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Preference of Priors for the Generalized Inverse Rayleigh distribution under Different Loss Functions

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Abstract: The main objective of our research problem is to study the Bayesian analysis of the unknown scale parameter of Generalized Inverse Rayleigh distribution under non-informative and informative priors via simulated and real life data. For the posterior distribution of this parameter, we consider Uniform, inverse Levy, inverse Gamma and Gumbel type II priors. The four loss functions taken up are Squared Error Loss function, Quadratic Loss Function, LINEX loss function and Entropy Loss function. Mean square error simulations are performed to compare the performance of these Bayes estimates under various priors and Loss functions in R Software.

Keywords: Bayesian Analysis, GIRD, Non-Informative and Informative Priors, Loss functions, R- Software.

1 Introduction

In probability and statistics, the Generalized Inverse Rayleigh distribution (GIRD) is commonly used to study the life of electronic components and wind speed. Similarly, in physics and signal processing, it is used in the study of various types of radiations, sounds and lights. Due to its diverse applications in different fields and special case of the Weibull distribution, many statisticians consider it for different types of data sets. It is widely used in communication engineering, reliability analysis and applied statistics. Various applications of this distribution are given in Siddiqui (1962). It presents a flexible family in the varieties of shapes and is suitable for modeling data with different types of hazard rate function: increasing, decreasing and upside down bathtub shape. The Generalized Inverse Rayleigh distribution (GIRD) has a nice connection with other distributions like Gamma, Weibull and Exponential distributions. The probability density function (p.d.f.) of the G*IRD* with scale parameter θ is

$$f(x;\theta,k)=\frac{k}{\theta^{\frac{1}{k}}\Gamma\left(\frac{1}{k}\right)}e^{\frac{-x^k}{\theta}} \qquad for \quad x\geq 0, k\; and\; \theta>0 \tag{1.1}$$
$$=0, otherwise$$

Where k and θ are shape and scale parameters respectively. Inferences for the Inverse Rayleigh distribution have been discussed by a number of authors. Dey and Das (2007), Howlader and Khan, Dey and Maiti (2012) derived Baye's estimator of the Rayleigh parameter and its associated risk based on extended Jeffrey's prior. Kazmiet al (2012*)* compare the class of life time distributions for Bayesian analysis. They studied properties of Bayes estimators of the parameter using different loss functions via simulated and real life data. Reshi et al. (2014) considered the estimation of scale parameter of Generalized Inverse Rayleigh distribution. They obtained Bayes estimator for parameters of GIR distribution by using Jeffrey's and extension of Jeffrey's prior under squared error loss function, Al-Bayyati's loss function and precautionaryloss function. They also compared the classical method with Bayesian method by using mean square error through simulation study with varying sample sizes. Recently Afaq et al. (2015) compare the priors for the exponentiated exponential distribution under different loss functions.

2 Likelihood Function for the Generalised Inverse Rayleigh Distribution

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Let $\underline{x} = x_1, x_2, ..., x_n$ be a random sample of size n taken from the Generalized Inverse Rayleigh distribution. Then the likelihood function for the given sample observation is

$$L(x,\theta) = \frac{k^n}{\theta^{\frac{n}{k}} \Gamma\left(\frac{1}{k}\right)^n} e^{\frac{-\sum_{i=1}^{n} x_i^k}{\theta}} \tag{2.1}$$

3 Prior and Posterior Distributions

The Bayesian deduction requires appropriate choice of priors for the parameters. Arnold & Press (1983) pointed out that, from a Bayesian viewpoint, there is clearly no way in which one can say that one prior is better than any other. Presumably one has one's own subjective prior and must live with all of its lumps and bumps. But if we have enough information about the parameter(s) then it is better to make use of the informative prior(s) which may certainly be preferred over all other choices. Otherwise it may be suitable to alternative to use non informative or vague priors (see Uppadhyay et al. (2001), Singpurwalla (2006)).In this paper, we consider three informative priors and one non-informative prior were used to derive the corresponding posterior distributions.

3.1 Posterior distribution under the assumption of uniform prior

The standard Uniform distribution is assumed as non-informative prior for the parameter θ. The Uniform prior for θ is

$$p_1(\theta) \propto 1, \qquad 0 < \theta < \infty \ , \tag{3.1}$$

Using the likelihood function (2.1) and the prior (3.1), the posterior distribution for the parameter θ takes the form

$$p_1(\theta \mid \underline{x}) \propto L(x,\theta)\, p_1(\theta),$$

$$p_1(\theta \mid \underline{x}) \propto \frac{k^n}{\theta^{\frac{n}{k}} \Gamma(\frac{1}{k})^n} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i^k} ,$$

Hence the posterior density of θ is given as

$$p_1(\theta \mid \underline{x}) = \frac{T^{\frac{n}{k}-1}}{\Gamma(\frac{n}{k}-1)\theta^{(\frac{n}{k})}} e^{-\frac{T}{\theta}} \ , \ \theta > 0 \ . \tag{3.2}$$

$(\frac{n}{k}-1)$ *and* $\sum_{i=1}^{n} x_i^k = T$ are the parameters of the posterior distribution similar to the inverted gamma distribution $G(T, \frac{n}{k}-1)$

3.2 Posterior distribution under the assumption of Inverse levy prior

Second prior is assumed to be inverse levy prior of θ with hyper parameter'd'

$$p_2(\theta) \propto \sqrt{\frac{d}{2\pi}}\, \theta^{\frac{-3}{2}} e^{\frac{-d}{2\theta}} \quad , \quad d > 0, \ \theta > 0, \tag{3.3}$$

Combining the prior distribution (3.3) and the likelihood function (2.1), the posterior distribution for θ takes the form

$$p_2(\theta\,|\,\underline{x}) \propto \frac{k^n}{\Gamma(\frac{1}{k})^n\ \theta^{\left(\frac{n}{k}+\frac{1}{2}+1\right)}} e^{\frac{-\sum_{i=1}^{n} x_i^{\,k}+\frac{d}{2}}{\theta}},$$

Hence the posterior density of θ is given as

$$p_2(\theta\,|\,\underline{x}) = \frac{T^{\left(\frac{n}{k}+\frac{1}{2}\right)}}{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)\theta^{(\frac{n}{k}+\frac{1}{2}+1)}} e^{-\frac{T}{\theta}}, \quad \theta > 0\,. \tag{3.4}$$

Which is the kernel density of the inverted gamma distribution with parameters $\left(\frac{n}{k}+\frac{1}{2}\right)$ $and \sum_{i=1}^{n} x_i^{\,k} + \frac{d}{2} = T$. So the posterior distribution of the parameter θ is the inverted gamma distribution $G\left(T, \frac{n}{k}+\frac{1}{2}\right)$

3.3 Posterior distribution under the assumption of conjugate prior

The inverted gamma distribution is used as a conjugate prior for θ with hyper parameters a and b, which is also a conjugate prior for the generalized inverse Rayleigh distribution, so the prior distribution is

$$p_3(\theta) \propto \frac{a^b}{\Gamma b} e^{\frac{-a}{\theta}} \frac{1}{\theta^{(b+1)}}, \quad a, b, \theta > 0, \tag{3.5}$$

Combining the prior distribution (3.5) and the likelihood function (2.1), the posterior distribution for θ takes the form

$$p_3(\theta\,|\,\underline{x}) \propto \frac{a^b k^n}{\Gamma(b)\Gamma(\frac{1}{k})^n} e^{\frac{-\sum_{i=1}^{n} x_i^{\,k}+a}{\theta}} \frac{1}{\theta^{(\frac{n}{k}+b+1)}}, \quad a, b, \theta > 0,$$

Hence the posterior density of θ is given as

$$p_3(\theta\,|\,\underline{x}) = \frac{T^{(\frac{n}{k}+b)}}{\Gamma(\frac{n}{k}+b)\theta^{(\frac{n}{k}+b+1)}} e^{-\frac{T}{\theta}}, \quad \theta > 0\ . \tag{3.6}$$

Which is the kernel density of the inverted gamma distribution with parameters $(\frac{n}{k}+b)$ $and \sum_{i=1}^{n} x_i^{\,k} + a = T$. So the posterior distribution of the parameter θ is the inverted gamma distribution $G(T, \frac{n}{k}+a)$

3.4 Posterior distribution under the assumption of Gumbel Type II prior

It is assumed that the prior distribution of θ is the Gumbel Type II distribution with hyper parameters a_1 and b_1, which is given as under

$$p_4(\theta) \propto \frac{a_1 b_1}{\theta^{(b_1+1)}} e^{\frac{-a_1}{\theta^{b_1}}}, \quad a_1, b_1, \theta > 0, \tag{3.7}$$

As a special case, when $b_1 = 1,$ the prior is

$$p_4(\theta) \propto \frac{a_1}{\theta^2} \; e^{\frac{-a_1}{\theta}} , \qquad a_1, \theta > 0, \tag{3.8}$$

The posterior distribution of parameter θ for the given likelihood function (2.1) and prior (3.8), is

$$p_4(\theta \mid \underline{x}) \propto \frac{a_1 k^n}{\Gamma(\frac{1}{k})^n \; \theta^{(\frac{n}{k}+2)}} e^{\frac{-\sum_{i=1}^{n} x_i^{\,k} + a_1}{\theta}} ,$$

Hence the posterior density of θ is given as

$$p_4(\theta \mid \underline{x}) = \frac{T^{(\frac{n}{k}+1)}}{\Gamma(\frac{n}{k}+1)\theta^{(\frac{n}{k}+1+1)}} \; e^{-\frac{T}{\theta}} \;, \quad \theta > 0 \;. \tag{3.9}$$

Which is the kernel density of the inverted gamma distribution with parameters $(\frac{n}{k}+1)$ $and \sum_{i=1}^{n} x_i^{\,k} + a_1 = T$. So the posterior distribution of the parameter θ is the inverted gamma distribution $G(T, \frac{n}{k}+1)$

4 Bayesian Estimation under Different Loss Functions

A loss function should be an appropriate for the decision of problem under consideration. The selection of a loss function can be difficult and its choice is often made for reason of mathematical convenience, without any particular decision problem of current interest. This section presents the derivation of different loss functions for the posterior distributions derived under the priors. In order to estimate Bayes estimates, four loss functions i.e. squared error loss function, Quadratic loss function, LINEX loss function and Entropy loss function are used here.

4.1 Bayesian estimation by using uniform prior under different Loss Functions

4.1.1 Baye's estimator under SELF

By using squared error loss function $L(\hat{\theta},\theta) = c(\hat{\theta}-\theta)^2$ for some constant c the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty c(\hat{\theta}-\theta)^2 \; \frac{T^{\frac{n}{k}-1}}{\Gamma(\frac{n}{k}-1)\theta^{\frac{n}{k}}} \; e^{-\frac{T}{\theta}} \, d\theta$$

$$= \frac{cT^{\frac{n}{k}-1}}{\Gamma(\frac{n}{k}-1)} \left[\hat{\theta}^2 \int_0^\infty \frac{1}{\theta^{\frac{n}{k}-1+1}} e^{\frac{-T}{\theta}} d\theta - 2\hat{\theta} \int_0^\infty \frac{1}{\theta^{\frac{n}{k}-2+1}} e^{\frac{-T}{\theta}} d\theta + \int_0^\infty \frac{1}{\theta^{\frac{n}{k}-3+1}} e^{\frac{-T}{\theta}} d\theta \right]$$

$$= c\hat{\theta}^2 - 2c\hat{\theta} \frac{T}{(\frac{n}{k}-2)} + c \frac{T^2}{(\frac{n}{k}-2)(\frac{n}{k}-3)}$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta}_{1US} = \frac{T}{(\frac{n}{k}-2)} \quad . \tag{4.1}$$

4.1.2 Baye's estimator under QLF

By using quadratic loss function $L(\hat{\theta},\theta)=\left(\frac{\hat{\theta}-\theta}{\theta}\right)^2$, the risk function is given

$$R(\hat{\theta},\theta)=\int_0^\infty\left(\frac{\hat{\theta}-\theta}{\theta}\right)^2 \frac{T^{\frac{n}{k}-1}}{\Gamma(\frac{n}{k}-1)\theta^{\frac{n}{k}}}\, e^{-\frac{T}{\theta}}\, d\theta$$

$$=\frac{T^{\frac{n}{k}-1}}{\Gamma(\frac{n}{k}-1)}\left[\hat{\theta}^2\int_0^\infty \frac{1}{\theta^{\frac{n}{k}+1+1}} e^{-\frac{T}{\theta}} d\theta - 2\hat{\theta}\int_0^\infty \frac{1}{\theta^{\frac{n}{k}+1}} e^{\frac{-T}{\theta}} d\theta + \int_0^\infty \frac{1}{\theta^{\frac{n}{k}-1+1}} e^{\frac{-T}{\theta}} d\theta\right]$$

$$=\hat{\theta}^2\,\frac{\frac{n}{k}(\frac{n}{k}-1)}{T^2}-2\hat{\theta}\,\frac{(\frac{n}{k}-1)}{T}+1$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}_{1UQ}=\frac{T}{(\frac{n}{k})} \qquad (4.2)$$

4.1.3 Baye's estimator under linex loss function

By using linex loss function $l(\hat{\theta},\theta)=\exp\left\{c_1\left(\frac{\hat{\theta}}{\theta}-1\right)\right\}-c_1\left(\frac{\hat{\theta}}{\theta}-1\right)-1$ for some constant c_1 the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty\left(\exp\left\{c_1\left(\frac{\hat{\theta}}{\theta}-1\right)\right\}-c_1\left(\frac{\hat{\theta}}{\theta}-1\right)-1\right)\frac{T^{\frac{n}{k}-1}}{\Gamma(\frac{n}{k}-1)\theta^{\frac{n}{k}}}\, e^{\frac{-T}{\theta}}\, d\theta$$

$$=\frac{T^{\frac{n}{k}-1}}{\Gamma(\frac{n}{k}-1)}\left[e^{-c_1}\,\frac{\Gamma(\frac{n}{k}-1)}{(T-c_1\hat{\theta})^{\frac{n}{k}-1}}-c_1\hat{\theta}\,\frac{\Gamma(\frac{n}{k})}{T^{\frac{n}{k}}}+c_1\,\frac{\Gamma(\frac{n}{k}-1)}{T^{\frac{n}{k}-1}}-\frac{\Gamma(\frac{n}{k}-1)}{T^{\frac{n}{k}-1}}\right]$$

$$=e^{-c_1}\left(\frac{T}{T-c_1\hat{\theta}}\right)^{\frac{n}{k}-1}-c_1\hat{\theta}\,\frac{(\frac{n}{k}-1)}{T}+c_1-1$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Baye's estimator as

$$\hat{\theta}_{1Ull}=\frac{T}{c_1}\left(1-e^{-\frac{c_1 k}{n}}\right). \qquad (4.3)$$

4.1.4 Baye's estimator under entropy loss function

By using entropy loss function $L(\delta)=c_2\left[\delta-\log\delta-1\right]$ for some constant c_2 the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty c_2\left(\delta-\log(\delta)-1\right)p_1(\theta\,|\,\underline{x})\,d\theta$$

$$=\int_0^\infty c_2\left(\frac{\hat{\theta}}{\theta}-\log\left(\frac{\hat{\theta}}{\theta}\right)-1\right)\frac{T^{\frac{n}{k}-1}}{\Gamma(\frac{n}{k}-1)}\frac{1}{\theta^{\frac{n}{k}}}\;e^{\frac{-T}{\theta}}\;d\theta$$

$$=c_2\left[\frac{\hat{\theta}(\frac{n}{k}-1)}{T}-\log(\hat{\theta})+\frac{\Gamma'(\frac{n}{k}-1)}{\Gamma(\frac{n}{k}-1)}-1\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}_{1UE}=\frac{T}{(\frac{n}{k}-1)}\;. \tag{4.4}$$

4.2 Bayesian estimation by using Inverse levy Prior under different Loss Functions

4.2.1 Baye's estimator under SELF

By using squared error loss function $L(\hat{\theta},\theta)=c(\hat{\theta}-\theta)^2$ for some constant c the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty c(\hat{\theta}-\theta)^2\frac{T^{\frac{n}{k}+\frac{1}{2}}}{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)\theta^{\frac{n}{k}+\frac{1}{2}+1}}\;e^{-\frac{T}{\theta}}\,d\theta$$

$$=\frac{c\,T^{\frac{n}{k}+\frac{1}{2}}}{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)}\left[\hat{\theta}^2\int_0^\infty\frac{1}{\theta^{\frac{n}{k}+\frac{1}{2}+1}}e^{-\frac{T}{\theta}}d\theta-2\hat{\theta}\int_0^\infty\frac{1}{\theta^{\frac{n}{k}-\frac{1}{2}+1}}\,e^{\frac{-T}{\theta}}\,d\theta+\int_0^\infty\frac{1}{\theta^{\frac{n}{k}-\frac{3}{2}+1}}\,e^{\frac{-T}{\theta}}\,d\theta\right]$$

$$=c\hat{\theta}^2-2c\hat{\theta}\frac{T}{\left(\frac{n}{k}-\frac{1}{2}\right)}+c\frac{T^2}{\left(\frac{n}{k}-\frac{1}{2}\right)\left(\frac{n}{k}-\frac{3}{2}\right)}$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}_{2lvS}=\frac{T}{\left(\frac{n}{k}-\frac{1}{2}\right)}\;. \tag{4.5}$$

4.2.2 Baye's estimator under QLF

By using quadratic loss function $L(\hat{\theta},\theta)=\left(\frac{\hat{\theta}-\theta}{\theta}\right)^2$, the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty\left(\frac{\hat{\theta}-\theta}{\theta}\right)^2\frac{T^{\frac{n}{k}+\frac{1}{2}}}{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)\theta^{\frac{n}{k}+\frac{1}{2}+1}}e^{-\frac{T}{\theta}}d\theta$$

$$=\frac{T^{\frac{n}{k}+\frac{1}{2}}}{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)\theta^{\frac{n}{k}+\frac{1}{2}+1}}\left[\hat{\theta}^2\int_0^\infty\frac{1}{\theta^{\frac{n}{k}+\frac{5}{2}+1}}e^{-\frac{T}{\theta}}d\theta-2\hat{\theta}\int_0^\infty\frac{1}{\theta^{\frac{n}{k}+\frac{3}{2}+1}}e^{\frac{-T}{\theta}}d\theta+\int_0^\infty\frac{1}{\theta^{\frac{n}{k}+\frac{1}{2}+1}}e^{\frac{-T}{\theta}}d\theta\right]$$

$$=\hat{\theta}^2\frac{\left(\frac{n}{k}+\frac{3}{2}\right)\left(\frac{n}{k}+\frac{1}{2}\right)}{T^2}-2\hat{\theta}\frac{\left(\frac{n}{k}+\frac{1}{2}\right)}{T}+1$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}_{2lvQ}=\frac{T}{\left(\frac{n}{k}+\frac{3}{2}\right)}\ . \tag{4.6}$$

4.2.3 Baye's estimator under linex loss function

By using linex loss function $l(\theta,\hat{\theta})=\exp\left\{c_1\left(\frac{\hat{\theta}}{\theta}-1\right)\right\}-c_1\left(\frac{\hat{\theta}}{\theta}-1\right)-1$ for some constant c_1 the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty\left(\exp\left\{c_1\left(\frac{\hat{\theta}}{\theta}-1\right)\right\}-c_1\left(\frac{\hat{\theta}}{\theta}-1\right)-1\right)\frac{T^{\frac{n}{k}+\frac{1}{2}}}{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)\theta^{\frac{n}{k}+\frac{1}{2}+1}}e^{\frac{-T}{\theta}}d\theta$$

$$=\frac{T^{\frac{n}{k}+\frac{1}{2}}}{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)}\left[e^{-c_1}\frac{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)}{(T-c_1\hat{\theta})^{\frac{n}{k}+\frac{1}{2}}}-c_1\hat{\theta}\frac{\Gamma\left(\frac{n}{k}+\frac{3}{2}\right)}{T^{\frac{n}{k}+\frac{3}{2}}}+c_1\frac{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)}{T^{\frac{n}{k}+\frac{1}{2}}}-\frac{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)}{T^{\frac{n}{k}+\frac{1}{2}}}\right]$$

$$=e^{-c_1}\left(\frac{T}{T-c_1\hat{\theta}}\right)^{\frac{n}{k}+\frac{1}{2}}-c_1\hat{\theta}\frac{\left(\frac{n}{k}+\frac{1}{2}\right)}{T}+c_1-1$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Baye's estimator as

$$\hat{\theta}_{2lvll} = \frac{T}{c_1}\left(1 - e^{\frac{-c_1}{\left(\frac{n}{k}+\frac{3}{2}\right)}}\right) \quad . \tag{4.7}$$

4.2.4 Baye's estimator under entropy loss function

By using entropy loss function $L(\delta) = c_2\left[\delta - \log\delta - 1\right]$ for some constant c_2 the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty c_2\left(\delta - \log(\delta) - 1\right) p_2(\theta\,|\,\underline{x})\, d\theta$$

$$= c_2 \frac{T^{\frac{n}{k}+\frac{1}{2}}}{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)} \int_0^\infty \left(\frac{\hat{\theta}}{\theta} - \log\left(\frac{\hat{\theta}}{\theta}\right) - 1\right) \frac{1}{\theta^{\frac{n}{k}+\frac{1}{2}+1}}\, e^{\frac{-T}{\theta}}\, d\theta$$

$$= c_2\left[\frac{\hat{\theta}\left(\frac{n}{k}+\frac{1}{2}\right)}{T} - \log(\hat{\theta}) + \frac{\Gamma'\left(\frac{n}{k}+\frac{1}{2}\right)}{\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)} - 1\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta}_{2lvE} = \frac{T}{\left(\frac{n}{k}+\frac{1}{2}\right)} \quad . \tag{4.8}$$

4.3 Bayesian estimation by using Inverse gamma Prior under different Loss Functions

4.3.1 Baye's estimator under SELF

By using squared error loss function $L(\hat{\theta},\theta) = c(\hat{\theta}-\theta)^2$ for some constant c the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty c(\hat{\theta}-\theta)^2 \frac{T^{\frac{n}{k}+b}}{\Gamma(\frac{n}{k}+b)\theta^{\frac{n}{k}+b+1}}\; e^{-\frac{T}{\theta}}\, d\theta$$

$$= \frac{c\,T^{\frac{n}{k}+b}}{\Gamma(\frac{n}{k}+b)}\left[\hat{\theta}^2\int_0^\infty \frac{1}{\theta^{\frac{n}{k}+b+1}} e^{-\frac{T}{\theta}} d\theta - 2\hat{\theta}\int_0^\infty \frac{1}{\theta^{\frac{n}{k}+b-1+1}} e^{\frac{-T}{\theta}} d\theta + \int_0^\infty \frac{1}{\theta^{\frac{n}{k}+b-2+1}} e^{\frac{-T}{\theta}} d\theta\right]$$

$$= c\hat{\theta}^2 - 2c\hat{\theta}\frac{T}{(\frac{n}{k}+b-1)} + c\frac{T^2}{(\frac{n}{k}+b-1)(\frac{n}{k}+b-2)}$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta}_{3CS} = \frac{T}{(\frac{n}{k}+b-1)}. \tag{4.9}$$

4.3.2 Baye's estimator under QLF

By using quadratic loss function $L(\hat{\theta},\theta)=\left(\frac{\hat{\theta}-\theta}{\theta}\right)^2$, the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty \left(\frac{\hat{\theta}-\theta}{\theta}\right)^2 \frac{T^{\frac{n}{k}+b}}{\Gamma(\frac{n}{k}+b)\theta^{\frac{n}{k}+b+1}}\, e^{-\frac{T}{\theta}}\, d\theta$$

$$=\hat{\theta}^2\frac{(\frac{n}{k}+b)(\frac{n}{k}+b+1)}{T^2}-2\hat{\theta}\frac{(\frac{n}{k}+b)}{T}+1$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}_{3CQ} = \frac{T}{(\frac{n}{k}+b+1)}. \tag{4.10}$$

4.3.3 Baye's estimator under linex loss function

By using linex loss function $l(\theta,\hat{\theta})=\exp\left\{c_1\left(\frac{\hat{\theta}}{\theta}-1\right)\right\}-c_1\left(\frac{\hat{\theta}}{\theta}-1\right)-1$ for some constant c_1 the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty\left(\exp\left\{c_1\left(\frac{\hat{\theta}}{\theta}-1\right)\right\}-c_1\left(\frac{\hat{\theta}}{\theta}-1\right)-1\right)\frac{T^{\frac{n}{k}+b}}{\Gamma(\frac{n}{k}+b)\theta^{\frac{n}{k}+b+1}}\, e^{\frac{-T}{\theta}}\, d\theta$$

$$=e^{-c_1}\left(\frac{T}{T-c_1\hat{\theta}}\right)^{\frac{n}{k}+b}-c_1\hat{\theta}\frac{(\frac{n}{k}+b)}{T}+c_1-1$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Baye's estimator as

$$\hat{\theta}_{3Cll} = \frac{T}{c_1}\left(1-e^{\frac{-c_1}{\frac{n}{k}+b+1}}\right). \tag{4.11}$$

4.3.4 Baye's estimator under entropy loss function

By using entropy loss function $L(\delta)=c_2[\delta-\log\delta-1]$ for some constant c_2 the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty c_2\left(\delta-\log(\delta)-1\right)p_3(\theta\,|\,\underline{x})\,d\theta$$

$$= c_2 \frac{T^{\frac{n}{k}+b}}{\Gamma(\frac{n}{k}+b)} \int_0^\infty \left(\frac{\hat{\theta}}{\theta} - \log\left(\frac{\hat{\theta}}{\theta}\right) - 1 \right) \frac{1}{\theta^{\frac{n}{k}+b+1}} e^{\frac{-T}{\theta}} \, d\theta$$

$$= c_2 \left[\frac{\hat{\theta}(\frac{n}{k}+b)}{T} - \log(\hat{\theta}) + \frac{\Gamma'(\frac{n}{k}+b)}{\Gamma(\frac{n}{k}+b)} - 1 \right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta}_{3CE} = \frac{T}{(\frac{n}{k}+b)}. \tag{4.12}$$

4.4 Bayesian estimation by using Gumbel Type II Prior under different Loss Functions

4.4.1 Baye's estimator under SELF

By using squared error loss function $L(\hat{\theta},\theta) = c(\hat{\theta}-\theta)^2$ for some constant c the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty c(\hat{\theta}-\theta)^2 \frac{T^{\frac{n}{k}+1}}{\Gamma(\frac{n}{k}+1)\theta^{\frac{n}{k}+1+1}} e^{-\frac{T}{\theta}} \, d\theta$$

$$= c\hat{\theta}^2 - 2c\hat{\theta}\frac{T}{(\frac{n}{k})} + c\frac{T^2}{(\frac{n}{k})(\frac{n}{k}-1)}$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_{4GS} = \frac{T}{(\frac{n}{k})}. \tag{4.13}$$

4.4.2 Baye's estimator under QLF

By using quadratic loss function $L(\hat{\theta},\theta) = \left(\frac{\hat{\theta}-\theta}{\theta}\right)^2$, the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty \left(\frac{\hat{\theta}-\theta}{\theta}\right)^2 \frac{T^{\frac{n}{k}+1}}{\Gamma(\frac{n}{k}+1)\theta^{\frac{n}{k}+1+1}} e^{-\frac{T}{\theta}} \, d\theta$$

$$= \hat{\theta}^2 \frac{(\frac{n}{k}+2)(\frac{n}{k}+1)}{T^2} - 2\hat{\theta}\frac{(\frac{n}{k}+1)}{T} + 1$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta}_{4GQ} = \frac{T}{(\frac{n}{k}+2)}. \qquad (4.14)$$

4.4.3 Baye's estimator under linex loss function

By using linex loss function $l(\theta,\hat{\theta}) = \exp\left\{c_1\left(\frac{\hat{\theta}}{\theta}-1\right)\right\} - c_1\left(\frac{\hat{\theta}}{\theta}-1\right) - 1$ for some constant c_1 the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty \left(\exp\left\{c_1\left(\frac{\hat{\theta}}{\theta}-1\right)\right\} - c_1\left(\frac{\hat{\theta}}{\theta}-1\right) - 1\right) \frac{T^{\frac{n}{k}+1}}{\Gamma(\frac{n}{k}+1)\theta^{\frac{n}{k}+1+1}} e^{\frac{-T}{\theta}} d\theta$$

$$= \frac{T^{\frac{n}{k}+1}}{\Gamma(\frac{n}{k}+1)}\left[e^{-c_1}\frac{\Gamma(\frac{n}{k}+1)}{(T-c_1\hat{\theta})^{\frac{n}{k}+1}} - c_1\hat{\theta}\frac{\Gamma(\frac{n}{k}+2)}{T^{\frac{n}{k}+2}} + c_1\frac{\Gamma(\frac{n}{k}+1)}{T^{\frac{n}{k}+1}} - \frac{\Gamma(\frac{n}{k}+1)}{T^{\frac{n}{k}+1}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_{4Gll} = \frac{T}{c_1}\left(1 - e^{\frac{-c_1}{\frac{n}{k}+2}}\right) \qquad (4.15)$$

4.4.4 Baye's estimator under entropy loss function

By using entropy loss function $L(\delta) = c_2[\delta - \log\delta - 1]$ for some constant c_2 the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty c_2\left(\frac{\hat{\theta}}{\theta} - \log\left(\frac{\hat{\theta}}{\theta}\right) - 1\right)\frac{T^{\frac{n}{k}+1}}{\Gamma(\frac{n}{k}+1)}\frac{1}{\theta^{\frac{n}{k}+1+1}} e^{\frac{-T}{\theta}} d\theta$$

$$= c_2\left[\frac{\hat{\theta}(\frac{n}{k}+1)}{T} - \log(\hat{\theta}) + \frac{\Gamma'(\frac{n}{k}+1)}{\Gamma(\frac{n}{k}+1)} - 1\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta}_{4GE} = \frac{T}{(\frac{n}{k}+1)}. \qquad (4.16)$$

5 Predictive Distributions

5.1 Prior predictive distribution

The prior predictive distribution or in other words the marginal distribution of an unobserved data value is the product of the prior for θ and the single variable p.d.f., integrating out this parameter. This makes intuitive sense as uncertainty in θ is averaged out to reveal a distribution for the data point. It is defined as:

$$p(y)=\int_0^\infty p(\theta)\,f(y;\theta)d\theta$$

Here Y is the random variable of the model with unknown parameter θ.

$$f(y;\theta,k)=\frac{k}{\theta^{\frac{1}{k}}\Gamma(\frac{1}{k})}e^{\frac{-y^k}{\theta}} \qquad for \quad y\geq 0, k\, and\, \theta>0$$
$$=0, otherwise \tag{5.1}$$

5.1.1 Prior predictive distribution using uniform prior (UP)

The prior predictive distribution using uniform prior for a random variable Y combining equation (3.1) and (5.1) is:

$$p_1(y)=\int_0^\infty 1\frac{k}{\theta^{\frac{1}{k}}\Gamma\left(\frac{1}{k}\right)}e^{\frac{-y^k}{\theta}}d\theta$$

$$p_1(y)=\frac{k}{(\frac{1}{k}-1)(y^k)^{\frac{1}{k}-1}}, \quad y>0\,. \tag{5.2}$$

5.1.2 Prior predictive distribution using Inverse levy prior (LP)

The prior predictive distribution using Inverse levy prior for a random variable Y combining equation (3.3) and (5.1) is:

$$p_2(y)=\int_0^\infty\sqrt{\frac{d}{2\pi}}\,\theta^{\frac{-3}{2}}e^{\frac{-d}{2\theta}}\;\frac{k}{\theta^{\frac{1}{k}}\Gamma\left(\frac{1}{k}\right)}e^{\frac{-y^k}{\theta}}d\theta$$

$$p_2(y)=\sqrt{\frac{d}{2\pi}}\,\frac{k\Gamma\left(\frac{1}{k}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{k}\right)(y^k+\frac{d}{2})^{\frac{1}{k}+\frac{1}{2}}}, \quad y>0. \tag{5.3}$$

5.1.3 Prior predictive distribution using Conjugate prior (CP)

The prior predictive distribution using conjugate prior for a random variable Y combining equation (3.5) and (5.1) is:

$$p_3(y)=\int_0^\infty\frac{a^b}{\Gamma(b)\theta^{\alpha+1}}e^{-\frac{a}{\theta}}\;\frac{k}{\theta^{\frac{1}{k}}\Gamma\left(\frac{1}{k}\right)}e^{\frac{-y^k}{\theta}}d\theta$$

$$p_3(y)=\frac{a^b k\Gamma\left(b+\frac{1}{k}\right)}{\Gamma(a)\Gamma\left(\frac{1}{k}\right)(y^k+a)^{b+\frac{1}{k}}} \qquad y>0 \tag{5.4}$$

5.1.4 Prior predictive distribution using Gumbel Type II prior (GP)

The prior predictive distribution using Gumbel Type II prior for a random variable Y combining equation (3.8) and (9.1) is:

$$p_4(y) = \int_0^\infty \frac{a_1}{\theta^2} e^{-\frac{a_1}{\theta}} \frac{k}{\theta^{\frac{1}{k}} \Gamma\left(\frac{1}{k}\right)} e^{\frac{-y^k}{\theta}} d\theta$$

$$p_4(y) = \frac{a_1 k \Gamma\left(1 + \frac{1}{k}\right)}{\Gamma\left(\frac{1}{k}\right)(y^k + a_1)^{1+\frac{1}{k}}}, \quad y > 0 . \tag{5.5}$$

5.2 Posterior predictive distributions

5.2.1 Posterior predictive distribution under Uniform prior

The posterior predictive distribution for $Y_{n+1} = y_{n+1}$ *given* $Y = y_1, y_2, \ldots, y_n$ under Uniform prior is

$$p_1(y_{n+1} / Y) = \int_0^\infty p_1(\theta / y) f(y; \theta, k) d\theta$$

$$= \frac{kT^{\frac{n}{k}-1}}{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\frac{n}{k}-1\right)} \left[\int_0^\infty \frac{1}{\theta^{\frac{n+1}{k}-1+1}} e^{\frac{-(T+y^k)}{\theta}} d\theta \right]$$

$$p_1(y_{n+1} / Y) = \frac{kT^{\frac{n}{k}-1} \Gamma\left(\frac{n+1}{k} - 1\right)}{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\frac{n}{k}-1\right)(T + y^k)^{\frac{n+1}{k}-1}}, \quad y > 0. \tag{5.6}$$

5.2.2 Posterior predictive distribution under Inverse Levy prior

The posterior predictive distribution for $Y_{n+1} = y_{n+1}$ *given* $Y = y_1, y_2, \ldots, y_n$ under Inverse Levy prior is

$$p_2(y_{n+1} / Y) = \int_0^\infty p_2(\theta / y) f(y; \theta, k) d\theta$$

$$= \frac{kT^{\left(\frac{n}{k}+\frac{1}{2}\right)}}{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)} \left[\int_0^\infty \frac{1}{\theta^{\frac{n+1}{k}+\frac{1}{2}+1}} e^{\frac{-(T+y^k)}{\theta}} d\theta \right]$$

$$p_2(y_{n+1}/Y)=\frac{kT^{\left(\frac{n}{k}+\frac{1}{2}\right)}\Gamma\left(\frac{n+1}{k}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\frac{n}{k}+\frac{1}{2}\right)(T+y^k)^{\frac{n+1}{k}+\frac{1}{2}}}\qquad y>0 \tag{5.7}$$

5.2.3 Posterior predictive distribution under Conjugate prior

The posterior predictive distribution for $Y_{n+1}=y_{n+1}\,given\;Y=y_1,y_2,\ldots,y_n$ under Conjugate prior is

$$p_3(y_{n+1}/Y)=\int_0^\infty p_3(\theta/y)f(y;\theta,k)d\theta$$

$$=\frac{kT^{\frac{n}{k}+b}}{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\frac{n}{k}+b\right)}\left[\int_0^\infty\frac{1}{\theta^{\frac{n+1}{k}+b+1}}e^{\frac{-(T+y^k)}{\theta}}d\theta\right]$$

$$p_3(y_{n+1}/Y)=\frac{kT^{\frac{n}{k}+b}\Gamma\left(\frac{n+1}{k}+b\right)}{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\frac{n}{k}+b\right)(T+y^k)^{\frac{n+1}{k}+b}},\qquad y>0. \tag{5.8}$$

5.2.4 Posterior predictive distribution under Gumbel Type II

The posterior predictive distribution for $Y_{n+1}=y_{n+1}\,given\;Y=y_1,y_2,\ldots,y_n$ under Gumbel Type II prior is

$$p_4(y_{n+1}/Y)=\int_0^\infty p_4(\theta/y)f(y;\theta,k)d\theta$$

$$=\frac{kT^{\frac{n}{k}+1}}{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\frac{n}{k}+1\right)}\left[\int_0^\infty\frac{1}{\theta^{\frac{n+1}{k}+1+1}}e^{\frac{-(T+y^k)}{\theta}}d\theta\right]$$

$$p_4(y_{n+1}/Y)=\frac{kT^{\frac{n}{k}+1}\Gamma\left(\frac{n+1}{k}+1\right)}{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\frac{n}{k}+1\right)(T+y^k)^{\frac{n+1}{k}+1}}\qquad y>0 \tag{5.9}$$

6 Simulation and real life example

6.1 Simulation Study

In our simulation study, we chose a sample size of n=25, 50 and 100 to represent small, medium and large data set. The scale parameter is estimated for Generalized Inverse Rayleigh distribution Bayesian using Uniform prior, inverse Levy prior, inverse Gamma prior and Gumbel Type II prior. For the scale parameter we have considered $\theta=1.0, 1.5\,\text{and}\,2.0$. The shape parameter k has been fixed at 0.5. The values of hyper parameters were

$a_1, a, d = 0.5 \& 1.0$ and $b = 1.0 \& 0.5$. The value of loss parameter $c_1 = \pm 0.5$. This was iterated 10000 times and the scale parameter for each method was calculated. A simulation study was conducted R-software to examine and compare the performance of the estimates for different sample sizes with different values of loss functions. The results are presented in tables for different selections of the parameters.

Table 1: Mean Squared Error for $\hat{\theta}$ under uniform prior

N	k	θ	$\hat{\theta}_{SL}$	$\hat{\theta}_{QL}$	$\hat{\theta}_{LL}$		$\hat{\theta}_{EL}$
					C_1=0.5	C_1= -0.5	
25	0.5	1.0	0.43445	0.45274	0.45495	0.45052	0.44374
		1.5	1.07149	1.11057	1.11529	1.10583	1.09134
		2.0	2.18142	2.24394	2.25148	2.23637	2.21321
50	0.5	1.0	0.41595	0.42520	0.42634	0.42406	0.42061
		1.5	1.06005	1.07956	1.08196	1.07716	1.06988
		2.0	2.16693	2.19819	2.202037	2.19435	2.18269
100	0.5	1.0	0.38244	0.38718	0.38777	0.38659	0.38482
		1.5	1.11213	1.12155	1.12272	1.12038	1.116864
		2.0	2.21917	2.23441	2.23630	2.23252	2.22682

Table 2: Mean Squared Error for $\hat{\theta}$ under inverse levy prior

N	k	θ	d	$\hat{\theta}_{SL}$	$\hat{\theta}_{QL}$	$\hat{\theta}_{LL}$		$\hat{\theta}_{EL}$
						C_1=0.5	C_1=-0.5	
25	0.5	1.0	0.5	0.4421311	0.4597413	0.4618737	0.4576001	0.4510669
			1.0	0.4911568	0.5077548	0.5097613	0.5057392	0.4995851
		1.5	0.5	1.074251	1.112109	1.116688	1.107511	1.093472
			1.0	1.014216	1.053331	1.058065	1.048576	1.034067
		2.0	0.5	2.167658	2.228687	2.236053	2.221285	2.19867
			1.0	2.200235	2.26048	2.267749	2.253175	2.230852
50	0.5	1.0	0.5	0.377033	0.386449	0.387609	0.385287	0.3817738
			1.0	0.416213	0.425318	0.426439	0.424195	0.420798
		1.5	0.5	1.050135	1.069465	1.071842	1.067083	1.059874
			1.0	1.064331	1.08351	1.08586	1.08114	1.07399
		2.0	0.5	2.11308	2.14456	2.14843	2.14068	2.12894
			1.0	2.17569	2.20638	2.21015	2.20260	2.19116
100	0.5	1.0	0.5	0.40483	0.40944	0.410020	0.408874	0.40714
			1.0	0.41273	0.41731	0.41788	0.41674	0.41503
		1.5	0.5	1.11622	1.12555	1.12671	1.12440	1.12091
			1.0	1.11195	1.12131	1.12246	1.12014	1.11664
		2.0	0.5	2.27088	2.28567	2.28750	2.28383	2.27830

Table 3: Mean Squared Error for $\hat{\theta}$ under inverse gamma prior

N	k	θ	a	b	$\hat{\theta}_{SL}$	$\hat{\theta}_{QL}$	$\hat{\theta}_{LL}$		$\hat{\theta}_{EL}$
							C_1=0.5	C_1=-0.5	
25	0.5	1.0	0.5	1.0	0.43998	0.45746	0.45958	0.45534	0.44885
			1.0	0.5	0.47741	0.49432	0.49636	0.49227	0.48600
		1.5	0.5	1.0	1.07350	1.11100	1.11554	1.10644	1.09253
			1.0	0.5	0.99427	1.03379	1.03857	1.02898	1.01432
		2.0	0.5	1.0	2.16846	2.22888	2.23617	2.22155	2.19916
			1.0	0.5	2.17065	2.23166	2.23902	2.22426	2.201659
50	0.5	1.0	0.5	1.0	0.37634	0.38572	0.38687	0.38456	0.381067
			1.0	0.5	0.40983	0.41899	0.42012	0.41786	0.41444
		1.5	0.5	1.0	1.04988	1.06912	1.07148	1.06675	1.05957
			1.0	0.5	1.054062	1.073352	1.07572	1.07097	1.06378
		2.0	0.5	1.0	2.11375	2.14507	2.14891	2.14121	2.12953
			1.0	0.5	2.16096	2.19184	2.19563	2.18804	2.17653
100	0.5	1.0	0.5	1.0	0.40440	0.40900	0.40957	0.40843	0.40671

			1.0	0.5	0.40953	0.41413	0.41470	0.41356	0.41184
		1.5	0.5	1.0	1.11592	1.12523	1.12639	1.12408	1.12060
			1.0	0.5	1.10668	1.11607	1.11724	1.11491	1.11140
		2.0	0.5	1.0	2.27082	2.28558	2.28741	2.28375	2.27823
			1.0	0.5	2.21534	2.23050	2.23238	2.22862	2.22295

Table4: Mean Squared Error for $\hat{\theta}$ under inverse Gumble Type II prior

N	k	θ	a_1	$\hat{\theta}_{SL}$	$\hat{\theta}_{QL}$	$\hat{\theta}_{LL}$		$\hat{\theta}_{EL}$
						C_1=0.5	C_1=-0.5	
25	0.5	1.0	0.5	0.43998	0.45746	0.45958	0.45534	0.44885
			1.0	0.48158	0.49823	0.50024	0.496208	0.49003
		1.5	0.5	1.07350	1.11100	1.11554	1.10644	1.09253
			1.0	1.00418	1.04311	1.04783	1.03838	1.02393
		2.0	0.5	2.16846	2.22888	2.23617	2.22155	2.19916
			1.0	2.18603	2.24605	2.25330	2.23877	2.21653
50	0.5	1.0	0.5	0.37634	0.38572	0.38687	0.38456	0.38106
			1.0	0.41211	0.42120	0.42232	0.42008	0.41669
		1.5	0.5	1.04988	1.06912	1.071486	1.06675	1.05957
			1.0	1.05889	1.07803	1.08038	1.07567	1.06853
		2.0	0.5	2.11375	2.14507	2.14891	2.14121	2.12953
			1.0	2.16872	2.19935	2.20311	2.19558	2.18416
100	0.5	1.0	0.5	0.40440	0.40901	0.40957	0.40843	0.40671
			1.0	0.41068	0.41526	0.41583	0.41469	0.41298
		1.5	0.5	1.11592	1.12523	1.12639	1.12408	1.12060
			1.0	1.10903	1.11839	1.11955	1.11723	1.11373
		2.0	0.5	2.27082	2.28558	2.28741	2.28375	2.27823
			1.0	2.21914	2.23424	2.23611	2.23236	2.22672

6.2 Real life example

The following data represent 39 liver cancer's patients taken from El Minia Cancer Center Ministry of Health in Egypt [see Attia et al. (2004)]. The ordered life times (in day) are: 10, 14, 14, 14, 14, 14, 15, 17,18, 20, 20, 20, 20, 20, 23, 23, 24, 26, 30, 30, 31, 40, 49, 51, 52, 60, 61, 67, 71, 74, 75, 87, 96, 105, 107, 107, 107, 116, 150.

The table below provides the posterior mean and posterior variance under four priors, viz. uniform prior, inverse Levy prior, inverse Gamma prior and GumbleTypeII prior

Table 5: Posterior Mean and Posterior Variance of a GIR Distribution using different priors

k	θ	***Hyper Parameters a_1=a =d***	***Hyper Parameter b***	***Mean/P.V***	***Uniform prior***	***Inverse Levy prior***	***Inverse gamma***	***Gumbel Type II***
0.5	1.0	0.5	1.0	**Mean**	3.33041	3.26918	3.25143	3.25143
				post.var	0.04382	0.04218	0.04168	0.04168
		1.0	0.5	**Mean**	3.33041	3.27241	3.27885	3.25784
				post.var	0.04382	0.04222	0.04231	0.04176

0.5	1.5	0.5	1.0	**Mean**	3.33041	3.26918	3.25143	3.25143
				post.var	0.04382	0.04218	0.04168	0.04168
		1.0	0.5	**Mean**	3.33041	3.27241	3.27885	3.25784
				post.var	0.04382	0.04222	0.04231	0.04176
0.5	2.0	0.5	1.0	**Mean**	3.33041	3.26918	3.25143	3.25143
				post.var	0.04382	0.04218	0.04168	0.04168
		1.0	0.5	**Mean**	3.33041	3.27241	3.27885	3.25784
				post.var	0.04382	0.04222	0.04231	0.04176

7 Results

i. From Tables 1to 4, the comparison of mean square error under different loss functions using non-informative as well as informative priors has been made through which we conclude that within each loss function informative prior (the Gumbel Type II prior) provides less mean square error so it is more suitable for the Generalized Inverse Rayleigh distribution and amongst loss functions, SELF loss function, is more preferable as compared to all other loss functions which are provided here because under this loss function mean square error is small for each and every value of parameter .

ii. The posterior mean and posterior variance under the assumed priors is calculated by assuming the different values of hyper parameters. From table 5, it is clear that the posterior variance under the Gumbel Type II prior are less as compared to other assumed priors, which shows that this prior is efficient as compared to other assumed priors and this less variation in posterior distribution helps in making more precise Bayesian estimation of the true unknown parameter θ of GIR distribution.

8 Conclusion

We consider the Bayesian analysis of the GIR distribution using different priors and different loss functions each in the worked example as well as in the simulation study. After analysis we conclude that the Gumble Type II (informative) is compatible for the unknown parameter θ of the GIR distribution and preferable over all other competitive priors because of having less posterior Variance. As far as choice of loss function is concerned, one can easily observe based on evidence of different properties as discussed above that squared error loss function has smaller mean square error. Further as we increase sample size mean square error comes down.

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