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Two-Sided Cumulative Sum (cusum) Control Chart for Monitoring Shift in the Shape Parameter of the Pareto Distribution

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Abstract: In this study, we propose a Cumulative Sum control chart for monitoring shift in the shape parameter of the Pareto distribution when the scale parameter is known. The V-mask method of constructing Cumulative Sum control chart was employed together with the Sequential Probability Ratio Test. We investigated the features of the V-mask, and observed that the lead distance, the mask angle and the Average Run Length of the Cumulative Sum control chart changed considerably given a small shift in the shape parameter of the Pareto distribution.

Keywords: Cumulative Sum, parameter, V-mask, two-sided and sequential

1 Introduction

An organization will attract and maintain a large number of customers, if it is able to improve and control the quality of its products. This desired quality cannot be achieved without employing efficient and effective quality control measures. Statistical Process Control (SPC) charts are used widely by quality engineers for monitoring the stability of different process over time. A control chart helps to distinguish between normal process variation and unusual variation due to special causes. In order to monitor a process quality, practitioners frequently use the Shewhart's control charts like $\bar{x}-chart$, p-chart and R-charts among others. These charts are effective in detecting large shift (larger than 1.5 σ) in the process parameter. Industry players therefore cannot limit themselves to the use of the Shewhart control charts since it is not always a preferred choice when it comes to detecting small to moderate shift in the process parameter. Faced with this challenge, a more robust control chart with greater sensitivity is needed than what the Shewhart's charts can provide.

One such method is the use of Cumulative Sum (CUSUM) control chart. The CUSUM control charts have the ability to detect small shifts (that is less than 1.5 σ) by charting a statistic that incorporates current and previous data values from the process. It is also able to determine when a process shift has occurred. The CUSUM control chart is a natural fit for situations where process average is expected to shift from a specified target and there is the need to make timely process adjustments to bring the process back to the target. The sensitivity of a CUSUM control chart largely depends on the type of distribution used. This makes the Pareto distribution a suitable candidate in the construction of a CUSUM control chart because of its skewness and heavy tail. Any small change in the process affects the mean and variance of the distribution.

The CUSUM control chart was first proposed by [1] to address the shortfalls of the Shewhart control charts. [2] stated that CUSUM control charts are most sensitive SPC charts to signal a persistent small step change in a process parameter. A great deal of research has been done in the area of CUSUM control charts: [3] developed alpha-cut control chart for attributes data: This approach provided the ability of the CUSUM control chart in detecting the tightness of the inspection by selecting a suitable alpha level. [4] applied CUSUM control chart using sequential probability likelihood ratio test. [5], applied the CUSUM control chart in agriculture to check the quality of production and bulk tank somatic cell counts in milk. Also, [6] developed unified CUSUM control chart for monitoring simultaneous shifts in the parameters of the Elang-Truncated Exponential Distribution and therefore derived the parameters of the CUSUM chart proposed. Again, [7] applied the CUSUM and the Shewhart charts to monitor oestrus detection efficiency in dairy cows. In their analysis,

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the SPC charts detected changes in oestrus detection efficiency in time to be potentially useful in diary management. It was pointed out that the correct design of the used control chart is important to receive meaningful classification results. Recently, [8] developed a one-sided CUSUM control chart for monitoring the shape parameter of the Pareto distribution. And lately [11], developed a one sided CUSUM control chart for monitoring shift in the scale parameter δ of the new Weibull-Pareto distribution. In this study, we extend the study of [8] and therefore develop a two-sided CUSUM control chart for monitoring shifts in the shape parameter of the Pareto distribution.

2 Pareto Distribution

The Pareto distribution is named after an Italian civil engineer and economist Vilfredo Pareto. It is a skewed distribution with heavy or slowly decaying tail. It is a powerful probability distribution that is applied in economics, sociology, and statistics. It was used by Pareto in modelling national income and wealth distribution in any human endeavour. The Pareto distribution is used in modelling many situations which are not evenly distributed. If the random variable has a Pareto distribution, then the density function is given by;

$$f_x(x;\gamma,c) = \frac{\gamma c^{\gamma}}{x^{\gamma+1}} \tag{1}$$

where $x \geqslant c$ and the parameters $\gamma > 0$ and $c > 0$ are the shape and scale parameters respectively. The corresponding cumulative distribution function is given by;

$$F_x(x;\gamma,c) = 1 - \left(\frac{c}{x}\right)^{\gamma} \tag{2}$$

The mean and variance of the Pareto distribution are given by;

$$\mu = \frac{\gamma c}{\gamma - 1}, \gamma > 1 \tag{3}$$

and

$$\sigma^2 = \frac{\gamma c^2}{(\gamma-2)(\gamma-1)^2}, \gamma > 2 \tag{4}$$

3 Sequential Probability Ratio Test

The Sequential Probability Ratio Test (SPRT) plays a major role in the development of an acceptance sampling plan. [9] SPRT is a joint, subject-by-subject, Likelihood Ratio Test (LRT), where each subject constitutes a stage. According to[10], the CUSUM control charts are roughly equivalent to the SPRT. The SPRT has been used extensively in the development of an acceptance sampling plan and this acceptance sampling plan has been used in determining the in and out of control limits in CUSUM procedures. Suppose that we take a sample of m values $x_1, x_2, \ldots, x_m$ successively, from a population $f(x,\theta)$. Consider two hypotheses about θ, $H_0 : \theta = \theta_0$ and $H_1 : \theta = \theta_1$. The ratio of the probabilities of the sample is;

$$L_m = \frac{\Pi_{i=1}^{m} f(x_i,\theta_1)}{\Pi_{i=1}^{m} f(x_i,\theta_0)} \tag{5}$$

We select two numbers A and B , which are related to the desired α and β errors. The sequential test is set up as follows;

1.As long as $B < L_m < A$ we continue sampling
2.At the first i that $L_m \geqslant A$ we accept H_1
3.At the first i that $L_m \leqslant B$ we accept H_0

An equivalent way of computation is to use the logarithm of L_m. Then, the inequality becomes:

$$\ln B < \Sigma_{i=1}^{m} \ln f(x_i,\theta_1) - \Sigma_{i=1}^{m} \ln f(x_i,\theta_0) < lnA \tag{6}$$

The family of test is referred to as SPRT. if:

$$Z_i = \ln\left\{\frac{(x_i,\theta_1)}{(x_i,\theta_0)}\right\} \tag{7}$$

then the sampling terminates when

$$\sum z_i \geqslant \ln A \tag{8}$$

or

$$\sum z_i \leqslant \ln B \tag{9}$$

The z_i's are independent random variables with variance, say $\sigma^2 > 0$: Obviously, $\sum_{i=1}^{m} Z_i$ has a variance $m\sigma^2$. As m increases the dispersion becomes greater and the probability that a value of $\sum Z_i$ will remain within the limits $\ln B$ and $\ln A$ tends to zero. The mean, $\overline{Z}$, tends to a normal distribution with variance $\frac{\sigma^2}{m}$, and therefore the probability that it will fall between $\frac{\ln B}{m}$ and $\frac{\ln m}{m}$ tends to zero.

If we take a sample for which L_m lies between A and B for the first $n-1$ trials, then $L_m \geqslant A$ at the n^{th} trial, so that we accept H_1 (and reject H_0). Then by definition, the probability of getting such a sample is at least A times as large under H_1 as under H_0 : The probability of accepting H_1 when H_0 is true is, α, and that of accepting H_1 when H_1 is true is $1-\beta$. This is expressed by:

$$A \leqslant \frac{1-\beta}{\alpha} \tag{10}$$

similarly, if we accept H_0

$$B \geqslant \frac{\beta}{1-\alpha} \tag{11}$$

Wald(1947) showed that for all practical purposes10 and 11 hold as equalities. Thus

$$A = \frac{1-\beta}{\alpha} \tag{12}$$

and

$$B = \frac{\beta}{1-\alpha} \tag{13}$$

In practice α and β are small and the use of A and B can increase one of the errors by a very small amount.

4 CUSUM control chart for monitoring the shape parameter

The SPRT method is used to construct a two-sided CUSUM control chart for monitoring the shape parameter of the Pareto distribution, when the scale parameter is known. The Likelihood Ratio Test for testing the null hypothesis of no shift in the shape parameter as against the alternative hypothesis of a shift in the shape parameter is given by:

$$\frac{L_1 m}{L_0 m} = \frac{\Pi_{i=1}^{m} f(x_i, \gamma_1, c)}{\Pi_{i=1}^{m} f(x_i, \gamma_0, c)} \tag{14}$$

where $L_1 m$ is the likelihood of the new value of the shape parameter, γ and $L_0 m$ is the likelihood of the original value of the shape parameter,γ . The continuation region of the SPRT discriminating between the two hypotheses is given by:

$$\frac{\beta}{1-\alpha} < \frac{\Pi_{i=1}^{m} \frac{\gamma_1 c^{\gamma_1}}{x^{\gamma_1+1}}}{\Pi_{i=1}^{m} \frac{\gamma_0 c^{\gamma_0}}{x^{\gamma_0+1}}} < \frac{1-\beta}{\alpha} \tag{15}$$

where α and β are types I and II errors. taking logarithm of 15, the continuation region becomes

$$\ln\left(\frac{\beta}{1-\alpha}\right) < \ln\left(\frac{\Pi_{i=1}^{m} \frac{\gamma_1 c^{\gamma_1}}{x^{\gamma_1+1}}}{\Pi_{i=1}^{m} \frac{\gamma_0 c^{\gamma_0}}{x^{\gamma_0+1}}}\right) < \ln\left(\frac{1-\beta}{\alpha}\right) \tag{16}$$

simplifying 16 we obtain:

$$\ln\left(\frac{\beta}{1-\alpha}\right) < m\left[\ln\left(\frac{\gamma_1}{\gamma_0}\right) + (\gamma_1-\gamma_0)\ln c\right] + (\gamma_0-\gamma_1)\sum_{i=1}^{m} \ln x < \ln\left(\frac{1-\beta}{\alpha}\right) \tag{17}$$

$$\ln\left(\frac{\beta}{1-\alpha}\right) < m\left[\ln\left(\frac{\gamma_1}{\gamma_0}\right)+(\gamma_1-\gamma_0)\ln c\right]+(\gamma_0-\gamma_1)\sum_{i=1}^{m}\ln x < \ln\left(\frac{1-\beta}{\alpha}\right) \tag{18}$$

A shift in the shape parameter affects the mean and variance of the Pareto distribution. Assuming γ_0 is the target value and γ_1 ($\gamma_1 > \gamma_0$) is the changed values due to a shift in the value of the parameter, the SPRT will be terminated by rejecting or accepting the null hypothesis or by continuing the sampling. The sampling process is stopped by rejecting the null hypothesis if $\ln\frac{L_1 m}{L_0 m} \geqslant \ln A$; which gives a rejection line ($\gamma_1 > \gamma_0$) . On the other hand, if $\ln\frac{L_1 m}{L_0 m} \leqslant \ln A$ is obtained for the case of ($\gamma_1 < \gamma_0$) , then another rejection line is obtained. These two rejection lines give a symmetrical masking: The observations in the sample are used with the mask in a sequential pattern. Consideration (18) and setting $\beta = 0$ the inequality in (18) reduces to:

$$m\left[\ln\left(\frac{\gamma_1}{\gamma_0}\right)+(\gamma_1-\gamma_0)\ln c\right]+(\gamma_0+\gamma_1)\sum_{i=1}^{m}\ln x < -\ln\alpha \tag{19}$$

Re-arranging (19) and keeping the term containing x at the left-hand side of the inequality, we obtain:

$$\sum_{i=1}^{m}\ln x \leqslant \frac{-m\left[\ln\left(\frac{\gamma_1}{\gamma_0}\right)+(\gamma_1-\gamma_0)\ln c\right]+(-\ln\alpha)}{(\gamma_0-\gamma_1)} \tag{20}$$

which implies

$$\sum_{i=1}^{m}\ln x \leqslant \frac{-m\left[\ln\left(\frac{\gamma_1}{\gamma_0}\right)+(\gamma_1-\gamma_0)\ln c\right]+(\ln\alpha)}{(\gamma_1-\gamma_0)} \tag{21}$$

now equation 21 can be wriiten in the form of a straight line as:

$$\sum_{i=1}^{m}\ln x_i \leqslant m f^* + g^* \tag{22}$$

where

$$f^* = \frac{\ln\left(\frac{\gamma_1}{\gamma_0}\right)+(\gamma_1-\gamma_0)\ln c}{\gamma_1-\gamma_0} \tag{23}$$

and

$$g^* = \frac{-\ln\alpha}{\gamma_1-\gamma_0} \tag{24}$$

Let $X_1, X_2, \ldots, X_m$ be a sample from a Pareto distribution. If the points $(m, \sum_{i=1}^{m}\ln x_i)$are plotted with a suitable scale, then the ordinates of the points represent the cumulative sum of the data. Equations 22 and 24 are the effects of a shift in the population parameter,γ. Figure 1 shows a sizeable shift in the parameters if $\sum_{i=1}^{m}\ln x_i$ falls outside the lines G_1H_1 *and* $G_{-1}H_{-1}$. The chart is interpreted by placing the mask over the last plotted point as shown in figure 1 If any of the points lies below $G_{-1}H_{-1}$, then it indicates a decrease in γ and if any of the points falls above, G_1H_1 then it shows an increase in γ . If all the points plotted are within the two lines G_1H_1 and $G_{-1}H_{-1}$ of the V-mask, then the process is in control: If there is a point below G_1H_1 then it is concluded that the process is out of control and has a positive shift of the process mean and if there is a point found above $G_{-1}H_{-1}$ then the process is out of control and has a negative shift of the process mean.

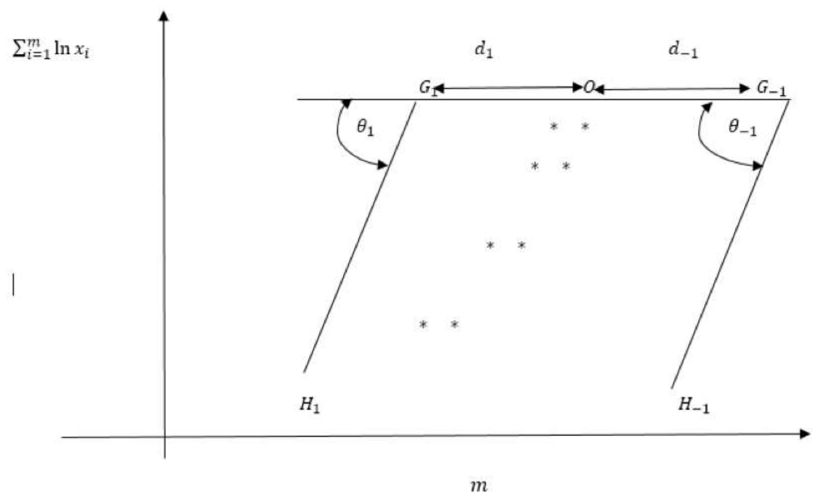

Fig. 1: : v- mask for two-sided cusum

The parameters of the CUSUM chart, known as the angle of the mask and the lead distance, are obtained. From Figure 1, $\tan\theta_1$= slope of the line $G_1H_1 = f$ and $\tan\theta_{-1}$= slope of the line $G_{-1}H_{-1} = f^*$, hence:

$$\theta_1 = \tan^{-1}\left[\frac{\ln\left(\frac{\gamma_1}{\gamma_0}\right) + (\gamma_1 - \gamma_0)\ln c}{(\gamma_1 - \gamma_0)}\right] \tag{25}$$

where $\gamma_1 > \gamma_0$ and

$$\theta_{-1} = \tan^{-1}\left[\frac{\ln\left(\frac{\gamma_1}{\gamma_0}\right) + (\gamma_1 - \gamma_0)\ln c}{(\gamma_1 - \gamma_0)}\right] \tag{26}$$

where where $\gamma_1 < \gamma_0$

Various values of the mask angle, $theta$, are shown in 1 for various values of the scale parameter, c , and shifts in the shape parameter: The results show that the values of the angle, $theta$, decreases with an increase in the value of $(\gamma_1 - \gamma_0)$ for any given value of c . Also, as the value of $(\gamma_1 - \gamma_0)$ increases the value of $theta$ increases with increasing value of c . Again increasing values of $\frac{\gamma_1}{\gamma_0}$, decreases the value of $theta$

Table 1: Values of the angles θ for controlling the parameters

		c		
γ_0	γ_1	1.5	2	2.5
1.5	3	40.94	49.12	54.04
1.5	3.5	39.66	48.16	53.27
1.5	4	38.58	47.35	52.61
1.5	4.5	37.66	46.65	52.06
1.5	5	36.85	46.05	51.57
1.5	5.5	36.14	45.51	51.14
1.5	6	35.51	45.04	50.76
1.5	6.5	34.94	44.61	50.42
1.5	7	34.43	44.22	50.11
1.5	7.5	33.97	43.87	49.83

Table 2: Value of the lead distance for the parameters of the Pareto distribution

		$c = 1.5$						
		α						
γ_0	γ_1	0.5	0.05	0.025	0.005	0.1	0.01	0.001
1.5	3	0.53	2.30	2.83	4.07	1.85	3.69	5.54
1.5	3.5	0.42	1.80	2.23	3.20	0.91	1.82	2.73
1.5	4	0.35	1.50	1.85	2.66	0.60	1.20	1.79
1.5	4.5	0.30	1.29	1.59	2.29	0.44	0.88	1.33
1.5	5	0.26	1.14	1.41	2.02	0.35	0.70	1.05
1.5	5.5	0.24	1.03	1.26	1.81	0.29	0.57	0.86
1.5	6	0.22	0.93	1.15	1.65	0.24	0.48	0.73
1.5	6.5	0.20	0.86	1.06	1.52	0.21	0.42	0.63
1.5	7	0.18	0.80	0.97	1.41	0.18	0.37	0.55
1.5	7.5	0.17	0.74	0.91	1.31	0.16	0.33	0.49
		$c = 2$						
γ_0	γ_1	0.5	0.05	0.025	0.005	0.1	0.01	0.001
1.5	3	0.40	1.73	2.13	3.06	1.33	2.66	3.99
1.5	3.5	0.31	1.34	1.65	2.37	1.03	2.06	3.09
1.5	4	0.26	1.10	1.36	1.95	0.85	1.70	2.55
1.5	4.5	0.22	0.94	1.16	1.67	0.73	1.45	2.17
1.5	5	0.19	0.83	1.02	1.46	0.63	1.27	1.90
1.5	5.5	0.17	0.74	0.91	1.30	0.57	1.13	1.70
1.5	6	0.15	0.67	0.82	1.18	0.51	1.02	1.53
1.5	6.5	0.14	0.61	0.75	1.07	0.47	0.93	1.40
1.5	7	0.13	0.56	0.69	0.99	0.43	0.86	1.29
2.5	7.5	0.12	0.52	0.64	0.92	0.40	0.80	1.20
		$c = 2.5$						
γ_0	γ_1	0.5	0.05	0.025	0.005	0.1	0.01	0.001
2.5	3.0	0.34	1.45	1.78	2.56	1.11	2.23	3.34
2.5	3.5	0.26	1.12	1.38	1.98	0.86	1.72	2.58
2.5	4.0	0.21	0.92	1.13	1.62	0.70	1.41	2.11
2.5	4.5	0.18	0.78	0.96	1.38	0.60	1.19	1.80
2.5	5.0	0.16	0.68	0.84	1.20	0.52	1.04	1.57
2.5	5.5	0.14	0.60	0.74	1.08	0.46	0.93	1.39
2.5	6.0	0.13	0.54	0.67	0.96	0.42	0.84	1.25
2.5	6.5	0.12	0.50	0.61	0.88	0.38	0.76	1.14
2.5	7.0	0.11	0.46	0.56	0.81	0.35	0.70	1.05
2.5	7.5	0.10	0.42	0.52	0.75	0.32	0.65	0.97

5 Average Run Length of the two-sided CUSUM control chart

The Average Run Length for the two-sided CUSUM control chart is given by:

$$ARL = \frac{-\ln\alpha}{\ln\left(\frac{\gamma_1}{\gamma_0}\right) + \frac{\gamma_0-\gamma_1}{\gamma_1}} \tag{27}$$

Proof by using:

$$ARL = \frac{-\ln\alpha}{E[Z]_{\gamma=\gamma_1}} \tag{28}$$

where

$$Z = \frac{f(x,\gamma_1,c)}{f(x,\gamma_0,c)} \tag{29}$$

Then:

$$Z = \frac{\gamma_1 c^{\gamma_1}}{x^{\gamma_1+1}} \div \frac{\gamma_0 c^{\gamma_0}}{x^{\gamma_0+1}} \tag{30}$$

and

$$Z = \left(\frac{\gamma_1}{\gamma_0}\right) c^{(\gamma_0-\gamma_1)} x^{(\gamma_0-\gamma_1)} \tag{31}$$

Taking natural logarithm of 31:

$$\ln Z = \ln\left(\frac{\gamma_1}{\gamma_0}\right)(\gamma_1 - \gamma_0)\ln c + (\gamma_0 - \gamma_1)\ln x \tag{32}$$

hence:

$$E[\ln Z] = \ln\left(\frac{\gamma_1}{\gamma_0}\right) + (\gamma_1 - \gamma_0)\ln c + (\gamma_0 - \gamma_1)E[\ln x] \tag{33}$$

where

$$E[\ln x] = \int_c^{\infty} \ln x f(x,\gamma_1,c)dx \tag{34}$$

and

$$E[\ln x] = \int_c^{\infty} \frac{\gamma_1 c^{\gamma_1}}{x^{\gamma_1+1}} \ln x dx \tag{35}$$

Then:

$$E[\ln x] = \gamma_1 c^{\gamma_1}\left[0 - \left(\frac{-c}{\gamma_1^2} - \frac{c\ln c}{\gamma_1}\right)e^{-}(\gamma_1+1)\ln c\right] \tag{36}$$

and

$$E[\ln x] = \left[\frac{\gamma_1 + \gamma_1^2 \ln c}{\gamma_1^2}\right] \tag{37}$$

substituting 31 into 29 we obtain:

$$E[\ln Z] = \ln\left(\frac{\gamma_1}{\gamma_0}\right) + \frac{\gamma_0 - \gamma_1}{\gamma_1} \tag{38}$$

and substituting 36 into 26, the ARL is obtained as:

$$ARL = \frac{-\ln\alpha}{\ln\left(\frac{\gamma_1}{\gamma_0}\right) + \frac{\gamma_0-\gamma_1}{\gamma_1}} \tag{39}$$

Various values of the ARL are given in 3. From the results, for a given value of α , and an increase in the value of $(\gamma_1 - \gamma_0)$, the value of the ARL decreases. Also, an increase in the ratio, $\frac{\gamma_1}{\gamma_0}$, with a given α , decreases the value of the ARL and a decrease in the value of α and a fixed value of $\frac{\gamma_1}{\gamma_0}$, increases the ARL.

Table 3: Average Run Length (ARL) for controlling the parameter γ

		α						
γ_0	γ_1	0.5	0.05	0.005	0.025	0.1	0.01	0.001
1.5	3.0	3.59	15.51	27.43	2.83	1.85	3.69	5.54
1.5	3.5	2.51	10.85	19.21	2.23	0.91	1.82	2.73
1.5	4.0	1.95	8.42	14.89	1.85	0.60	1.20	1.79
1.5	4.5	1.61	6.94	12.27	1.59	0.44	0.88	1.33
1.5	5.0	1.38	5.94	10.51	1.41	0.35	0.70	1.05
1.5	5.5	1.21	5.23	9.26	1.26	0.29	0.57	0.86
1.5	6	1.09	4.71	8.33	1.15	0.24	0.48	0.73
1.5	6.5	0.99	4.30	7.60	1.06	0.21	0.42	0.63
1.5	7.0	0.92	3.97	7.02	0.97	0.18	0.37	0.55
1.5	7.5	0.86	3.70	6.55	0.91	0.16	0.33	0.49

6 Practical Demonstration of the use of the Two-sided CUSUM control chart

In this section, the application of the proposed CUSUM control chart is illustrated using hypothetical data simulated from the Pareto distribution. The first ten observations were simulated with γ_0 = 2.5 and c=1.5; the last five observations were simulated with γ_1=5 . Table 4 displays the simulated data and their cumulative sum. The parameters of the V-mask were calculated using γ_0 = 2.5 c=1.5; γ_1 = 5 and α=0.01. The lead distance and the angle of the mask were 2.7 and 53^0 respectively. The sample number(m) was plotted against the cumulative sum of the data. The V-mask was then placed at the last plotted point to monitor whether the process is in control or out of control as shown in figure 2. It can be

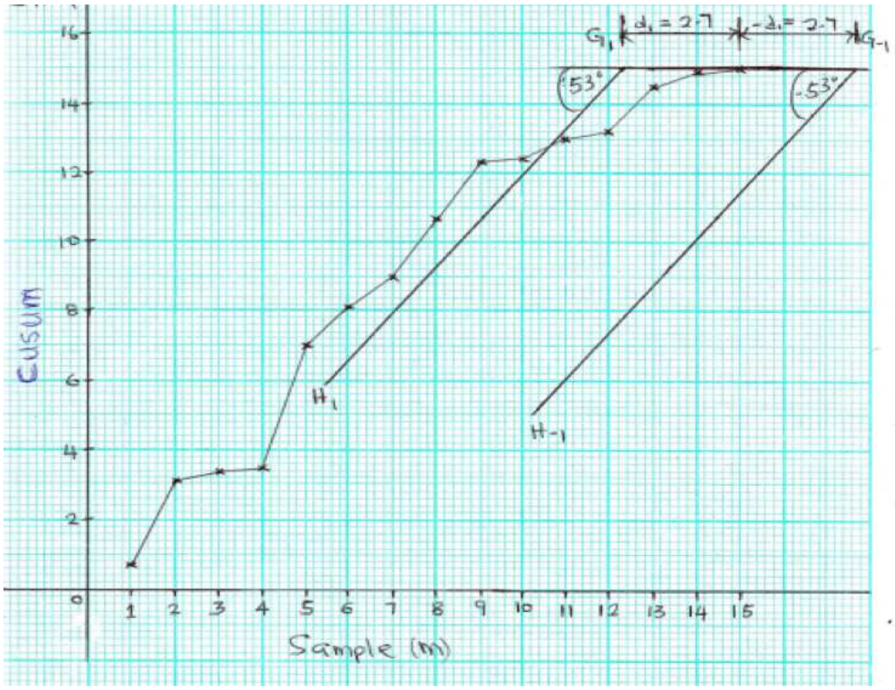

Fig. 2: two-sided cusum plot for the simulated hypothetical data

established from figure 2 that the process was out of control as observations 1 to 5 fell above line G_1H_1 indicating an increase in γ . This means an action must be taken in order to bring the process back to control.

Table 4: Simulated data for the two-sided CUSUM control chart

sample	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Data(x)	2.1	10.7	1.3	1.1	33.2	3.1	2.4	5.6	5.2	1.1	1.8	1.2	3.7	1.5	1.1
$\ln x$	0.7	2.4	0.3	0.1	3.5	1.1	0.9	1.7	1.6	0.1	0.6	0.2	1.3	0.4	0.1
CUSUM	0.7	3.1	3.4	3.5	7.0	8.1	9.0	10.7	12.3	12.4	13.0	13.2	14.5	14.9	15.0

7 Conclusion

Control charts play a key role as long as the quality of a product is concerned in the industrial establishment. Therefore proposing control charts for monitoring the quality of products is not trivial. This CUSUM will particularly be helpful in determining when the assignable cause has occurred and in situations in which two directions of a system are to be monitored. It also offers considerable performance improvement relative to the Shewhart charts. It can be applied in the chemical and process industries. The limitation of this CUSUM is that when the mask angle and the lead distance are not well determined, can lead to inappropriate system adjustments. Further studies can be done on this distribution by looking at simultaneous shift in both the shape and scale parameters of the distribution.

The results of the study showed that the size of the mask angle Θ decreases as the value of $(\gamma_1 - \gamma_0)$ increases with any given value of the scale parameter c . However, given any fixed value of $(\gamma_1 - \gamma_0)$, the angle Θ increases for an increase in the value of c. With regards to the lead distance, it can be observed that an increase in the value of $(\gamma_1 - \gamma_0)$ and given the values of α and c , the value of d decreases. Again the value of d increases with fixed value of c and α. However, d decreases as c increases in value, if α and $(\gamma_1 - \gamma_0)$ values are maintained. On the ARL, it can be established that as the value of increases, the ARL tends to decrease with any given value of . Also, as the value of decreases, the ARL also increases for fixed values of $(\gamma_1 - \gamma_0)$.

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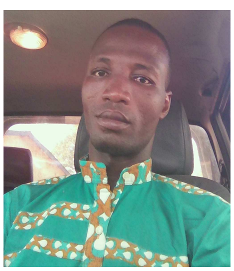

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