Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/110220

Some Contributions of Congruence Relations on Lattice of Fuzzy ℓ-ideals

P. Bharathi, J. Vimala, L. Vijayalakshmi and J. Arockia Reeta*

Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.

Received: 2 Dec. 2016, Revised: 26 Jan. 2017, Accepted: 28 Jan. 2017
Published online: 1 Mar. 2017

Abstract: The main objective of this paper is to introduce the congruence relations on the set of all fuzzy ℓ - ideals of ℓ-group. Let F be the set of all fuzzy ℓ-ideals defined on the lattice ordered group G. We introduce the congruence relations on F and derived some intresting results on the relation between F and its congruence relations. Also we established some important results on congruence relations by using the operations on fuzzy ℓ-ideals.

Keywords: lattice ordered group, fuzzy ℓ-ideal, congruence, fuzzy congruence

1 Introduction

To generalize the classical notion of set theory, [19] initiated the study of fuzzy set as a mapping from any non empty set into the unit interval [0,1]. Then many algebraists took interest to introduce fuzzy theory in various algabraic structures by fuzzyfying the formal theory. [2,3,16] developed the theory of fuzzy groups. In [1,12] fuzzy lattices were studied. Subsequently [11,18] introduced fuzzy ℓ-idelas and produced some interesting results. In [7,9,10,11] fuzzy algera was studied. [6] applied the theory of fuzzy ideals to robotics motion planning. In [8,21,22] the theory of (ε , $\varepsilon\vee$ q) fuzzy ideals is applied to medical diagonosis system. Now a days the study of congruence relations is important for its applications in the field of logic-based process to uncertainity. In fuzzy automata theory congruence relations are widely used. [12,13,14,15] introduced the concept of idelas in ℓ-groups and they discussed about the concept of congruence relations on the family of fuzzy ideals. Using the congruence relations they derived a characterization theorem for distributive ℓ-ideals.Fuzzy equivalence relations and fuzzy congruence relations are the main tools in the research area of fuzzy algebra. [17] initiated the notion of L-Fuzzy ℓ-ideals and gave some prominent results. He proved that the set of all L-Fuzzy ℓ-ideals of an ℓ-group form a complete lattice. Also he initiated the study of fuzzy congruence in ℓ-groups and derived some main results on the relation between fuzzy ℓ-ideals and fuzzy congruence. In this paper, we introduce the congruence relation on the set of all fuzzy ℓ-ideals of ℓ-group. In section 2 , we gave some preliminary definitions and results. In section 3, we discussed about the relation between the congruence and the set of all fuzzy ℓ-ideals. Also we obtained an important result on the relation between the congruence and fuzzy congruence on the family of fuzzy ℓ-ideals

2 Preliminaries

In this section we presented some preliminary definitions and results which will be used for subsequent discussions.

Definition 1.[5] A non-empty set G is called a ℓ-group iff
(i) (G,+) is a group.
(ii)(G, $\leq$) is a lattice.
(iii) x $\leq$ y implies a+x+b $\leq$ a+y+b for all x,y,a,b in G.

Definition 2.[5] A non-empty set G is called a ℓ-group iff
(i) (G,+) is a group.
(ii) (G,$\vee$, $\wedge$) is a lattice.
(iii) a +(x$\vee$y)=(a+x) $\vee$ (a+y) and a+(x$\wedge$y)=(a+x)$\wedge$(a+y)for all x,y,a,b in G.

Theorem 1.[5] The above two definitions of ℓ-group are equivalent.

* Corresponding author e-mail: vimaljey@alagappauniversity.ac.in

Definition 3.[19] Let X be any non empty set and let I=[0,1]. Then the map μ : X $\to$ I is called a fuzzy subset of X.

Definition 4.[20] Let μ be a fuzzy subset on a non empty set X and t $\in$ [0,1].Then the set μ_t = { x $\in$ X / μ (x) $\geq$ t} is called the level set of μ.

Definition 5.[20] Let μ be a fuzzy subset on a non empty set X. Then the set {μ(x)/x $\in$ X} is called the image of μ and is denoted by Im(μ).

Definition 6.[20] Let μ be a fuzzy subset on a non empty set X. The set { x / x$\in$ X , μ(x)> 0 } is called the support of μ and it is denoted by supp(μ).

Definition 7.[4] Let G= (G, +, $\wedge$, $\vee$) be a ℓ-group.A fuzzy set μ : G $\to$ [0,1] is called a fuzzy ℓ-ideal of G if
(i)μ(x - y) $\geq$ μ(x) $\wedge\mu$(y)
(ii)μ(x $\vee$ y) $\geq$ μ(x) $\wedge\mu$(y)
(iii)μ(x $\wedge$ y) $\geq$ μ(x) $\wedge\mu$(y)
(iv) 0<x<a $\Rightarrow$ μ(x)$\geq$ $\mu(a)$ for x,y,a,b $\in$ G.

Result[4][Characterization Theorem] Let G be a ℓ - group. A fuzzy set μ of G is a fuzzy ℓ-ideal of G if and only if the set μ_t={x $\in$G/μ(x)$\geq$t } is an ℓ-ideal of G for all t$\in$[0,1] with $\mu_t \neq \phi$. μ_t is known as level ℓ-ideal of G.

Definition 8.[4] The union of two fuzzy ℓ-ideals μ_1 and μ_2 of a ℓ-group G denoted by ($\mu_1 \cup \mu_2$) is a fuzzy subset of G defined by
($\mu_1 \cup \mu_2$) (x) = max{ μ_1(x) , μ_2(x) } for all x $\in$G.
The intersection of two fuzzy ℓ-ideals μ_1 and μ_2 of a commutative ℓ-group G denoted by ($\mu_1 \cap \mu_2$) is a fuzzy subset of G defined by
($\mu_1 \cap \mu_2$) (x) = min{ μ_1(x) , μ_2(x) } for all x $\in$G.

Definition 9.[4] Let μ_1 and μ_2 be any two fuzzy ℓ-ideals of a ℓ-group G. Then μ_1 is said to be contained in μ_2 denoted by $\mu_1 \subseteq \mu_2$ if μ_1(x) $\leq$ μ_2(x) for all x $\in$ G. If μ_1(x)=μ_2(x) for all x $\in$ G then μ_1 and μ_2 are said to be equal and we can write μ_1 = μ_2.

Result[4] Let μ_1 and μ_2 be any two fuzzy ℓ-ideals of a ℓ-group G. If $\mu_1 \subseteq \mu_2$ then $\mu_1 \cup \mu_2$=μ_2 and $\mu_1 \cap \mu_2$=μ_1.

Definition 10.[4] If μ_1 and μ_2 are any two fuzzy ℓ-ideals of the ℓ-group G then $\mu_1 \vee \mu_2$ is defined by($\mu_1 \vee \mu_2$)(x) = $\sup_{x=y\vee z}$ { min { μ_1(y) , μ_2(z) } } and $\mu_1 \wedge \mu_2$ is defined by
($\mu_1 \wedge \mu_2$)(x) = $\sup_{x=y\wedge z}$ { min { μ_1(y) , μ_2(z) } } where x,y z $\in$ G.

Result[4] Let μ_1 and μ_2 be any two fuzzy ℓ-ideals of a ℓ-group G.
Then (i)$\mu_1 \vee \mu_2$ = $\mu_2 \vee \mu_1$ and $\mu_1 \wedge \mu_2$= $\mu_2 \wedge \mu_1$.
(ii) ($\mu_1 \vee \mu_2$) $\vee\mu_3$ = $\mu_1\vee$ ($\mu_2 \vee \mu_3$) and ($\mu_1 \wedge \mu_2$) $\wedge\mu_3$ = $\mu_1\wedge$ ($\mu_2 \wedge \mu_3$)

Definition 11.[4] A Binary Relation θ on a ℓ-group G is called congruence relation if

1. θ is reflexive: x $\equiv$x(θ) for all x $\in$ G.
2. θ is symmetric : x $\equiv$ y(θ) $\Rightarrow$ y $\equiv$ x(θ) for all x,y $\in$ G.
3. θ is transitive: x $\equiv$ y(θ) and y $\equiv$ z(θ) $\Rightarrow$ x $\equiv$ z(θ)for all x,y,z $\in$ G.
4. θ satisfies substitution property:x $\equiv$ x_1(θ) and y $\equiv$ y_1 (θ) $\Rightarrow$ x $\wedge$ y $\equiv$ x_1 $\wedge y_1$ (θ) and x $\vee$ y $\equiv$ x_1 $\vee y_1$ (θ)

Definition 12.[17] Let G be the ℓ-group.A Fuzzy relation μ on G is a mapping from G X G to [0,1].

Definition 13.[17] Let G be the ℓ-group. The fuzzy relation μ on G is called the fuzzy equivalence relation on G if the following conditions are satisfied:
(i)μ(a,a) =1[Fuzzy Reflexive].
(ii)μ(a,b)=μ(b,a) [Fuzzy Symmetric].
(iii) ($\mu \circ \mu$)$\subseteq$ μ [Fuzzy Transitive].
Here ($\mu \circ \mu$)(x,y) = $\sup_{z\in G}$ [Min[μ(x,z), μ(z,y)]].

Definition 14.[17] Let G be the ℓ-group and μ be the fuzzy equivalence relation on G then μ is said to be the fuzzy congruence on G if

1. μ(a - x,b - y) $\geq$ μ(a,b) $\wedge\mu$(x,y).
2. μ(a $\wedge$ x , b $\wedge$ y)$\geq$ μ(a,b) $\wedge\mu$(x,y).
3. μ(a $\vee$ x , b $\vee$ y) $\geq$ μ(a,b) $\wedge\mu$(x,y) for all x,y,a,b $\in$ G..

3 Congruence on lattice of fuzzy ℓ-ideals

In this section we initiate the study of congruence relations on the family of fuzzy ℓ-ideals.First we derive the following proposition to introduce the congruence relation on the family of fuzzy ℓ-ideals of the ℓ-group.

Theorem 2. Let G be the ℓ-group and F be the set of all fuzzy ℓ-ideals on G.The binary relation θ_F defined on F such that $\mu_1 \equiv \mu_2$ (θ_F) if and only if $\theta \wedge \mu_1$ = $\theta \wedge \mu_2$ is a congruence relation for θ , μ_1, μ_2 $\in$ F and $\theta \subseteq \mu_1$, $\theta \subseteq \mu_2$.

Proof Let μ_1 , μ_2 $\in$ F.
Then the binary relation θ_F on F such that $\mu_1 \equiv \mu_2$ (θ_F) if and only if $\theta \wedge \mu_1$ = $\theta \wedge \mu_2$ is reflexive,symmetric and transitive.
Next to prove the substitution property,
Assume that $\mu_1 \equiv \mu_2(\theta_F)$ and $\mu_3 \equiv \mu_4(\theta_F)$
$\Rightarrow$ $\theta \wedge \mu_1$ = $\theta \wedge \mu_2$ and $\theta \wedge \mu_3$ = $\theta \wedge \mu_4$.
$\Rightarrow$ $\theta\wedge(\mu_1 \wedge \mu_3)$ = ($\theta \wedge \mu_1$) $\wedge\mu_3$.
$\Rightarrow$ = ($\theta \wedge \mu_2$) $\wedge\mu_3$.
$\Rightarrow$ = $\theta\wedge(\mu_2 \wedge \mu_3)$.
$\Rightarrow$ = $\theta\wedge(\mu_3 \wedge \mu_2)$.
$\Rightarrow$ = ($\theta \wedge \mu_3$)$\wedge\mu_2$.
$\Rightarrow$ = ($\theta \wedge \mu_4$)$\wedge\mu_2$.
$\Rightarrow$ = $\theta\wedge(\mu_4 \wedge \mu_2)$.
$\Rightarrow$ = $\theta\wedge(\mu_2 \wedge \mu_4)$.
$\Rightarrow$ $\mu_1 \wedge \mu_3 \equiv \mu_2 \wedge \mu_4(\theta_F)$
$\Rightarrow$ θ_F is a congruence relation. *Example 1.* Let G={0,a,b,1} where 0 < a< b < 1 and + is defined as follows:

+	0	a	b	1
0	0	a	b	1
a	a	0	1	1
b	b	1	0	1
1	1	1	1	0

Then (G,+,$\wedge$,$\vee$) is a ℓ-group.
Define μ_1 : G $\rightarrow$[0,1] by $\mu_1(0)$ = 0.7 and μ_1(a) =μ_1(b)=μ_1(1)= 0.5.
μ_2 : G $\rightarrow$[0,1] by $\mu_2(0)$ = 0.6 and μ_2(a) =μ_2(b)=μ_2(1)= 0.4.
μ_3 : G $\rightarrow$[0,1] by $\mu_3(0)$ = 0.5 and μ_3(a) =μ_3(b)=μ_3(1)= 0.3.
μ_4 : G $\rightarrow$[0,1] by $\mu_4(0)$ = 0.4 and μ_4(a) =μ_4(b)=μ_4(1)= 0.2.
θ : G $\rightarrow$[0,1] by $\theta(0)$ = 0.3 and θ(a) =θ(b)=θ(1)= 0.1.
Let F= { μ_1 , μ_2, μ_3, μ_4, θ.} Then the binary relation θ_F on F such that $\mu_1 \equiv \mu_2$ (θ_F) if and only if $\theta \wedge \mu_1 = \theta \wedge \mu_2$ is a congruence relation.

Throughout this section G be the ℓ-group and F be the set of all fuzzy ℓ-ideals defined on G.We derive the following propositions to establish some interesting results on congruence relation by using the operations on fuzzy ℓ-ideals.

Theorem 3. Let θ , μ_1, $\mu_2 \in$ F.If $\mu_1 \equiv \mu_2$ (θ_F) then F is distributive.

Proof Let μ_1, $\mu_2 \in$ F.
Define the congruence relation θ_F on F such that $\mu_1 \equiv \mu_2$ (θ_F) if and only if $\theta \wedge \mu_1 = \theta \wedge \mu_2$ for $\theta \subseteq \mu_1$ and $\theta \subseteq \mu_2$.
Now $\theta \wedge \mu_1$ =($\theta \wedge \theta$) $\wedge\mu_1$ = $\theta\wedge$ ($\theta \wedge \mu_1$) and $\theta \wedge \mu_2$ = ($\theta \wedge \theta$) $\wedge\mu_2$ = $\theta\wedge$ ($\theta \wedge \mu_2$)
$\Rightarrow \mu_1 \equiv \theta \wedge \mu_1(\theta_F)$ and $\mu_2 \equiv \theta \wedge \mu_2(\theta_F)$
$\Rightarrow \mu_1 \vee \mu_2 \equiv (\theta \wedge \mu_1) \vee (\theta \wedge \mu_2)$ (θ_F)
$\Rightarrow \theta\wedge$ ($\mu_1 \vee \mu_2$)= $\theta\wedge$ (($\theta \wedge \mu_1$) $\vee$ (($\theta \wedge \mu_2$)
$\Rightarrow \theta\wedge$ ($\mu_1 \vee \mu_2$)=(($\theta \wedge \mu_1$) $\vee$ ($\theta \wedge \mu_2$)
$\Rightarrow$ F is distribitive.

Theorem 4. Let μ_1 , $\mu_2 \in$ F. Assume that $\mu_1 \equiv \mu_2(\theta_F)$. If $\mu_1 \subseteq \mu_2$ then $\mu_1 \cup \mu_2 \equiv \mu_2(\theta_F)$ and $\mu_1 \cap \mu_2 \equiv \mu_1(\theta_F)$.

Proof Given that $\mu_1 \equiv \mu_2(\theta_F)$.
$\Leftrightarrow \theta \wedge \mu_1 = \theta \wedge \mu_2$ for $\theta \subseteq \mu_1$ and $\theta \subseteq \mu_2$.
$\Leftrightarrow(\theta \wedge \mu_1)$ (x) = ($\theta \wedge \mu_2$(x)) for x $\in$ G.
Now $\mu_1 \subseteq \mu_2 \Rightarrow \mu_1$ (x) $\leq \mu_2$ (x).
We have ($\mu_1 \cup \mu_2$) (x) = max{ μ_1(x) , μ_2(x) = μ_2(x)
($\mu_1 \cap \mu_2$) (x) = min{ μ_1(x) , μ_2(x) = μ_1(x).
Now ($\theta\wedge$ ($\mu_1 \cup \mu_2$))(x) = $\sup\limits_{x=y\wedge z}$ { min{θ(y),($\mu_1 \cup \mu_2$)(z)}}
$\Rightarrow$ = $\sup\limits_{x=y\wedge z}$ {min{θ(y),μ_2(z)}}
$\Rightarrow$ = [$\theta \wedge \mu_2$](x)
$\Rightarrow$ ($\theta\wedge$($\mu_1 \cup \mu_2$))(x) = [$\theta \wedge \mu_2$](x)
$\Rightarrow$ $\mu_1 \cup \mu_2 \equiv \mu_2(\theta_F)$
Also ($\theta\wedge$ ($\mu_1 \cap \mu_2$)) (x) = $\sup\limits_{x=y\wedge z}$ {min{ θ(y) , ($\mu_1 \cap \mu_2$)(z)}}
$\Rightarrow$ = $\sup\limits_{x=y\wedge z}$ { min { θ(y) , μ_1(z)}}
$\Rightarrow$ = [$\theta \wedge \mu_1$] (x)
$\Rightarrow$ ($\theta\wedge$ ($\mu_1 \cap \mu_2$)) (x) = [$\theta \wedge \mu_1$] (x)
$\Rightarrow$ $\mu_1 \cap \mu_2 \equiv \mu_1(\theta_F)$

Theorem 5. Let μ_1 , $\mu_2 \in$ F and $\mu_1 \equiv \mu_2(\theta_F)$.
Then $\mu_1 \vee \mu_2 \equiv \mu_1 \cap \mu_2$ (θ_F).

ProofGiven that $\mu_1 \equiv \mu_2(\theta_F)$.
$\Rightarrow$ ($\theta \wedge \mu_1$) = ($\theta \wedge \mu_2$)
Now ($\mu_1 \wedge \mu_2$) (x) = $\sup\limits_{x=y\wedge z}$ { min{θ(y),($\mu_1 \cup \mu_2$)(z)}}
$\geq$ min[mu_1(x), μ_2(x)] for x= x$\vee$x.
= ($\mu_1 \cap \mu_2$)(x)
$\Rightarrow \mu_1 \wedge \mu_2 \geq \mu_1 \cap \mu_2$.
Since $\theta \subseteq \mu_1$ and $\theta \subseteq \mu_2$,
$\theta\wedge$ ($\mu_1 \wedge \mu_2$) $\geq \theta\wedge$ ($\mu_1 \cap \mu_2$)....(1)
Let x = p$\wedge$q.
$\Rightarrow$ x $\leq$ p and x$\leq$ q.
Since μ_1 is a fuzzy ℓ-ideal, μ_1(x) $\geq \mu_1$(p).
Since μ_2 is a fuzzy ℓ-ideal, μ_2(x) $\geq \mu_2$(q).
$\Rightarrow$ min { μ_1 (x) , μ_2(x) } $\geq$ min{ μ_1(p), μ_2(q)}.
$\Rightarrow$ ($\mu_1 \cap \mu_2$) (x) $\geq$ min{ μ_1(p), μ_2(q)}.
$\Rightarrow$ ($\mu_1 \cap \mu_2$) (x) $\geq$ $\sup\limits_{x=y\wedge z}$ [min{ μ_1(p), μ_2(q)}.
$\Rightarrow$ ($\mu_1 \cap \mu_2$) (x) $\geq \mu_1 \wedge \mu_2$.
Since $\theta \subseteq \mu_1$ and $\theta \subseteq \mu_2$,
$\theta \wedge \mu_1 \cap \mu_2$) (x) $\geq \theta \wedge \mu_1 \cap \mu_2$.....(2)
From (1) and (2)($\theta \wedge \mu_1 \cap \mu_2$) (x) = $\theta \wedge \mu_1 \cap \mu_2$.
$\Rightarrow \mu_1 \vee \mu_2 \equiv \mu_1 \cap \mu_2$ (θ_F).

The following proposition shows the existence of fuzzy congruence on the family of fuzzy ℓ-ideals.

Theorem 6. Let μ_1 , $\mu_2 \in$ F and θ_F be the congruence relation on F. If $\mu_1 \equiv \mu_2$ (θ_F) for $\theta \subseteq \mu_1$, $\theta \subseteq \mu_2$ then there exist a fuzzy congruence $\bar{\theta}$ on θ_t for t $\in$ [0,1] such that $\bar{\theta}(x,y) = \begin{cases} \theta(x) \wedge \theta(y) & if\, x \neq y \\ 1 & if\, x = y \end{cases}$

Proof Assume that $\mu_1 \equiv \mu_2$ (θ_F).
$\Rightarrow$There exist $\theta \in$F such that $\theta \wedge \mu_1 = \theta \wedge \mu_2$ for $\theta \in$F.
Let θ_t = { x $\in$ G / θ(x) $\geq$ t } and x,y,z $\in \theta_t$.
Let Min{ $\bar{\theta}$(x,z) , $\bar{\theta}$(z,y) } = t.
Now $\bar{\theta}$(x,x) = 1.
$\Rightarrow$ $\bar{\theta}$ is fuzzy reflexive.
$\bar{\theta}$(x,y) = θ (x) $\wedge\theta$ (y) = θ (y) $\wedge\theta$ (x) = $\bar{\theta}$(y,x).
$\Rightarrow$ $\bar{\theta}$ is fuzzy symmetric.
Now $\bar{\theta}$(x,y) = θ(x)$\wedge\theta$(y)$\geq$ t.
($\bar{\theta} \circ \bar{\theta}$)(x,y)= $\sup\limits_{z\in\theta_t}$ Min[$\bar{\theta}$(x,z),$\bar{\theta}$(z,y)] = t $\leq \bar{\theta}$(x,y).
$\Rightarrow$ ($\bar{\theta} \circ \bar{\theta}$) $\subseteq \bar{\theta}$.
$\Rightarrow$ $\bar{\theta}$ is fuzzy transitive.
Now $\bar{\theta}$(a - x , b - y) = θ(a-x) $\wedge\theta$(b-y)
$\geq$ θ(a)$\wedge\theta$(x)$\wedge\theta$(b)$\wedge\theta$(y)
=θ(a)$\wedge\theta$(b)$\wedge\theta$(x)$\wedge\theta$(y)
,since θ is an fuzzy ℓ-ideal.
=$\bar{\theta}$ (a,b) $\wedge\bar{\theta}$ (x,y).
$\Rightarrow$ $\bar{\theta}$(a - x , b - y)$\geq$ $\bar{\theta}$ (a,b)$\wedge\bar{\theta}$ (x,y).
Similarly $\bar{\theta}$(a $\wedge$ x , b $\wedge$ y)$\geq$ $\bar{\theta}$ (a,b)$\wedge\bar{\theta}$ (x,y) and
$\bar{\theta}$(a $\vee$ x , b $\vee$ y)$\geq$ $\bar{\theta}$ (a,b)$\wedge\bar{\theta}$ (x,y).

Hence $\bar{\theta}$ is Fuzzy Congruence.

4 Conclusion

In this paper we initiated the study of congruence relations on the set of all fuzzy ℓ-ideals of ℓ-group G. Also we showed the existence of fuzzy congruence on the family of fuzzy ℓ-ideals. In future the study of relation between congruence and fuzzy congruence on the family of fuzzy ℓ-ideals can be extended.

Acknowledgements

Authors are sincerely grateful to the valuable reviewers for their helpful comments to improve this research work.

References

[1] N. Ajmal and K.V Thomas, Information Sciences **79**, 271-291 (1994).
[2] M. Bakhshi, On Fuzzy Convex Lattice Ordered Subgroups, Iranian Journal of Fuzzy Systems **10**, 159-172 (1992).
[3] S.K Bhakat and P. Das, Iranian Journal of Fuzzy Systems **10**, 159-172 (2013).
[4] P. Bharathi and J. Vimala, Global Journal of Pure and Applied Mathematics **12**, 2067-2074 (2016).
[5] G. Birkhof, Annals of Mathematics **43**, 298-331 (1942).
[6] S.H Dhanani and Y.S Pawar, International Journal of Fuzzy Systems and Rough Systems **4**, 99-102 (2011).
[7] G. Gratzer, Lattice Theory: Foundation, Springer, 2011.
[8] R. Khyalappa, Y.S Pawar and S.H Dhanani, International Journal of Engineering Research and Applications **3**, 873-878 (2013).
[9] J.N Mordeson and D.S Malik, Fuzzy Commutative Algebra, World Scientific publishing, 1998.
[10] D.S Malik and Mordeson, Fuzzy Sets and Systems **45**, 245-251 (1992).
[11] T.K Mukherjee and M.K Sen, Fuzzy Sets and systems **21**, 99-104 (1987).
[12] Nanda, Bulletin of the Calcutta Mathematical Society **81**, (1989).
[13] R. Natarajan and J. Vimala, Acta Ciencia Indica Mathematics **33**, 517 (2007).
[14] R. Natarajan and J. Vimala, Acta Ciencia Indica Mathematics **33**, 1765 (2007).
[15] Rajeshkumar, Fuzzy Algebra, publication Division, University of Delhi, 1993.
[16] A. Rosenfeld, Journal of Mathematical Analysis and Applications **35**, 512-517 (1971).
[17] G.S.V Sathya Saibaba, Southeast Asian Bulletin of Mathematics **32**, 749-766 (2008).
[18] U.M Swamy and D.V. Raju, Fuzzy sets and systems **95**, 249-253 (1998).
[19] L.A Zadeh, Information and Control **8**, 69-78 (1965).
[20] H.J Zimmermann, Wiley Interdisciplinary Reviews: Computational Statics **2**, 317-332 (2010).
[21] A. Yajnik and R. Ali, DJ Journal of Engineering and Applied mathematics **1**, 17-22(2015).
[22] G. Vasanti and B.V Rao, DJ Journal of Engineering and Applied Mathematics **2**, 1-6(2016)

P. Bharathi is a research scholar of Mathematics in Alagappa University, Karaikudi. Her research interest is Fuzzy Algebra and Fuzzy Decision Mak- ing theory. She published various research articles in reputed international journals. She presented her research work in many International / National conferences. She completed UGC Minor research project on Fuzzy Decision Making theory during the period 2013-15.

J. Vimala had obtained her Ph.D. degree during 2007 in the area of Algebra Lattice Theory, from Alagappa University. In her Ph.D. work, she solved long standing open problem, :Generalize the concept of distributive $\ell-$ideals to convex $\ell-$subgroups, given by Gratzer,G. At present, she is working as Assistant Professor in the Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu. She is a reviewer of Journal of Engineering Applied Mathematics, World Academy of Science, Engineering and Technology and also reviewed research articles in the International Conference on soft Computing in Data Science 2015(SCDS 2015) , Putrajaya. She organized one National Conference on Pure and Applied Mathematics(NCPAM 2016) in Alagappa University.

L. Vijayalakshmi is a research scholar of Mathematics in Alagappa University, Karaikudi. Her research interest is Soft lattice theory and soft -groups. She presented her research work in many international conferences. Her paper was accepted for International Journal of Pure and Applied Mathematics", Academic Publications, Volume: 112 (2017) Issue: 1.

J. Arockia Reeta is the research scholar of Mathematics in Alagappa University, Karaikudi. Her research interest is Lattice ordered fuzzy soft group. She presented her research work in many conferences. Her paper was published in International journal of Applied Mathematical Sciences Vol 9, Number 1(2016).