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Design of Controller for a Higher Order System Without Using Model Reduction Methods

Pritesh Shah[1,*] *and Sudhir Agashe*[2]

[1] Symbiosis Institute of Technology, Symbiosis International University, Pune, India
[2] Department of Instrumentation and Control, College of Engineering, Pune, India

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Abstract: In the industry, many plants are described by higher order systems. Most of the time, higher order systems are approximated with the lower order system using model reduction method, and then the appropriate controllers are designed. In this paper, the controller for higher order system is designed without using model reduction methods. Instead, a fractional PID (FPID) controller is designed for higher order system. In simulations, ten different plants were examined, ranging from order 3 to order 7, with and without delay. The time-domain optimal tuning of higher order systems was carried out using integrated squared error (ISE) as the performance index. Results indicate that the controller for higher order system can be designed without model reduction methods by using FPID controller. The results of FPID controllers are also compared with classical PID controller. The FPID controller displayed robust performance; better gain and phase margin. The complementary sensitivity and sensitivity functions are better achieved with FPID controller. The FPID controller exhibits an iso-damping property (flat response around gain crossover frequency) for higher order systems.

Keywords: Higher order control, fractional calculus, fractional PID controller, model reduction methods, fractional order controller.

1 Introduction

Fractional calculus is one of the growing fields in the area of science and engineering. A fractional order controller is one of the applications of fractional calculus in the field of the control system. In this paper, controllers for higher order systems are designed using a fractional PID (FPID) controller without using model reduction methods.

The FPID controller contains two more parameters in comparison to a simple PID controller. In the FPID controller, the order of differential (λ) and integration (μ) are non-integer number, in other words, fractional order. Podlubny proposed this type of controller, and made a remark that this type of a controller will provide better response than simple PID controller for a fractional order system [1,2,3,4,5]. Three different generations of CRONE (Commande Robuste d'Ordre Non Entier meaning robust control of non-integer order) controller are also discussed in the literature as non-integer controller [6].

The higher order systems can be found in multifarious industries like oil, cement, chemical, pharmaceutical, aircraft system, atomic nuclear plant, flexible robot manipulator [7], quadrotor with a variable degree of freedom (DOF) [8], fuel injector and spark timing of automobiles, etc. [9]. Many times, nonlinear systems are linearized at different operating points, and turn out to be higher order systems [10,11]. Also, sometimes during modeling of the system, one may tend to get higher order system using first principle method or finite-element model [12,13]. Mostly, these models are inappropriate for many applications like analysis of system, optimization or control system design. In large-scale systems, the system complexity makes the computation impractical owing to memory and time limitations as well as the ill-conditioning [14]. Designing of the controller for this class of systems is always a challenging task.

If the order of the system is more than two, it is referred as higher order control system [15]. S. Das et al. [12] in 2011 designed fractional order controller with a model reduction method for higher order systems using optimization method. In 1998, B. Bandyopadhyay et al. proposed a technique for designing a stabilizing controller for the stable higher order system via its reduced order model. In 2014, RituRani et al. proposed fuzzy self-tuning PID controller for higher order system [16]. Even in 2007, Vaishnav et al. proposed fuzzy PID controller for this category of systems [17].

* Corresponding author e-mail: pritesh.ic@gmail.com

In most of these work, higher order systems are approximated to lower order system by means of model reduction methods like Pade approximation methods, Routh approximation techniques, model reduction by impulse/step error minimization, principal component analysis method [18], and balanced-truncation method [19]. These methods are preferred in the case of limited computational power, accuracy and storage capabilities. These reduced models need to prevent essential characteristics of the original system. However, it is not always possible to capture these characteristics. Hence, reduced order model is not appropriate for different applications if it is not capturing important characteristics. The disadvantage of some methods is that they do not guarantee stable reduced models even though the original system is stable. In this paper, higher order systems are designed using the FPID controller without using model reduction methods.

In this study, ten different plants are considered for the simulations with order 3 to 7 without delay and with a delay of 5 and 15 unit time. The complementary sensitivity function $T(s)$ and sensitivity function $C(s)$ for one of the plants have been carried out. The complementary sensitivity and sensitivity function indicate noise and disturbance rejection capabilities respectively. The FPID controller is designed using optimization method. The Nelder-Mead method is used for optimization of FPID controller parameters. In this approach, integrated squared error (ISE) has been used as a performance index for calculation of the cost function. The FPID controllers are providing much better responses compared to classical PID controller. The PID controller is designed using *PID tuner* application of MATLAB toolbox. The closed loop responses for all plants are plotted by varying the gain of the system (K), which shows iso-damping property of the controller. Iso-damping property means that the controller will demonstrate the same kind of response even after changing gain of the system in a certain range. In other words, it indicates robustness characteristics of the controller.

This paper is organized as follows. In Section 2, the introduction of fractional calculus and FPID controller with different tuning methods and stability analysis are presented briefly. In Section 3, the FPID controller is designed for higher order systems using optimization method for various plants. In Section 4, results are tabulated regarding FPID controller parameters, and their frequency specifications are also mentioned. The closed loop responses of higher order systems are plotted. Discussion of results is also included in this section. In the last Section, conclusions are made regarding this work, followed by references.

2 Introduction to Fractional Calculus and Fractional PID Controller

2.1 Fractional calculus

Fractional calculus is a field of mathematical study, which comes out of the traditional calculus of integral and derivative operators [20]. During last few decades, many researchers have worked in the different areas of science and engineering using fractional calculus [21,22,23].

There are various definitions of fractional differentiation-integral available in many books as well as research articles on fractional calculus. The Caputo definition is more popular for engineering applications [24,25,26], as it is a straightforward association between the type of the initial conditions and the type of the fractional derivative. As stated by I. Podlubny [1], this definition allows initial conditions such as $y''(0), y'(0)$ etc., not like fractional condition such as $y^{0.28}(0)$. Derivative of constant is bounded in the case of Caputo definition. This definition is defined as follows:

$$ {}_aD_t^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \tag{1} $$

where n is an integer number, which satisfies the condition $(n-1) \leq \alpha \leq n$, α is a real number and a and t are the limits of integration. For instant, if α is 0.9, then n would be 1 as $0 \leq 0.9 \leq 1$. $\Gamma(n)$ is the gamma function, which is defined by following equation:

$$ \Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx. \tag{2} $$

This function is not defined for value of n, which is not negative whole number and zero. This function is very beneficial for fractional calculus.

The Riemann-Liouville definition is given by:

$$ {}_aD_t^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \tag{3} $$

where n is an integer number, which satisfies the condition $(n-1) \leq \alpha \leq n$, α is real number, J is the integral operator and a and t are the limits of integration.

The Grunwald-Letnikov's definition is given by:

$$ {}_aD_t^{\alpha} = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{r=0}^{\left[\frac{t-a}{h}\right]} (-1)^r \binom{n}{r} f(t-rh), \tag{4}$$

where n is an integer number, which satisfies the condition $(n-1) \leq \alpha \leq n$, a and t are the limits of differentiation, h is the step size for differentiation, $[\frac{t-a}{h}]$ is integer part, and $\binom{n}{r}$ is the binomial coefficient.

2.2 Stability of Fractional Order System

A general fractional differentiation equation is given by:

$$ a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \cdots + a_0 D^{\alpha_0} y(t) = b_m D^{\beta_n} y(t) + b_{m-1} D^{\beta_{n-1}} y(t) + \cdots + b_0 D^{\beta_0} y(t), \tag{5}$$

where D^{α} denotes the fractional differentiation order with time limit of 0 to t. a_n, a_{n-1}, $\cdots$, a_0, b_m, b_{m-1}, $\cdots$, b_0 are real constants, and $\alpha_n, \alpha_{n-1}, \cdots, \alpha_0$,
$\beta_m, \beta_{m-1}, \cdots, \beta_0$ are positive real numbers.

The above equation can be converted into transfer function using Laplace transfer using following property [27],

$$ L\{D^{\gamma} x(t)\} = s^{\gamma} X(s);\ \text{if x(t) =0 } \forall t < 0. \tag{6}$$

By using this property, it can be presented in the following form of transfer function:

$$ G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + s_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} = \frac{Q(s^{\alpha})}{P(s^{\alpha})}. \tag{7}$$

Eq. (5) can be called commensurate order system if α_k and β_k are arithmetical progression with the same difference. Mathematically,

$$ \alpha_k = k \times \alpha; \text{where k=0,1,2,} \cdots, n $$

,

$$ \beta_k = k \times \alpha; \text{where k=0,1,2,} \cdots, m $$

and the value of α is between 0 to 1.

Theorem 1.*A commensurate order system is stable if and only if*

$$ |arg(\lambda_i)| > \pi/2; \forall i, \tag{8}$$

where λ_i is the i th root of $P(s^{\alpha})$.

Eq. (5) can be called incommensurate order, if α_k and β_k are not in the integer multiple. The incommensurate system can be decoupled by following model form of the fractional form:

$$ F(s) = \sum_{i=1}^{N} \sum_{k=1}^{n_k} \frac{A_{i,k}}{(s^{q_i} + \lambda_i)^k}, \tag{9}$$

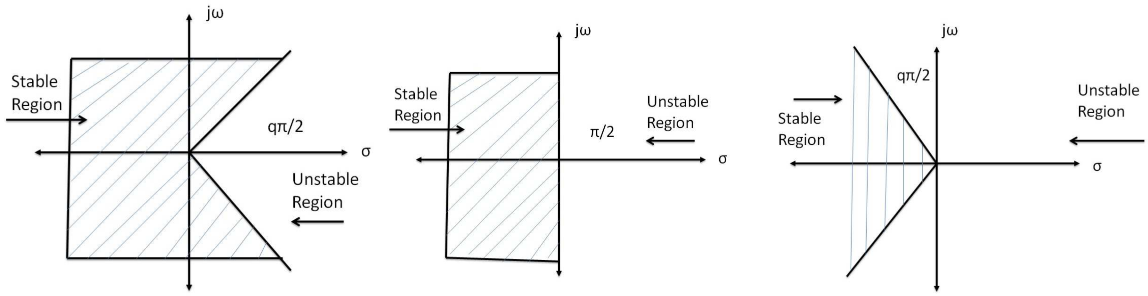
where n_k is the positive integer, N is the total roots of the pseudo-polynomial $P(s^{\alpha})$, and $A_{i,k}, \lambda_i$ are complex numbers.

Theorem 2.*The incommensurate order system is BIBO stable if and only if*

$$ 0 < q_i < 2 \tag{10}$$

$$ |arg(\lambda_i)| < pi(1 - q_i/2); \forall i. \tag{11}$$

The stability region for fractional order system (s^q) is shown in Fig. 1. The stability region depends upon the value of q. The stability of the fractional system can be computed using Riemann surface. The Riemann surface of $s^{1/4}$ is shown in Fig. 2, which shows four Riemann sheets in the complex plane.



(a) For $0 < q < 1$ **(b)** For q=1 **(c)** For $1 < q < 2$

Fig. 1: Stability region of the fractional order system.

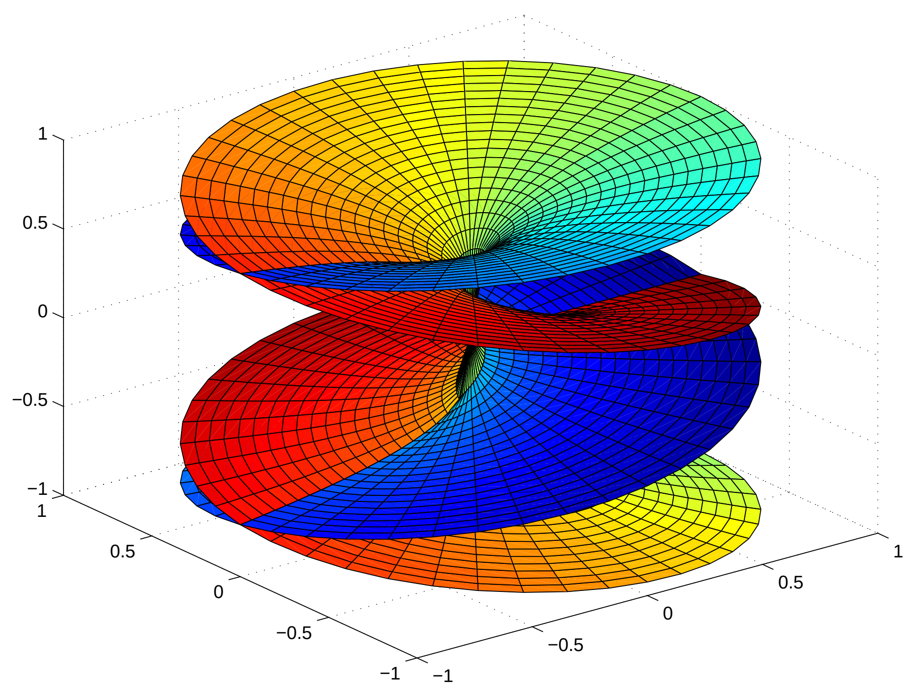


Fig. 2: Riemann surface for $s^{1/4}$.

2.3 Fractional PID Controller

A fractional order controller was proposed by I. Podlubny for fractional order systems [1,2]. The attractive feature of FPID controller is that it is less sensitive to changes of parameters of a controlled system and controller [28]. FPID controller has five parameters to tune as shown in following Eq. (12). Fig. 3 shows a block diagram of FPID controller. The structure of FPID controller is in the parallel form [29,30,31]

$$G_C(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s^\lambda} + K_D s^\mu; (0 \leq \lambda, \mu \leq 2), \tag{12}$$

where $G_C(s)$ is the controller transfer function, $U(s)$ is the Laplace of control signal, $E(s)$ is the Laplace of error signal, K_P is the proportional constant gain, K_I is the integration constant gain, K_D is the derivative constant gain, λ is the order of integration, and μ is the order of differentiator.

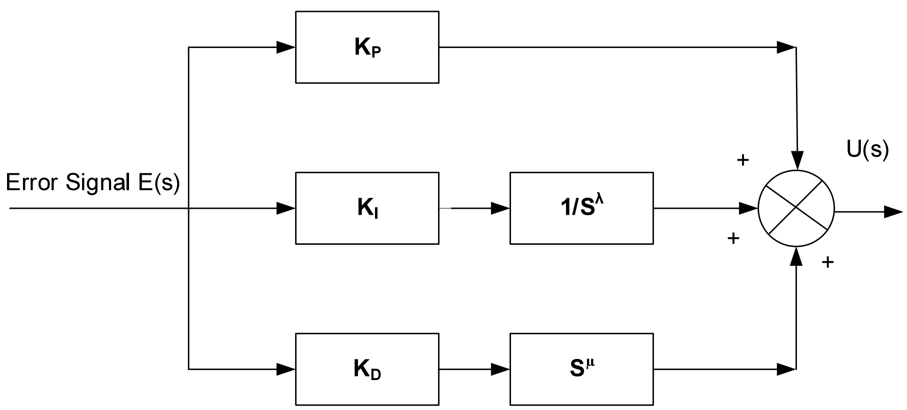


Fig. 3: Block diagram of FPID controller.

The fractional order system can be approximated by various methods [32,33,34]. Oustaloup recursive approximation is the most popular method for approximate fractional order [6,35,36]. It is given by:

$$s^v \approx K \prod_{k=-N}^{N} \frac{1 + s/\omega_k}{1 + s/\omega_k'}. \tag{13}$$

Above approximation equation can by calculated using following equations:

$$\omega_u = \sqrt{\omega_h \omega_b}$$

where ω_h, ω_b are the frequency bound for approximation.

$$\omega_0' = \alpha^{-0.5}\omega_u; \omega_0 = \alpha^{0.5}\omega_u;$$

$$\frac{\omega_{k+1}'}{\omega_k'} = \frac{\omega_{k+1}}{\omega_k} = \alpha\eta > 1,$$

$$\frac{\omega_{k+1}'}{\omega_k} = \eta > 0; \frac{\omega_k}{\omega_k'} = \alpha > 0$$

$$N = \frac{\log(\omega_N/\omega_0)}{\log(\alpha\eta)}.$$

FPID controller is also implemented in real time applications, using analog as well as digital approximation methods. The order of integrator and differentiator are in the range of 0 to 2 [37].

2.4 Tuning of Fractional PID Controller

Tuning of FPID controllers is challenging task, as it has two more parameters to tune as compared to classical PID controller. According to D. Valerio and J. Costa [38], tuning methods can be divided into three different categories as follow:

–Rule-based methods
–Analytical methods
–Numerical methods

Apart from the above methods, auto-tuning and internal model control (IMC) methods are also used for tuning of fractional order controllers [23].

Ziegler Nichols type rules for FPID controller were proposed by D. Valerio and J. Costa in 2006 [39]. This method is only applicable, where plant response is similar to S-shaped behavior for a step input. In [40], Ziegler-Nichols method was used for tuning the gain parameters of the FPID controller (K_P, K_I, K_D). Other parameters (λ, μ) were tuned manually by varying these parameters and observing the effect on the required specifications.

A tuning method for the fractional order system based on analytical method was proposed by C. Zhao et al. in 2005 [41]. Parameters of the FPID controller were obtained by solving equations that were obtained from the desired specifications. Results of this method were validated using two examples. The tuning method based on the specifications as well as the proposed auto-tuning method for FPID controller based on the relay test was described in 2007 by B. Vinagre et al. [42] and 2008 C. Monje et al.[23]. It allows the requirements of robustness constraints for the FPID system using simple relations among its parameters.

The tuning of fractional order controllers based on numerical methods has been proposed by many researchers. Based on a genetic algorithm [43], J. Cao et al. proposed tuning of FPID using integral of time-weighted absolute error (ITAE) and control input as a performance index. L. Chang and H. Chen [37] also suggested tuning of FPID using an adaptive genetic algorithm for the active magnetic bearing system. Based on the genetic algorithm [44], tuning rules have been developed using time domain performance index. The enhanced particle swarm optimization (PSO) method has also been used for designing of fractional order controllers [45,46,47]. The FPID controllers were also designed by different methods like an improved differential evolution optimization approach by A. Biswas et al. [48] in 2009, electromagnetism-like algorithm [49] by C. Lee and F. Chang, MIGO (M_s constrained integral gain optimization) method by Y. chen et al. [50] in 2008. In this work, numerical based method is used for designing of FPID controller, as mathematical models are available for higher order systems.

The auto-tuning method for FPID controller was developed based on relay [23] and modulus, phase and phase slope of the process [51]. IMC based tuning for the FPID controller was proposed by few researchers [52,53,54,55].

2.5 Performance of FPID Controller

The performance of FPID controller is demonstrated in this subsection. The first order transfer function plant is considered. It has following transfer function:

$$G_P(s) = \frac{K}{s\tau + 1} \tag{14}$$

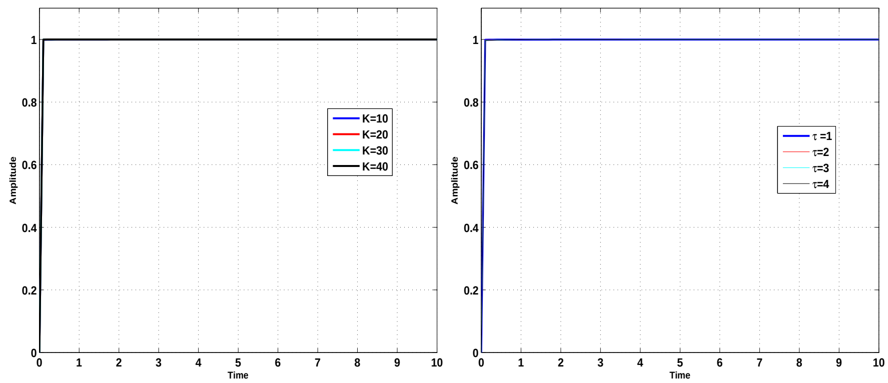
A FPID controller is designed for $K = 10$ and $\tau = 1$ using optimization method. The obtained parameters for FPID controller are $K_P = 100, K_I = 100, K_D = 100, \lambda = 0.01$ and $\mu = 0.90$. The values of K are varied between 10 to 40. Similarly, the values of τ are varied between 1 to 4. The closed loop responses are plotted in Fig. 4. The closed loop responses show similar kind of behaviors for different values of K and τ.

3 Design of Fractional PID Controller for Higher Order Systems

A general transfer function of higher order system is given by:

$$G_P(s) = \frac{a_0 + a_1 s + \cdots + a_{m-1}s^{m-1} + a_m s^m}{b_0 + b_1 s + \cdots + b_{n-1}s^{n-1} + b_n s^n} \tag{15}$$

$$G_P(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}, \tag{16}$$



(a) Different values of K **(b)** different values of τ

Fig. 4: Response of the FPID controller for different values of K and τ.

where a_i and b_i are scalars, $n > m$ and $n > 2$.

The various plants are considered for designing of FPID controller. There test bench process plants are referred from [12, 56, 57]. In these plants, order of systems is varying from 3 to 7 with and without delay (plant 2 and plant 9).

$$P_1(s) = \frac{1}{s(s+1)^3} \tag{17}$$

$$P_2(s) = \frac{e^{-5s}}{(s+1)^3} \tag{18}$$

$$P_3(s) = \frac{1}{s(s+1)(1+0.2s)(1+0.04s)(1+0.0008s)} \tag{19}$$

$$P_4(s) = \frac{1}{(s+1)^4} \tag{20}$$

$$P_5(s) = \frac{1}{(s+1)^5} \tag{21}$$

$$P_6(s) = \frac{1}{(s+1)^6} \tag{22}$$

$$P_7(s) = \frac{1}{(s+1)^7} \tag{23}$$

$$P_8(s) = \frac{1-2s}{(s+1)^3} \tag{24}$$

$$P_9(s) = \frac{e^{-15s}}{(s+1)^3} \tag{25}$$

$$P_{10}(s) = \frac{9}{(s+1)(s^2+2s+9)} \tag{26}$$

The step responses of above higher order systems are plotted in Fig. 5, and open loop frequency responses are tabulated in Table 1. The plant 1 and plant 3 are integrating higher order system. Plant 8 has one zero at left hand side of s-plane.

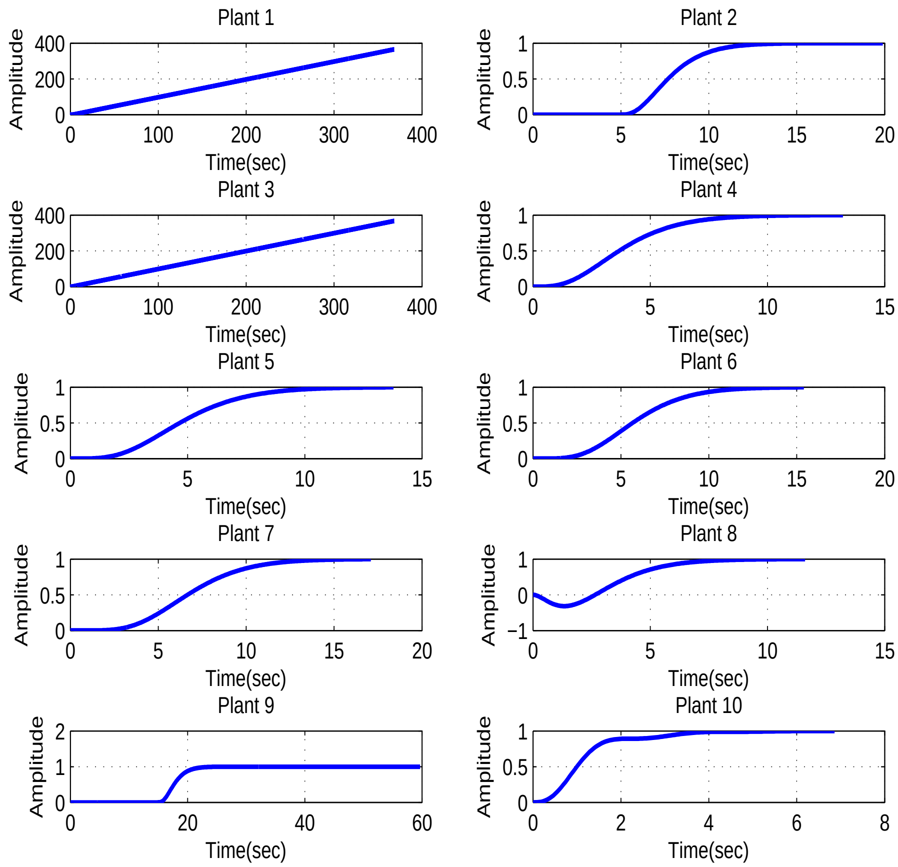


Fig. 5: Step responses of higher order systems.

Table 1: Open loop frequency response.

Plant No	**GM in dB**	**PM in deg**	ω_{pc} **rad/sec**	ω_{gc} **rad/sec**
P_1	-1.02	-4.97	0.577	0.617
P_2	1.93	-180	0.4	0
P_3	13.4	41.1	1.97	0.779
P_4	12	-180	1	0
P_5	9.2	-180	0.727	0
P_6	7.5	-180	0.577	0
P_7	6.34	-180	0.481	0
P_8	1.16	45.8	0.845	0.55
P_9	0.392	-180	0.175	0
P_{10}	8.52	-180	3.32	0

3.1 FPID Design for Without Delay System

The parameters of FPID controller are obtained using numerical method, as this method is suitable for all kinds of process model. The ISE performance index is considered because it penalizes highly for significant error. It can also easily discriminate between excessively overdamped and underdamped system. It is calculated by following equation:

$$ISE = \int_0^t e^2(t)\ \mathrm{dt} \tag{27}$$

where $e(t)$ is the error signal, and it is given for unity feedback system considering unit step input,

$$e(t) = 1 - L^{-1}\left(\frac{1}{s}\frac{G_P(s)G_C(s)}{1+G_P(s)G_C(s)}\right). \tag{28}$$

Note: $L^{-1}\{F(s)\}$ represents the inverse Laplace transform of $F(s)$.

In optimization, following constrains have been considered [58,59],

$$0 \le K_P, K_I, K_D \le 20 \text{ and } 0 \le \lambda, \mu \le 2. \tag{29}$$

Also, this cost function is subjected to following specifications.

1. Gain margin (GM) means in-system gain changes that make the system marginally stable. It is defined as

$$GM \text{ in dB} = 20 \times log_{10}\frac{1}{|G_C(j\omega_P)G_P(j\omega_P)|}, \tag{30}$$

where G_C is the controller transfer function, and G_P is the plant transfer function.

2. A phase crossover frequency ω_P is given by:

$$\arg[G_C(j\omega_P)G_P(j\omega_P)] = -\pi. \tag{31}$$

3. Phase margin (ϕ_m) is the amount of phase shift at frequency ω_{gc} that would be needed to produce instability. It is given by:

$$\phi_m = \arg[G_C(j\omega_{gc})G_P(j\omega_{gc})] + \pi \tag{32}$$

4. Gain crossover frequency ω_{gc} is given by

$$|G_C(j\omega_{gc})G_P(j\omega_{gc})| = 1 \tag{33}$$

5. Complementary sensitivity function $T(s)$ is indication of noise rejection ratio, which maps the noise input to the output, and it is given by

$$T(s) = \frac{G_P(s)G_C(s)}{1+G_P(s)G_C(s)}. \tag{34}$$

The frequency noise rejection with X, the desired noise attenuation for frequencies $\omega \ge \omega_t$ rad/sec is given by:

$$|T(j\omega)| \le X dB. \tag{35}$$

6. Sensitivity function $S(s)$ is indication of disturbance rejection ratio, which maps the disturbance to the output, and it is given by:

$$S(s) = \frac{1}{1+G_P(s)G_C(s)}. \tag{36}$$

To ensure good output disturbance rejection with Y, the desired noise attenuation for frequencies $\omega \le \omega_s$ rad/sec is given by:

$$|S(j\omega)| \le Y dB. \tag{37}$$

Nelder-Mean method is used for tuning of the FPID controller for higher order systems [60]. It is a heuristic search method, which converges to non-stationary solution. This method was proposed by John Nelder & Roger Mead (1965). It is based on concept of simplex approach (sort, reflection, expansion, contraction, shrink).

3.2 Design for Delay System

For designing of delay system of higher order system, smith predictor structure is used for control purpose [61]. It is one of the best known schemes for controlling systems with time-delay. It is a feedback control scheme that has an inner loop as well as an outer loop. The inner loop functions to eliminate the actual delayed output as well as to feed the predicted output to the controller. By using this scheme, it is possible to design the controller assuming no time-delay in the control loop. The model of smith predicator is shown in Fig. 6. For delay system, the controller is designed using same method as described for without delay system.

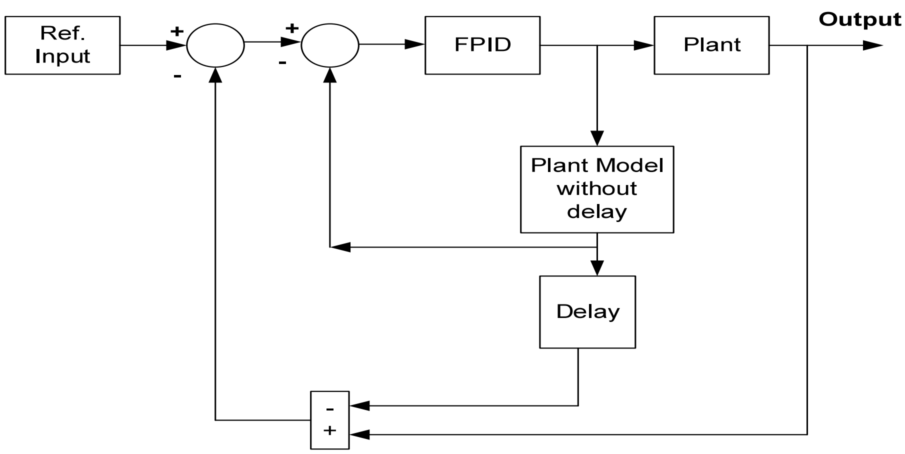


Fig. 6: Smith predictor model for time delay system.

4 Results and Discussions

A fractional PID controller is designed with following specifications using method described in previous section.

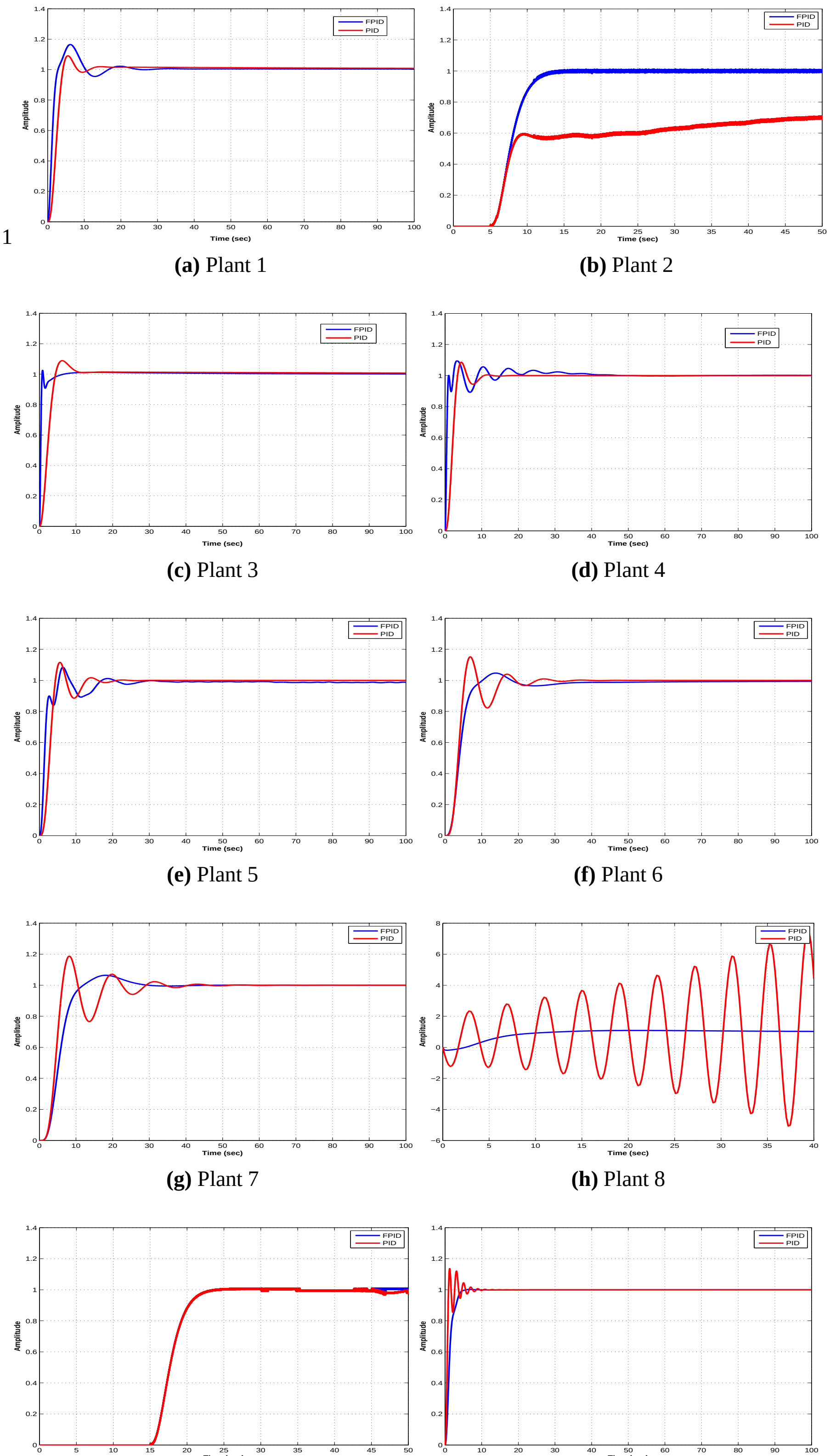
–Minimize ISE
–GM $\geq$ 10 dB
–$\phi_m \geq$ 45 deg
–$T(s)+S(s) \approx 1$
–$\omega_{gc} \geq$ 0.1 rad/sec
–$\omega_{pc} \geq$ 0.5 rad/sec

The closed loop responses of systems are plotted in Fig. 7. In addition, performance of FPID controller is compared with classical PID controller. The classical PID controllers are tuned using *PID tuner* application in MATLAB version of R2014a. The results of this controller are tabulated in Table 2 and 3. The phase margin (ϕ_m) and damping ratio (ζ) are correlated with each other. For robust performance, phase margin should be around 45 $^\circ$. Similarly, gain margin is also required to be 10 to 20 dB. For all the plants, the value of gain and phase margins are better. The responses of FPID controllers are better than classical PID controller for most of the plants. The FPID controllers have very less overshoot and quick response compared to classical PID controller. For plant 8 (one zero at left hand side of s-plane), the response of classical PID controller is unstable, interestingly, a FPID controller response is stable for plant 8.

For one of the plants, the complementary sensitivity transfer function and complementary sensitivity function are shown in Fig. 8. It is showing good noise and disturbance rejection characteristics of the FPID controller.

Using smith predictor, design of controller for time delay systems become simple and straight forward. The smith predictor structure also works for higher order system with delay.

The variation of gain (K) is introduced for all the plants up to 50 %. For different values of K, the closed loop responses of higher order systems are shown in Fig. 9. It yields the iso-damping property (flat response around gain



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(a) Plant 1 **(b)** Plant 2

(c) Plant 3 **(d)** Plant 4

(e) Plant 5 **(f)** Plant 6

(g) Plant 7 **(h)** Plant 8

(i) Plant 9 **(j)** Plant 10

Fig. 7: FPID and PID controller responses for higher order system.

Table 2: Design specifications of system with FPID controller.

Plant	Design specifications			
	ϕ_m deg.	ω_{gc} rad/sec	GM in dB	ω_{pc} rad/sec
P_1	60.3	0.562	21.8	1.87
P_2	60.2	1.5	19.3	4.96
P_3	60	3.07	17.3	10.7
P_4	60	0.966	28.2	15.6
P_5	59.8	0.512	10.1	2.17
P_6	60.1	0.142	10	0.839
P_7	60	0.178	10	0.622
P_8	59.9	0.161	10	0.826
P_9	60.2	1.5	19.3	4.96
P_{10}	75.8	0.725	9.99	3.02

Table 3: Parameters for FPID controller

Plant	FPID controller parameters with cost function					
	K_P	K_I	λ	K_D	μ	ISE
P_1	1.3498	0.0094982	1.0023	2.8752	1.5642	1.00159
P_2	0.23594	1.7476	0.98745	5.1258	0.63837	0.65466
P_3	1.647	0.041962	0.9997	3.7827	0.99995	0.295583
P_4	15.387	0.50503	1.529	14.249	1.83332	0.39317
P_5	4.0416	0.36081	1.1254	7.6198	1.6718	1.18302
P_6	0.70904	0.35967	0.88653	2.1096	1.0008	3.18
P_7	0.72433	0.20129	1.0003	1.6126	0.99975	5.18651
P_8	0.51498	0.11961	1.1597	0.23558	1.4822	5.4517
P_9	0.23594	1.7476	0.98745	5.1258	0.63837	0.65466
P_{10}	0.45347	0.76373	0.98988	0.16165	0.18618	0.89986

crossover frequency) for higher order systems. In other words, the overshoots of the closed-loop step responses will remain almost constant for different values of the gain. The iso-damping property can be described by

$$\left(\frac{d}{d\omega}(Arg[G(j\omega)])\right)_{\omega=\omega_{gc}} = 0. \tag{38}$$

5 Conclusion

The FPID controllers are designed for higher order systems without using any model reduction methods. Different plants are simulated ranging from order 3 to 7 without delay and with delay. The closed loop responses are also compared with classical PID controller. The FPID controllers show better response regarding less overshoot, better transient responses, as well as gain and phase margin. In addition, the FPID controller has better noise and disturbance rejection ratio.

For higher order control system, the FPID controller exhibited iso-damping property. It shows that the FPID controller is a kind of robust controller for higher order system.

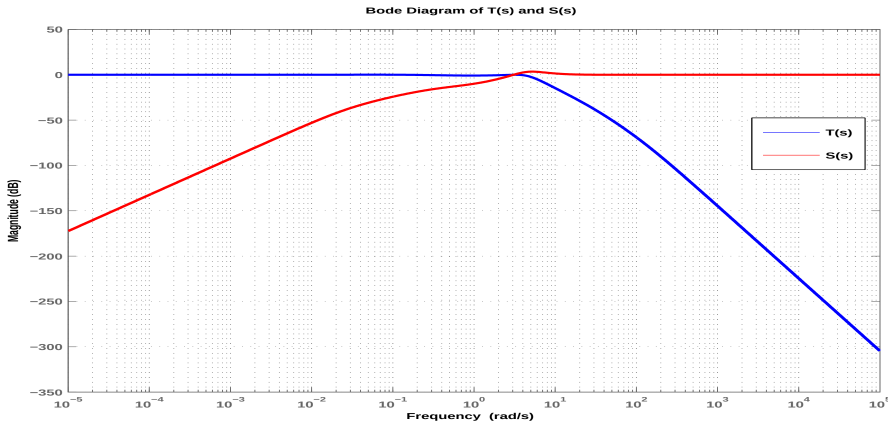

Fig. 8: Complementary and sensitivity function for plant 3.

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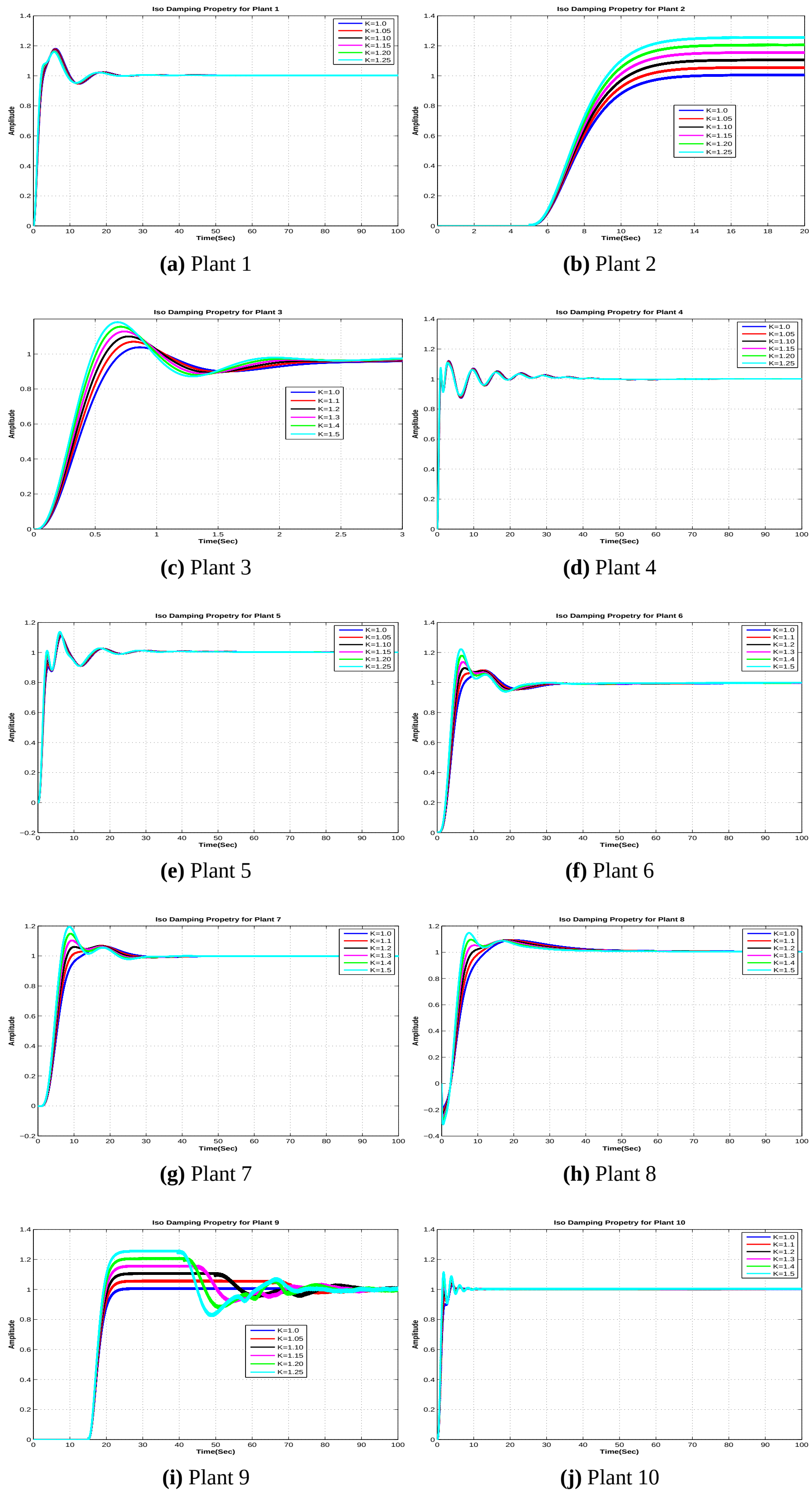

(a) Plant 1 **(b)** Plant 2

(c) Plant 3 **(d)** Plant 4

(e) Plant 5 **(f)** Plant 6

(g) Plant 7 **(h)** Plant 8

(i) Plant 9 **(j)** Plant 10

Fig. 9: Iso damping properties for higher order systems.

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