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Study on Non Response and Measurement Error using Double Sampling Scheme

Sunil Kumar[1,*] *and Sandeep Bhougal*[2]

[1] Department of Statistics, University of Jammu, Jammu, J and K, India.
[2] Department of Mathematics, Shri Mata Vaishno Devi University, Kakryal, Jammu, J and K, India.

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Abstract: In this paper, a ratio-product type estimator has been developed for estimating the population mean of the study variable using auxiliary information under double sampling scheme in the presence of non-response and measurement error on both the variables. The optimum property of the suggested estimator has been identified. A theoretical and empirical study has been done to demonstrate the efficiency of the proposed estimator over other estimators.

Keywords: Non-response, Measurement error, Ratio estimator, Product estimator, Double sampling.

1 Introduction

In surveys, response rates have fallen over the last three decades, [2]. As per [3], researchers should concentrate to maximize response rates and to minimize risk of non-response errors. [4], [5], [6] hypothesized that reluctant sample persons, should successfully brought into the respondent pool through persuasive efforts, and may provide data filled with measurement error. Two questions arise in this situation as – first has to do with the quality of a statistic (eg. Means, correlation coefficients) calculated from a survey i.e. does the mean square error of a statistic increase when sample persons who are less likely to be contacted or corporate are incorporated into the respondent pool? Secondly, question has to do with methodological inquiries for detecting non-response bias, see [7].
Many researchers have studied the properties of estimators in the presence of non-response and measurement errors independently, respectively, viz [8], [9], [10], [11], [12], [13], etc.

In general, the researchers who have studied non-response have not considered the presence of measurement error and vice versa. On this situation [1] on his Ph. D. thesis studied the properties of estimators for estimating the population mean of study variable in the presence of non-response and measurement error using single auxiliary variable. For estimating the parameter's, [14] and [15] have extended the work of [1] on estimating the population mean of the study variable in the presence of non-response and measurement error, respectively when the population mean of auxiliary variable is known. In the present study, we have proposed a generalized estimator in the situation when non-response and measurement errors are present on both study and auxiliary variables under double sampling scheme.

2 The suggested Estimator

In situations with unknown population mean of the auxiliary variable$X$, a two-phase sampling scheme is adopted where$\overline{X}$, the population mean of the variable X is replaced by $\bar{x}_1$, the first-phase sample estimator for the population mean. A larger sample of size n_1 is taken from a population of size N at the first-phase by a simple random sampling without replacement (SRSWOR) method. Information on the auxiliary variable is obtained from the sample drawn at the first-phase. At second phase, a smaller sample of size$n$is taken from the units of the first-phase sample using a simple random sampling without replacement (SRSWOR) method and data on the variable of interest are collected. At first phase, we assume that there

* Corresponding author e-mail: sunilbhougal06@gmail.com

is complete response and no measurement error. Letx_{1i} be the observed values and X_{1i} be the true values on auxiliary characteristic associated with the $i^{th}\,(i=1,\,2,\,\ldots,\,n_1)$ unit in the first-phase sample. Since we have assumed that there are no measurement errors at first-phase sample, therefore$x_{1i}=X_{1i}$. Let$(x_{1i},\,y_i)$ be the observed values and$(X_i,\,Y_i)$ be the true values on two characteristics $(x,\,y)$respectively associated with the $i^{th}\,(i=1,\,2,\,\ldots,\,n)$ unit of the second-phase sample.
In surveys, there are many potential sources of non-sampling errors. The greater the impact of these sources of error, the greater the difference will be between our survey (or census) estimate and the true values. Based on above situation, I have suggested one general class of estimator under the situation when there is non-response and measurement error on study as well as auxiliary variable under double sampling scheme

$$T=\bar{y}^*\left[k\left(\frac{\bar{x}_1^{*\delta}}{\bar{x}_1}\right)\left(\frac{\bar{x}_1^{*\delta}}{\bar{x}^*}\right)+(1-k)\left(\frac{\bar{x}_1}{\bar{x}_1^{*\delta}}\right)\left(\frac{\bar{x}^*}{\bar{x}_1^{*\delta}}\right)\right], \tag{1}$$

where $\bar{x}_1^{*\delta}=\frac{n_1\bar{x}_1-n\bar{x}^*}{n_1-n}$.
In order to obtain the bias and mean square error (MSE) of suggested estimator, the following are some notations.
Let

$$\omega_{X_1}=\frac{1}{\sqrt{n_1}}\sum_{i=1}^{n_1}\left(x_{1i}-\bar{X}\right),$$

$$\omega_Y^*=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(Y_i^*-\bar{Y}\right),$$

$$\omega_U^*=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}U_i^*,$$

$$\omega_X^*=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(X_i^*-\bar{X}\right),$$

and

$$\omega_V^*=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}V_i^*.$$

Multiplying both sides of ω_{X_1} by$\frac{1}{\sqrt{n_1}}$, we have

$$\frac{1}{\sqrt{n_1}}\omega_{X_1}=\frac{1}{n_1}\sum_{i=1}^{n_1}\left(x_{1i}-\bar{X}\right),$$

or $\bar{x}_1=\bar{X}+\frac{1}{\sqrt{n_1}}\omega_{X_1}$.
Adding $\omega_Y^*$and ω_U^*, we have

$$\omega_Y^*+\omega_U^*=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(Y_i^*-\bar{Y}\right)+\frac{1}{\sqrt{n}}\sum_{i=1}^{n}U_i^*.$$

Multiplying both sides by$\frac{1}{\sqrt{n}}$, we have

$$\frac{1}{\sqrt{n}}\left(\omega_Y^*+\omega_U^*\right)=\frac{1}{n}\sum_{i=1}^{n}\left(Y_i^*-\bar{Y}\right)+\frac{1}{n}\sum_{i=1}^{n}U_i^*,$$

or $\frac{1}{\sqrt{n}}\left(\omega_Y^*+\omega_U^*\right)=\bar{y}^*-\bar{Y}$,
or $\bar{y}^*=\bar{Y}+\frac{1}{\sqrt{n}}\left(\omega_Y^*+\omega_U^*\right)$.
Similarly, we have

$$\bar{x}^*=\bar{X}+\frac{1}{\sqrt{n}}\left(\omega_X^*+\omega_V^*\right).$$

Further

$$E\left(\frac{\omega_{X_1}}{\sqrt{n_1}}\right)^2=\lambda_1 S_X^2=B_1;$$

$$E\left(\frac{\omega_Y^*+\omega_U^*}{\sqrt{n}}\right)^2=\lambda_2\left(S_Y^2+S_U^2\right)+\theta\left(S_{Y(2)}^2+S_{U(2)}^2\right)=A;$$

$$E\left(\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right)^2=\lambda_2\left(S_X^2+S_V^2\right)+\theta\left(S_{X(2)}^2+S_{V(2)}^2\right)=B;$$

$$E\left\{\left(\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right)\left(\frac{\omega_Y^*+\omega_U^*}{\sqrt{n}}\right)\right\}=\lambda_2\rho_{YX}S_YS_X+\theta\rho_{YX(2)}S_{Y(2)}S_{X(2)}=C;$$

$$E\left\{\left(\frac{\omega_Y^*+\omega_U^*}{\sqrt{n}}\right)\left(\frac{\omega_{X_1}}{\sqrt{n_1}}\right)\right\}=\lambda_1\rho_{YX}S_YS_X=C_1;E\left\{\left(\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right)\left(\frac{\omega_{X_1}}{\sqrt{n_1}}\right)\right\}=\lambda_1S_X^2;$$

$$\psi_1=n_1^2B_1+n^2B-2nn_1B_1;\psi_2=3n_1B_1-nB-2B_1;\psi_3=C+2C_1;\psi_4=n_1C_1-nC.$$

Expressing T in terms of $\omega_i^*;;i=x,\ y,\ U,\ V$; we have

$$T=\bar{y}^*\left[k\left(\frac{n_1\bar{x}_1-n\bar{x}^*}{\bar{x}_1(n_1-n)}\right)\left(\frac{n_1\bar{x}_1-n\bar{x}^*}{\bar{x}^*(n_1-n)}\right)+(1-k)\left(\frac{\bar{x}_1(n_1-n)}{n_1\bar{x}_1-n\bar{x}^*}\right)\left(\frac{\bar{x}^*(n_1-n)}{n_1\bar{x}_1-n\bar{x}^*}\right)\right]$$

$$T=\left(\overline{Y}+W_Y\right)\left[k\left\{1+\frac{n_1W_{X_1}-nW_X}{(n_1-n)\overline{X}}\right\}^2\left\{1+\frac{(n_1-n)W_{X_1}}{(n_1-n)\overline{X}}\right\}^{-1}\left\{1+\frac{W_X}{\overline{X}}\right\}^{-1}\right.$$

$$\left.+(1-k)\left\{1+\frac{W_{X_1}}{\overline{X}}\right\}\left\{1+\frac{W_X}{\overline{X}}\right\}\left\{1+\frac{n_1W_{X_1}-nW_X}{(n_1-n)\overline{X}}\right\}^{-2}\right]$$

where$W_Y=\frac{1}{\sqrt{n}}\left(\omega_Y^*+\omega_U^*\right)$ and $W_X=\frac{1}{\sqrt{n}}\left(\omega_X^*+\omega_U^*\right)$.
Simplifying and ignoring terms of order greater than two, one can obtain

$$\begin{aligned}\left(T-\overline{Y}\right)=&RW_{X_1}-2R\left(\frac{n_1W_{X_1}-nW_X}{n_1-n}\right)+\frac{3R}{\overline{X}}\left(\frac{n_1W_{X_1}-nW_X}{n_1-n}\right)^2\\&-2R\frac{n_1W_{X_1}^2-nW_XW_{X_1}}{(n_1-n)\overline{X}}+k\left\{\frac{R}{\overline{X}}W_X^2+\frac{R}{\overline{X}}W_{X_1}^2-RW_X-2RW_{X_1}\right.\\&\left.+4R\left(\frac{n_1W_{X_1}-nW_X}{n_1-n}\right)-\frac{2R}{\overline{X}}\left(\frac{n_1W_{X_1}-nW_X}{n_1-n}\right)^2-\frac{2R}{\overline{X}}\left(\frac{n_1W_XW_{X_1}-nW_X^2}{n_1-n}\right)\right\}\\&+W_Y+\frac{W_YW_{X_1}}{\overline{X}}-\frac{2\left(n_1W_YW_{X_1}-nn_1W_XW_Y\right)}{(n_1-n)\overline{X}}-k\left\{\frac{W_YW_X}{\overline{X}}\right.\\&\left.+2\frac{W_YW_{X_1}}{\overline{X}}+4\frac{\left(n_1W_YW_{X_1}-nn_1W_XW_Y\right)}{(n_1-n)\overline{X}}\right\}.\end{aligned}\tag{2}$$

Taking expectation on both sides of (2), one can get the expression of bias as

$$\begin{aligned}B(T)=&\frac{(3-2k)R}{\overline{X}}\left\{\frac{n_1(n_1-2n)\lambda_1S_X^2+n\left(\lambda_2S_X^2+\theta S_{X(2)}^2\right)+n\left(\lambda_2S_V^2+\theta S_{V(2)}^2\right)}{(n_1-n)^2}\right\}\\&-\frac{2(1+2k)}{\overline{X}}\left\{\frac{(n_1-n)\lambda_1\rho_{YX}S_YS_X-n\theta\rho_{YX(2)}S_{Y(2)}S_{X(2)}}{(n_1-n)}\right\}\\&+\frac{(\lambda-2\lambda_1k)}{\overline{X}}\rho_{YX}S_YS_X-2\frac{R}{\overline{X}}\lambda_1S_X^2+k\left[\frac{R}{\overline{X}}\left\{\lambda_2S_X^2+\theta S_{X(2)}^2+\lambda_2S_V^2+\theta S_{V(2)}^2\right\}\right.\\&\left.-\frac{R}{\overline{X}}\lambda_1S_X^2-\frac{\lambda_2\rho_{YX}S_YS_X+\rho_{YX(2)}S_{Y(2)}S_{X(2)}}{\overline{X}}\right].\end{aligned}\tag{3}$$

Squaring both sides of (2) ignoring terms of order greater than two and taking expectations, one can obtain MSE of T as

$$MSE\left(T\right)=\left[\left(1+R^{2}k^{2}\right)A+\left(1+8k^{2}\right)R^{2}B_{1}+\frac{4R^{2}(1-2k)^{2}}{(n_{1}-n)^{2}}\left(n_{1}^{2}B_{1}+n^{2}B-2nn_{1}B_{1}\right)\right.$$
$$+2RC_{1}-2k\left(RC+2RC_{1}\right)-\frac{4R\left(1-2k\right)}{\left(n_{1}-n\right)}\left(n_{1}C_{1}-nC\right)$$
$$\left.-6R^{2}kB_{1}-4R^{2}\left(1-2k\right)B_{1}+\frac{4Rk\left(1-2k\right)}{\left(n_{1}-n\right)}\left(3Rn_{1}B_{1}-nRB-2RB_{1}\right)\right]. \quad (4)$$

Minimize equation (4) with respect to k yields its optimum value as

$$k=\frac{\psi_{3}-\left(\frac{4}{n_{1}-n}\right)\psi_{4}-RB_{1}+\frac{8R}{(n_{1}-n)^{2}}-\left(\frac{2R}{n_{1}-n}\right)\psi_{2}}{A+8B_{1}+\frac{16}{(n_{1}-n)^{2}}-\frac{8}{n_{1}-n}\psi_{2}}=\frac{W_{1}}{W_{2}}$$

Using the above optimum value of k, one can obtain the $min.\ MSE\ of\ T$ as

$$min.MSE\left(T\right)=\left[\left(1+R^{2}k_{opt}^{2}\right)A+\left(1+8k_{opt}^{2}\right)R^{2}B_{1}+\frac{4R^{2}(1-2k_{opt})^{2}}{(n_{1}-n)^{2}}\right.$$
$$\left(n_{1}^{2}B_{1}+n^{2}B-2nn_{1}B_{1}\right)+2RC_{1}-2k_{opt}\left(RC+2RC_{1}\right)$$
$$-\frac{4R\left(1-2k_{opt}\right)}{\left(n_{1}-n\right)}\left(n_{1}C_{1}-nC\right)-6R^{2}k_{opt}B_{1}-4R^{2}\left(1-2k_{opt}\right)B_{1}$$
$$\left.+\frac{4Rk_{opt}\left(1-2k_{opt}\right)}{\left(n_{1}-n\right)}\left(3Rn_{1}B_{1}-nRB-2RB_{1}\right)\right]. \quad (5)$$

3 Efficiency Comparison

For efficiency comparison, I have considered the following special cases:
Let $k=1$ in equation (1), the proposed estimator tends to the ratio estimator as

$$T_{R}=\overline{y}^{*}\left(\frac{\overline{x}_{1}^{*\delta}}{\overline{x}_{1}}\right)\left(\frac{\overline{x}_{1}^{*\delta}}{\overline{x}^{*}}\right)$$

with mean squared error as

$$MSE\left(T_{R}\right)=\left[\left(1+R^{2}\right)A+9R^{2}B_{1}+\frac{4R^{2}}{(n_{1}-n)^{2}}\psi_{1}+2RC_{1}-2R\psi_{3}+\frac{4R}{n_{1}-n}\psi_{4}-6R^{2}B_{1}\right.$$
$$\left.+4R^{2}B_{1}-\frac{4R^{2}}{n_{1}-n}\psi_{2}\right] \quad (6)$$

From (4) and (6), one can obtain

$$MSE\left(T_{R}\right)-MSE\left(T\right)\geq0$$

$$if\ \frac{-\phi_{2}-\sqrt{\phi_{2}^{2}-4R\phi_{1}\phi_{3}}}{2R\phi_{1}}\leq k\leq\frac{-\phi_{2}+\sqrt{\phi_{2}^{2}-4R\phi_{1}\phi_{3}}}{2R\phi_{1}}, \quad (7)$$

where $\phi_{1}=\frac{8}{n_{1}-n}\psi_{2}-\frac{16}{(n_{1}-n)^{2}}\psi_{1}-8B_{1}-R^{2}A;$

$$\phi_{2}=\frac{16R}{(n_{1}-n)^{2}}\psi_{1}-\frac{4R}{(n_{1}-n)}\psi_{2}-2RB_{1}-\frac{32}{(n_{1}-n)}\psi_{4}-2\psi_{3};$$

Table 1: Mean squared error (MSE) of the estimators for Population I

	Sample sizes	Estimator(s)	1/k			
			1/2	1/3	1/4	1/5
$\sigma^2_{y(2)} = 99.99174;\ \sigma^2_{x(2)} = 99.87471;\ \sigma^2_{U(2)} = 9.150544;\ \sigma^2_{V(2)} = 8.756592;\ \rho_{yx(2)} = 0.994916$	$n = 400;\ n_1 = 1000$	T_R	0.11647	0.175852	0.235233	0.294614
		T_0	1.028799	1.170743	1.312687	1.454631
		T	**0.004621**	**0.021996**	**0.037643**	**0.052127**
	$n = 500;\ n_1 = 1250$	T_R	0.131956	0.203214	0.274471	0.345729
		T_0	0.878781	1.049114	1.219447	1.38978
		T	**0.016496**	**0.034835**	**0.051338**	**0.066747**
	$n = 600;\ n_1 = 1500$	T_R	0.14234	0.221515	0.30069	0.379865
		T_0	0.778769	0.968028	1.157287	1.346546
		T	**0.023216**	**0.041699**	**0.058518**	**0.074434**
$\sigma^2_{y(2)} = 100.9428;\ \sigma^2_{x(2)} = 100.8224;\ \sigma^2_{U(2)} = 9.053862;\ \sigma^2_{V(2)} = 8.766538;\ \rho_{yx(2)} = 0.995535$	$n = 400;\ n_1 = 1000$	T_R	0.116788	0.176486	0.236184	0.295883
		T_0	1.03011	1.173366	1.316621	1.459877
		T	**0.004664**	**0.022058**	**0.037714**	**0.052202**
	$n = 500;\ n_1 = 1250$	T_R	0.132237	0.203975	0.275613	0.347251
		T_0	0.880355	1.052261	1.224168	1.396075
		T	**0.016533**	**0.034884**	**0.051393**	**0.066808**
	$n = 600;\ n_1 = 1500$	T_R	0.142763	0.222361	0.301959	0.381557
		T_0	0.780518	0.971525	1.162533	1.35354
		T	**0.023246**	**0.041738**	**0.058565**	**0.074489**
$\sigma^2_{y(2)} = 104.2711;\ \sigma^2_{x(2)} = 103.2349;\ \sigma^2_{U(2)} = 8.821278;\ \sigma^2_{V(2)} = 8.339179;\ \rho_{yx(2)} = 0.995472$	$n = 400;\ n_1 = 1000$	T_R	0.117218	0.177347	0.237476	0.297604
		T_0	1.033668	1.180482	1.327296	1.47411
		T	**0.004806**	**0.022335**	**0.038131**	**0.05277**
	$n = 500;\ n_1 = 1250$	T_R	0.132853	0.205008	0.277162	0.349317
		T_0	0.884625	1.060801	1.236978	1.413154
		T	**0.016699**	**0.03522**	**0.05192**	**0.067539**
	$n = 600;\ n_1 = 1500$	T_R	0.143337	0.223509	0.30368	0.383852
		T_0	0.785262	0.981014	1.176766	1.372517
		T	**0.023432**	**0.04213**	**0.059188**	**0.075361**

$$\phi_3 = RA + 10RB_1 - 2\psi_3 + \frac{4}{(n_1-n)}\psi_4 + \frac{16R}{(n_1-n)^2}\psi_4 - \frac{4R}{(n_1-n)}\psi_2.$$

Let $k = 0$ in equation (1), the proposed estimator tends to the ratio estimator as

$$T = \bar{y}^* \left(\frac{\bar{x}_1}{\bar{x}_1^{*\delta}}\right)\left(\frac{\bar{x}^*}{\bar{x}_1^{*\delta}}\right),$$

with mean squared error as

$$MSE\left(T_O\right) = \left[A + R^2B_1 + \frac{4R^2}{(n_1-n)^2}\psi_1 + 2RC_1 - \frac{4R}{n_1-n}\psi_4 - 4R^2B_1\right], \tag{8}$$

From (4) and (8), one can obtain

$$0 \le k \ge \left\{\frac{2RB_1 - 2\psi_3 - \frac{16R}{(n_1-n)^2}\psi_1 + \frac{8}{(n_1-n)}\psi_4}{\frac{4R}{(n_1-n)}\psi_2 - RA - 8RB_1 - \frac{16R}{(n_1-n)^2}\psi_1}\right\}. \tag{9}$$

If (7) and (9) holds true, the proposed estimator is more efficient estimator than $T_R$and T_O.

Table 2: Mean squared error (MSE) of the estimators for Population II

	Sample sizes	Estimator(s)	1/k			
			1/2	1/3	1/4	1/5
$\sigma^2_{y(2)} = 97.02783; \sigma^2_{x(2)} = 94.54578;\ \sigma^2_{U(2)} = 22.80557;\ \sigma^2_{V(2)} = 25.43263;\ \rho_{yx(2)} = 0.994546$	$n = 400;\ n_1 = 1000$	T_R	0.36564	0.44701	0.52839	0.60976
		T_0	1.0831	1.22932	1.37554	1.52175
		T	**0.10069**	**0.119299**	**0.137114**	**0.154383**
	$n = 500;\ n_1 = 1250$	T_R	0.33508	0.43273	0.53037	0.62802
		T_0	0.92226	1.09772	1.27318	1.44864
		T	**0.08949**	**0.10067**	**0.13098**	**0.15076**
	$n = 600;\ n_1 = 1500$	T_R	0.31476	0.42326	0.53175	0.64025
		T_0	0.81503	1.00999	1.20494	1.3999
		T	**0.08148**	**0.10412**	**0.12595**	**0.147329**
$\sigma^2_{y(2)} = 98.27616; \sigma^2_{x(2)} = 97.42674;\ \sigma^2_{U(2)} = 23.27837;\ \sigma^2_{V(2)} = 24.13829;\ \rho_{yx(2)} = 0.994992$	$n = 400;\ n_1 = 1000$	T_R	0.3658	0.44734	0.52887	0.61041
		T_0	1.08563	1.23438	1.38312	1.53187
		T	**0.10062**	**0.11917**	**0.13695**	**0.15418**
	$n = 500;\ n_1 = 1250$	T_R	0.33527	0.43311	0.53096	0.6288
		T_0	0.9253	1.10379	1.28228	1.46078
		T	**0.008942**	**0.110545**	**0.13082**	**0.15058**
	$n = 600;\ n_1 = 1500$	T_R	0.31498	0.42369	0.5324	0.64112
		T_0	0.81841	1.01673	1.21506	1.41338
		T	**0.08141**	**0.10401**	**0.125821**	**0.147195**
$\sigma^2_{y(2)} = 96.09359; \sigma^2_{x(2)} = 94.71923;\ \sigma^2_{U(2)} = 24.42978;\ \sigma^2_{V(2)} = 23.03076;\ \rho_{yx(2)} = 0.99467$	$n = 400;\ n_1 = 1000$	T_R	0.36394	0.44362	0.5233	0.60297
		T_0	1.08205	1.22722	1.37238	1.51755
		T	**0.10074**	**0.119482**	**0.137471**	**0.15494**
	$n = 500;\ n_1 = 1250$	T_R	0.33304	0.42865	0.52427	0.61988
		T_0	0.921	1.0952	1.2694	1.44359
		T	**0.089599**	**0.11099**	**0.131551**	**0.151611**
	$n = 600;\ n_1 = 1500$	T_R	0.3125	0.41873	0.52497	0.63121
		T_0	0.81363	1.00719	1.20074	1.39429
		T	**0.081645**	**0.104555**	**0.126698**	**0.14841**

4 Empirical Comparison

To examine the merits of the suggested class of estimators T over the other competitors, I have generated four populations from normal distribution with different choices of parameters by using R language program. The auxiliary information of variable X has been generated from N(5,10) population. This type of population is very relevant in most of the socio-economic situations with one interest and one auxiliary variable.

4.1 Population I:

$$X = N(5,\ 10);;Y = X + N(0,\ 1);;y = Y + N(1,\ 3);;x = X + N(1,3);;N = 5000;;$$
$$\mu_Y = 4.927167;;\ \mu_X = 4.924306;;\ \sigma_y^2 = 102.0075;;\ \sigma_x^2 = 101.4117;;\ \sigma_U^2 = 8.862114;;$$
$$\sigma_V^2 = 9.001304;;\ \rho_{yx} = 0.995059$$

Table 3: Mean squared error (MSE) of the estimators for Population III

	Sample sizes	Estimator(s)	1/k			
			1/2	1/3	1/4	1/5
$\sigma^2_{y(2)} = 102.7504; \sigma^2_{x(2)} = 101.2097; \sigma^2_{U(2)} = 9.095136; \sigma^2_{V(2)} = 8.8123; \rho_{yx(2)} = 0.995045$	$n = 400; n_1 = 1000$	T_R	0.11848	0.17784	0.23721	0.29658
		T_0	1.02074	1.16486	1.30899	1.45312
		T	**0.00727**	**0.02468**	**0.04039**	**0.05497**
	$n = 500; n_1 = 1250$	T_R	0.13318	0.20442	0.27566	0.3469
		T_0	0.8732	1.04615	1.2191	1.39205
		T	**0.01844**	**0.03687**	**0.05351**	**0.06908**
	$n = 600; n_1 = 1500$	T_R	0.14304	0.22219	0.30135	0.38051
		T_0	0.77484	0.96701	1.15917	1.35134
		T	**0.02471**	**0.04334**	**0.06036**	**0.0765**
$\sigma^2_{y(2)} = 99.55993; \sigma^2_{x(2)} = 99.49764; \sigma^2_{U(2)} = 9.233619; \sigma^2_{V(2)} = 8.805872; \rho_{yx(2)} = 0.995314$	$n = 400; n_1 = 1000$	T_R	0.11786	0.17661	0.23535	0.2941
		T_0	1.01761	1.15861	1.29961	1.44061
		T	**0.00697**	**0.0241**	**0.03954**	**0.05383**
	$n = 500; n_1 = 1250$	T_R	0.13244	0.20293	0.27343	0.34393
		T_0	0.86945	1.03865	1.20785	1.37705
		T	**0.01809**	**0.03618**	**0.05248**	**0.0677**
	$n = 600; n_1 = 1500$	T_R	0.14221	0.22055	0.29888	0.37721
		T_0	0.77067	0.95867	1.4667	1.33467
		T	**0.02433**	**0.04258**	**0.0592**	**0.07493**
$\sigma^2_{y(2)} = 105.4334; \sigma^2_{x(2)} = 103.8947; \sigma^2_{U(2)} = 9.277715; \sigma^2_{V(2)} = 9.072151; \rho_{yx(2)} = 0.995105$	$n = 400; n_1 = 1000$	T_R	0.12005	0.18098	0.24192	0.30286
		T_0	1.02454	1.17246	1.32039	1.46832
		T	**0.00774**	**0.02551**	**0.04152**	**0.05637**
	$n = 500; n_1 = 1250$	T_R	0.13506	0.20819	0.28131	0.35444
		T_0	0.87776	1.05527	1.23278	1.41029
		T	**0.01893**	**0.03773**	**0.05469**	**0.07057**
	$n = 600; n_1 = 1500$	T_R	0.14513	0.22638	0.30763	0.3888
		T_0	0.7799	0.97714	1.17437	1.37161
		T	**0.02521**	**0.04421**	**0.06157**	**0.07804**

4.2 Population II:

$$X = N(5,\ 10);;Y = X + N(0,\ 1);;y = Y + N(1,\ 5);;x = X + N(1,5);;N = 5000;;$$
$$\mu_Y = 4.996681;;\ \mu_X = 5.013507;;\ \sigma^2_y = 97.12064;;\ \sigma^2_x = 95.95803;;\ \sigma^2_U = 23.96055;;$$
$$\sigma^2_V = 24.19283;;\ \rho_{yx} = 0.994822$$

4.3 Population III:

$$X = N(5,\ 10);;Y = X + N(0,\ 1);;y = Y + N(2,\ 3);;x = X + N(2,\ 3);;N = 5000;;$$
$$\mu_Y = 4.730993;;\ \mu_X = 4.741928;;\ \sigma^2_y = 101.2633;;\ \sigma^2_x = 100.2288;;\ \sigma^2_U = 9.1025;;$$
$$\sigma^2_V = 9.052019;;\ \rho_{yx} = 0.995187$$

Table 4: Mean squared error (MSE) of the estimators for Population IV

	Sample sizes	Estimator(s)	1/k			
			1/2	1/3	1/4	1/5
$\sigma^2_{y(2)} = 103.5361; \sigma^2_{x(2)} = 102.1031; \sigma^2_{U(2)} = 25.31099; \sigma^2_{V(2)} = 22.84483; \rho_{yx(2)} = 0.35223$	$n = 400; n_1 = 1000$	T_R	0.88862	1.04958	1.21055	1.37151
		T_0	0.95469	1.06656	1.7842	1.29029
		T	**0.30793**	**0.33906**	**0.36947**	**0.39939**
	$n = 500; n_1 = 1250$	T_R	0.78591	0.97906	0.17222	1.36537
		T_0	0.80076	0.935	1.06924	1.20349
		T	**0.25397**	**0.2903**	**0.32586**	**0.36095**
	$n = 600; n_1 = 1500$	T_R	0.7175	0.93211	1.14673	1.36134
		T_0	0.69814	0.8473	0.99646	1.14561
		T	**0.21752**	**0.25711**	**0.29599**	**0.33448**
$\sigma^2_{y(2)} = 103.6790; \sigma^2_{x(2)} = 102.7446; \sigma^2_{U(2)} = 24.6859; \sigma^2_{V(2)} = 26.12337; \rho_{yx(2)} = 0.35223$	$n = 400; n_1 = 1000$	T_R	0.89257	1.05749	1.2224	1.38731
		T_0	0.9564	1.06998	1.18357	1.29715
		T	**0.30775**	**0.33864**	**0.36877**	**0.39841**
	$n = 500; n_1 = 1250$	T_R	0.79065	0.98855	1.18644	1.38434
		T_0	0.80281	0.93911	1.07541	1.21171
		T	**0.25372**	**0.28972**	**0.32493**	**0.35966**
	$n = 600; n_1 = 1500$	T_R	0.72276	0.94265	1.16253	1.38241
		T_0	0.70042	0.85187	1.00331	1.15475
		T	**0.21721**	**0.25642**	**0.2949**	**0.33298**
$\sigma^2_{y(2)} = 100.1031; \sigma^2_{x(2)} = 99.31665; \sigma^2_{U(2)} = 25.80394; \sigma^2_{V(2)} = 24.50468; \rho_{yx(2)} = 0.35223$	$n = 400; n_1 = 1000$	T_R	0.88718	1.0467	1.20621	1.36573
		T_0	0.95272	1.06263	1.17253	1.28243
		T	**0.30728**	**0.33776**	**0.36751**	**0.39677**
	$n = 500; n_1 = 1250$	T_R	0.78418	0.9756	1.16702	1.35844
		T_0	0.7984	0.93028	1.06217	1.19405
		T	**0.25319**	**0.28873**	**0.3235**	**0.3578**
	$n = 600; n_1 = 1500$	T_R	0.71557	0.92826	1.14095	1.35364
		T_0	0.69552	0.84206	0.98859	1.13513
		T	**0.21665**	**0.25536**	**0.29336**	**0.33096**

N_1	N_2	$\sigma^2_{y(2)}$	$\sigma^2_{x(2)}$	$\sigma^2_{U(2)}$	$\sigma^2_{V(2)}$	$\rho_{yx(2)}$
4500	500	99.99174	99.87471	9.150544	8.756592	0.994916
4250	750	100.9428	100.8224	9.053862	8.766538	0.995535
4000	1000	104.2711	103.2349	8.821278	8.339179	0.995472

N_1	N_2	$\sigma^2_{y(2)}$	$\sigma^2_{x(2)}$	$\sigma^2_{U(2)}$	$\sigma^2_{V(2)}$	$\rho_{yx(2)}$
4500	500	97.02783	94.54578	22.80557	25.43263	0.994546
4250	750	98.27616	97.42674	23.27837	24.13829	0.994992
4000	1000	96.09359	94.71923	24.42978	23.03076	0.99467

N_1	N_2	$\sigma^2_{y(2)}$	$\sigma^2_{x(2)}$	$\sigma^2_{U(2)}$	$\sigma^2_{V(2)}$	$\rho_{yx(2)}$
4500	500	102.7504	101.2097	9.095136	8.8123	0.995045
4250	750	99.55993	99.49764	9.233619	8.805872	0.995314
4000	1000	105.4334	103.8947	9.277715	9.072151	0.995105

4.4 Population IV:

$$X = N(5,\ 10);;Y = X + N(0,\ 1);;y = Y + N(2,\ 5);;x = X + N(2,53);;N = 5000;;$$
$$\mu_Y = 4.961081;;\ \mu_X = 4.96178;;\ \sigma_y^2 = 102.2408;;\ \sigma_x^2 = 100.868;;\ \sigma_U^2 = 25.94111;;$$
$$\sigma_V^2 = 25.03951;;\ \rho_{yx} = 0.394221$$

N_1	N_2	$\sigma^2_{y(2)}$	$\sigma^2_{x(2)}$	$\sigma^2_{U(2)}$	$\sigma^2_{V(2)}$	$\rho_{yx(2)}$
4500	500	103.5361	102.1031	25.31099	22.84483	0.394622
4250	750	103.6790	102.7446	24.6859	26.12337	0.395036
4000	1000	100.1031	99.31665	25.80394	24.50468	0.394778

The following points are noted from the above table 1 as:

1. Under population I, there are nine different situations, for each situation the performance of the proposed estimator in terms of mean squared error (MSE) is best for different values of k as compared with other estimators.
2. For all situations, with small sample sizes the performance of the proposed estimator (T) is always best as compared with larger samples sizes.

The following points are noted from the table 2:

1. Performance of the proposed estimator (T) under all different situations is best for different values of k among all the considered estimators.
2. It is further noted that for large sample sizes, proposed estimator performance better than the other estimators.
3. The MSE of the estimators has increased with the increase in the value of k.

It is envisaged from the table 3 that the performance of the proposed estimator (T) is best as compared to other estimators in all situations. Further, with the increase in the value of k, the MSE of the estimators also increases. Also, the MSE of the proposed estimator (T) has increase with the increase in the sample sizes.
From table 4, it is envisaged that the performance of the proposed estimator (T) is better than the other considered estimators for all the different situations in population IV. For large values of sample sizes, the proposed estimator is best in terms of MSE for different values of k. Also, with the increase in the value of k, the proposed estimator increases in all the situations.

5 Conclusion

In this paper, we have considered the situation when there is non-response and measurement error on study as well as auxiliary variable for estimating the population mean of the study variable under double sampling scheme. Based on this situation, I have proposed estimator and studied its properties in terms of bias and mean squared error. Comparison of the proposed estimator with other estimators also obtained. Theoretically, conditions have been obtained where the proposed estimator performance better than the other considered estimators. Next, I have considered the simulated data for different situations in different four populations. It is shown in section 4 of the paper that the performance of the proposed estimator is best among the other estimators for all different situations. So, I recommend the proposed estimator for the situation where both the non-response and measurement error is present on study as well as auxiliary variable when the population mean of the auxiliary variable is unknown.

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Sunil Kumar is Assistant Professor of Statistics at Department of Statistics, University of Jammu (India). Earlier worked as Assistant Professor of Statistics at Alliance University, Bangalore (India), Visiting Scientist/Professor at Indian Statistical Institute, Kolkata (India), Department of Statistics, University of Jammu (India). He received the Ph. D. degree in ”Statistics” at Vikram University (India). He is referee and editorial member of several International journals in the frame of Statistics and Management studies. His main research interests are: sample survey, non-response, estimation theory, consumer behav- ior,latent class analysis and applications.

Sandeep Bhougal is Assistant Professor of Statistics at Department of Math- ematics, Shri Mata Vaishno Devi, University, Katra. (India). He is PhD in ”Statistics”. He has authored many research papers published in reputed Inter- national Journals. His main research interests are: sample survey, non-response, Inference.