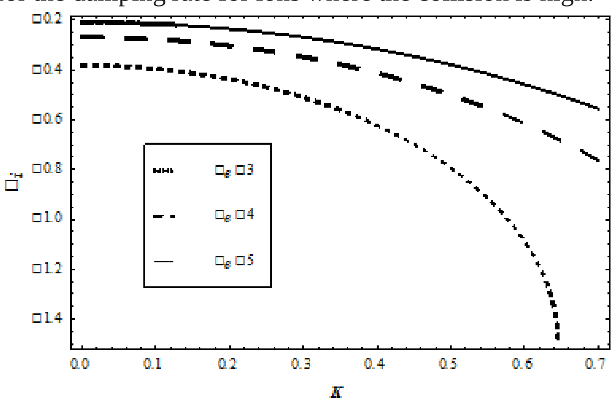

Fig. 1. Graph of the damping rate, for electron plasma $\overline{\omega}_i$ vs the wave number,$K_e$for$\overline{\Gamma}_e = 5$, solid line$\overline{\Gamma}_e = 4$dashed line, and $\overline{\Gamma}_e = 3$dotted line.

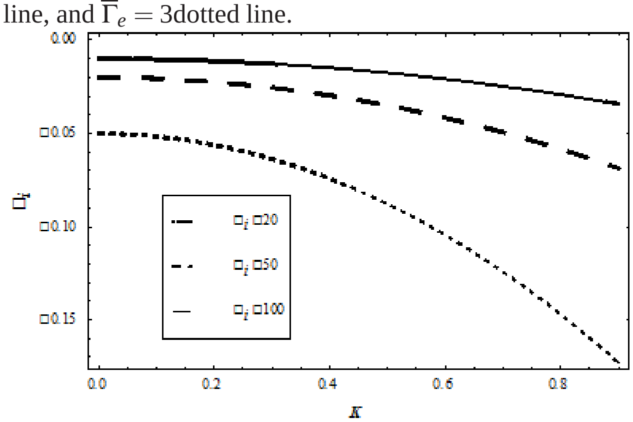

Fig. 2. Graph of the damping rate for ion plasma$\overline{\omega}_i$ vs the wave numberK_i, for$\overline{\Gamma}_i = 100$, solid line$\overline{\Gamma}_i = 50$,dashed line, and $\overline{\Gamma}_i = 20$,dotted line.

5 Conclusions:

In this paper, the dispersion relation of collisional electron at equilibrium ion and the dispersion relation of collisional ions at equilibrium electron are calculated. The frequency is plotted against the collision term. The instability due to collision term is studied. It is found that the collision term affects on the instability. The solutions for electron and ion wave propagating in three variables cylindrical geometry are calculated. These equations were separated into two 2^{nd} order differential equations; the first describes a finite wavelength transverse wave of electron and ion plasma, and the second describes the longitudinal electron and ion plasma waves. It's found that, the transverse wave of electrons and ions plasma are independent of the collision parameter $\Gamma_{e,i}$, while the longitudinal electron and ion plasma waves are strongly depend on the collision parameter. Graphs describe the variation of damping rate of electron plasma waves with the wave number,K_e and the damping rate of ion plasma waves with the wave number,K_i are plotted, at different values of electron collision frequency. It is found that the increase on the collision electron frequency or the increase on the collision ion frequency leads to increase of the damping rate which means that the instability is more affected by the collision frequency of electrons and ions.

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