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Quantum Search with Superconducting Qubits in Cavity QED

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Abstract: We present an effective way to implement two-superconducting-qubit Grover's quantum search algorithm, based on the qubit-qubit interaction with Cavity Quantum Electrodynamics. In the scheme, the qubits in the cavity are not required to be addressed individually were driven simultaneously by a strong classical field, and all the two-qubit conditional phase operations required to achieve the two-qubit Grover quantum search can be implemented directly in a time (nanosecond-scale) much shorter than decoherence time and dephasing time (microsecond-scale) via cavity. Numerical simulation shows that our proposal is realizable with high fidelity. Moreover, we propose a detailed procedure and analyze the experimental feasibility. Finally, we generalize our Implementation of two-qubit Grover search to the case of multi-qubits Grover search. So, our proposal can be experimentally realized in the range of current cavity QED techniques.

Keywords: Grover algorithm, superconducting qubit, cavity QED, qubit-qubit interaction

1 Introduction

Over the past few years, much attention has been paid to quantum computers, which are based on the fundamental quantum mechanical superposition principle. Quantum computers would be more powerful than their classical counterparts [1]. The quantum algorithms work much more efficiently than their classical counter parts due to quantum superposition and quantum interference. For example, consider the search of an one item among N items stored in an unsorted database. The database can be accessed by an "oracle", a "blackbox" comparing any item with the searched one. It gives the "yes" answer when the items match, "no" otherwise. The most efficient classical algorithm is to examine items one by one until the blackbox returns "yes". Classical computation requires $O(N)$ steps to carry out the search. However, In the search algorithm, multiple items are simultaneously examined using a superposition of the corresponding states. The search for the marked item requires only $O(\sqrt{N})$ steps [2,3]. Thus, Grover's algorithm represents a quadratic advantage over its classical counterpart. The efficiency of this algorithm has been tested experimentally in few-qubit cases by NMR [4,5], superconducting qubits [6,7] and by atom cavity QED systems [8].

Recently, Jiang [9] proposed a simple scheme to implement the two-qubit Grover search algorithm with trapped ions in thermal motion by applying a single standing-wave laser pulse during the two-qubit operation. Similarly, Wang et al. [10] proposed a scheme to implement two-qubit Grover quantum search by using dipole-dipole interaction (DDI) and the atom-cavity interaction (ACI) in cavity Quantum Electrodynamics (QED), driven simultaneously by a strong classical field. In Ref. [11], the authors presented a method to implement the Deutsch-Jozsa algorithm based on two atom interaction in a thermal cavity. The photon-number-dependent parts in the evolution operator are canceled with the strong resonant classical field added. Besides, large detuning between the atoms and the cavity is not necessary neither, leading to potential speed up of quantum operation.

The goal of this work is to implement two-superconducting-qubit Grover's algorithm using in cavity-QED. We propose a scheme of implementing this algorithm based by introducing the qubit-qubit interaction

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using the diffusion transform D. Moreover, the scheme is insensitive to both the cavity decay and the thermal field because the photon-number-dependent parts in the evolution operator are canceled with the assistance of a strong classical field. The Grover search algorithm can be implemented in a time much smaller than decoherence time and dephasing time in cavity QED. Numerical simulation under the implementation shows that the scheme could be achieved efficiently within current state-of-the-art technology.

The paper is organized as follows: In Sec.2, we concretely illustrate the way to implement the two-superconducting-qubit Grover quantum search using the qubit-qubit interaction in the cavity QED. In Sec.3, we study the fidelity of this implementation. In Sec.4, we calculate the implementation time and discuss the result. In Sec.5, we generalize our Implementation of two-qubit Grover search to the case of multi-qubits Grover search. A concluding summary is given in Sec.6.

2 Implementation of two-superconducting-qubit Grover's search algorithm

The superconducting charge qubit consists of a small box, connected to a symmetric superconducting quantum interference device (SQUID) with capacitance C_{J0}, and Josephson coupling energy E_{J0}, pierced by an external magnetic flux $\Phi = \Phi_0/2$ (Φ_0 is the flux quantum), permit tuning of the effective Josephson energy. Let us consider two identical superconducting charge qubits placed in a single-mode cavity QED are involved in the gate operation [12]. A control gate voltage V_g is connected to the system via a gate capacitor C_g. We suppose that $V_g = V_g^{dc} + V_g^{ac} + V_g^{qu}$, where V_g^{dc} (V_g^{ac}) is the dc (ac) part of the gate voltage and V_g^{qu} is the quantum part of the gate voltage, which is caused by the electric field of the resonator mode when the qubit is coupled to a resonator. Correspondingly, we have $n_g = n_g^{dc} + n_g^{ac} + n_g^{qu}$ where $n_g^{dc} = C_g V_g^{dc}/2e$, $n_g^{ac} = C_g V_g^{ac}/2e$ and $n_g^{qu} = C_g V_g^{qu}/2e$. The goal of this section is to demonstrate the manner to implement the two-superconducting-qubit Grover search algorithm via cavity QED in the case of apply a resonant pulse to each charge qubit. We Consider two qubits interacting with the cavity mode. Therefore, the total Hamiltonian of the system (assuming $\hbar = 1$) is [10,13,14]

$$H_{total} = E_z S_{z,j} + \omega_c a^+ a - E_J(\Phi) S_x + (a^+ + a) S_z + \varepsilon(a^+ e^{-i\omega_d t} + a^- e^{i\omega_d t}) + \sum_{\substack{i,j=1 \\ i\neq j}}^{2} \Gamma_{ij} \sigma_{x,i}\sigma_{x,j}, \quad (1)$$

S_z and S_x are the collective operators for the two qubits, where $S_z = \sum_{j=1}^{2} g_j \sigma_{z,j}$ and $S_x = \sum_{j=1}^{2} \sigma_{x,j}$, with Pauli operators $\sigma_{z,j} = \frac{1}{2}(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j|), \sigma_{x,j} = \frac{1}{2}(|+_j\rangle\langle -_j| + |-_j\rangle\langle +_j|), |e_j\rangle(|g_j\rangle)$ is the excited state (ground state) of the qubit, ω_c is the cavity mode frequency, ω_d is the frequency of the external drive, a^+, a are the creation and annihilation of the cavity mode, ε is the amplitude of the microwave, g_j is the coupling constant between the charge qubit and the resonator mode, and Γ_{ij} is the force qubit-qubit coupling. Note that the $E_z = -2E_c(1 - 2n_g^{dc})$ with the charge energy $E_c = e^2/2C_\Sigma$ ($E_{J0} \ll E_c$ with $C_\Sigma = C_g + 2C_{J0}$) and $n_g = C_g V_g/2e$. The effective Josephson coupling energy is given by $E_J(\Phi) = 2E_{J0} cos(\pi\Phi/\Phi_0)$. The qubits are capacitively coupled to the cavity is determined by the gate voltage, which contains both the dc contribution and a quantum part. The fourth term of Eq.(1) is given by $V_g^{qu} = V_0^{qu}(a + a^+)$ where $V_0^{qu} = (\hbar\omega_c)^{1/2}(Lc_0)^{-1/2}$. Finally, the coupling constant g_j is given by $g_j = 2E_c C_g V_0^{qu}/(\hbar e)$. For get useful logical gate rates, we work with large amplitude driving fields, in this case, the quantum fluctuations in the drive are very small with respect to the drive amplitude and the drive can be considered, for all practical purposes, as a classical field. Here, it is convenient to displace the field operators using the time-dependent displacement operator [15,16] $D(\alpha) = e^{(\alpha a^+ - \alpha^* a)}$.

Under this transformation, the field a and a^+ goes to $(\alpha + a)$ and $(a^+ + \alpha^*)$, respectively, where α is a complex number representing the classical part of the field. The displaced Hamiltonian reads [15,17]

$$\begin{aligned} H &= D(\alpha) H_{total} D(\alpha) - iD^+(\alpha)\dot{D}(\alpha) \\ &= E_z S_{z,j} + \omega_c a^+ a - E_J(\Phi) S_x + (a^+ + a) S_z - g(\alpha^* + \alpha) S_z \\ &\quad + \sum_{\substack{i,j=1 \\ i\neq j}}^{2} \Gamma_{ij} \sigma_{x,i}\sigma_{x,j}, \end{aligned} \quad (2)$$

we have chosen $\alpha(t)$ to satisfy $\dot{\alpha} = -i\omega_r\alpha - i\varepsilon e^{-i(\omega_d t + \varphi)}$, where φ the initial phase of the pulse. This choice of α is made so as to eliminate the direct drive of microwave field on the cavity mode, which is described by $\varepsilon(a^+ e^{-i\omega_d t} + a e^{i\omega_d t})$. Then, we get $\alpha = -\frac{\varepsilon}{\omega} e^{-i(\omega_d t + \varphi)}$, where $\omega = \omega_c - \omega_d$, and for convince, we assume that we can tuned the coupling strength Γ_{ij} and the coupling constant g_j at the same time to have $\Gamma_{ij} = \Gamma$ and $g_j = g$. Then the Hamiltonian H becomes [15,16]

$$H = E_z S_{z,j} + \omega_c a^+ a - E_J(\Phi) S_x + (a^+ + a) S_z + \Omega \cos(\omega_d t + \varphi) S_z + \Gamma \sum_{\substack{i,j=1 \\ i\neq j}}^{2} \sigma_{x,i}\sigma_{x,j}, \quad (3)$$

where $\Omega = 2g\varepsilon/\omega$ and $S_z = g\sum_{j=1}^{2} \sigma_{z,j}$. In addition, by setting $E_z = 0$ ($i.e.$, $n_g = 1/2$) for each qubit and defining $\omega_0 = E_J(\Phi)$, the Hamiltonian (3) reduces to

$$H = \omega_c a^+ a - \omega_0 S_x + (a^+ + a) S_z + \Omega \cos(\omega_d t + \varphi) S_z + \Gamma \sum_{\substack{i,j=1 \\ i \neq j}}^{2} \sigma_{x,i} \sigma_{x,j}, \quad (4)$$

Define the new basis [18,19] $|e_j\rangle = \frac{1}{\sqrt{2}}(|+_j\rangle + |-_j\rangle), |g_j\rangle = \frac{1}{\sqrt{2}}(|+_j\rangle - |-_j\rangle)$. Then, the Hamiltonian H becomes

$$\begin{aligned} H &= H_0 + H_1 + H_2 + H_3 \\ &= \omega_c a^+ a - \omega_0 S_z + (a^+ + a) S_x + \Omega \cos(\omega_d t + \varphi) S_x \\ &\quad + \Gamma \sum_{\substack{i,j=1 \\ i \neq j}}^{2} \sigma_{z,i} \sigma_{z,j}, \end{aligned} \quad (5)$$

with

$$H_0 = \omega_c a^+ a - \omega_0 S_z \quad (6)$$

$$H_1 = \Omega \cos(\omega_d t + \varphi) S_x \quad (7)$$

$$H_2 = (a^+ + a) S_x \quad (8)$$

$$H_3 = \Gamma \sum_{\substack{i,j=1 \\ i \neq j}}^{2} \sigma_{z,i} \sigma_{z,j}, \quad (9)$$

H_0 is the free Hamiltonian of the qubits and the cavity mode, H_1 is the interaction Hamiltonian between the qubits and the classical pulse, H_2 is the interaction Hamiltonian between the qubits and the cavity mode, and H_3 is the interaction Hamiltonian between qubits. In the interaction picture with respect to H_0, the Hamiltonians H_1, H_2 and H_3 are rewritten, respectively, as (under the assumption that $\omega_d = \omega_0$)

$$H_1 = \Omega \sum_{i,j=1}^{2} (e^{i\varphi} \sigma_j^- + \sigma_j^+ e^{-i\varphi}) \quad (10)$$

$$H_2 = g \sum_{i,j=1}^{2} (a \sigma_j^+ e^{i\delta t} + a^+ \sigma_j^- e^{-i\delta t}) \quad (11)$$

$$H_3 = \Gamma \sum_{\substack{i,j=1 \\ i \neq j}}^{2} \sigma_{z,i} \sigma_{z,j}, \quad (12)$$

where $\delta = \omega_c - \omega_0$ (the detuning between the atomic transition frequency ω_c and the frequency of the cavity mode ω_0), $\sigma_{x,j} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $\sigma_{z,j} = \frac{1}{2}(|g_j\rangle\langle e_j| + |e_j\rangle\langle g_j|)$, $\sigma_j^+ = |e_j\rangle\langle g_j|$, and $\sigma_j^- = |g_j\rangle\langle e_j|$. In the case of pulse phase $\varphi = 0$, the Hamiltonian H_1 becomes

$$H_1 = 2\Omega S_x, \quad (13)$$

where

$$S_x = \frac{1}{2} \sum_{j=1}^{2} (\sigma_j^- + \sigma_j^+) = \sum_{j=1}^{2} \sigma_{x,j}. \quad (14)$$

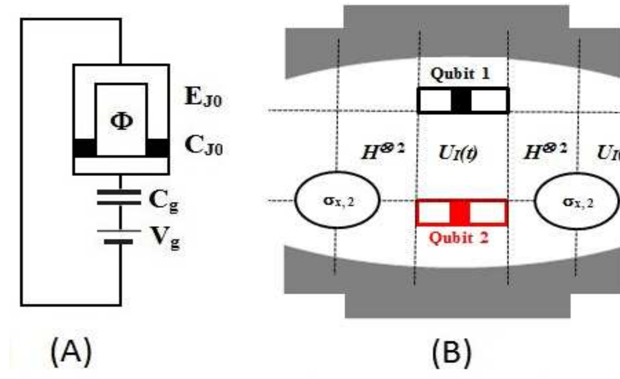

Fig. 1: (A) Diagram of a superconducting charge qubit, in the charge regime $\Delta \gg E_c \gg E_{J0} \gg k_B T$ (here, Δ, E_c, E_{J0}, k_B and T are the Gap, charging energy, Josephson coupling energy, boltzmann constant, and temperature, respectively). (B) Two superconducting qubits are placed in a microwave cavity and are coupled to each other via the cavity mode, where the two qubits are initially prepared in the average state, $H^{\otimes 2}, U_I(t), H^{\otimes 2}$ constitute the conditional phase gate $|g_1\rangle|g_2\rangle$ or $|e_1\rangle|e_2\rangle$, and the two NOT gates acting on qubit 2, i.e., $\sigma_{x,2}$, added on the two ends of above three operations are for generating $|g_1\rangle|e_2\rangle$ or $|e_1\rangle|g_2\rangle$. The next operation $U_I(t)$ is for the diffusion transform D.

By solving the Schrodinger equation

$$i \frac{d|\Psi(t)\rangle}{dt} = (H_2 + H_3)|\Psi(t)\rangle, \quad (15)$$

with

$$|\Psi(t)\rangle = e^{-iH_1 t}|\Psi'(t)\rangle, \quad (16)$$

we obtain

$$i \frac{d|\Psi'(t)\rangle}{dt} = H_I|\Psi'(t)\rangle, \text{(where } H_I = H_{I2} + H_{I3}) \quad (17)$$

with

$$\begin{aligned} H_{I2} &= e^{iH_1 t} H_2 e^{-iH_1 t} \\ &= g \sum_{j=1}^{2} a^+ e^{-i\delta t} \left[\sigma_{x,j} + \frac{1}{2} \left(\sigma_{z,j} + \frac{1}{2}\sigma_j^+ - \frac{1}{2}\sigma_j^- \right) e^{2i\Omega t} \right. \\ &\quad \left. - \frac{1}{2} \left(\sigma_{z,j} - \frac{1}{2}\sigma_j^+ + \frac{1}{2}\sigma_j^- \right) e^{-2i\Omega t} \right] + H.C, \end{aligned} \quad (18)$$

and

$$\begin{aligned} H_{I3} &= e^{iH_1 t} H_3 e^{-iH_1 t} \\ &= \Gamma \sum_{\substack{i,j=1 \\ i \neq j}}^{2} \sigma_{zi} \sigma_{zj}, \end{aligned} \quad (19)$$

In the strong driving region $2\Omega \gg \delta, g, \Gamma$, we can eliminate the terms oscillating fast. Then the Hamiltonian

H_I reduces to [20,21,22]

$$H_I^{'} = g(a^+ e^{i\delta t} + a e^{-i\delta t})S_x + \Gamma \sum_{\substack{i,j=1\\ i\neq j}}^{2} \sigma_{zi}\sigma_{zj}. \quad (20)$$

The evolution operator of the Hamiltonian $H_I^{'}$ can be written as [10,23]

$$U^{'}(t) = e^{-iA(t)S_x^2} e^{-iB(t)aS_x} e^{-iB^*(t)\,a^+S_x} e^{-C(t)X},\ (X = \sum_{\substack{i,j=1\\ i\neq j}}^{2} \sigma_{zi}\sigma_{zj}). \quad (21)$$

By solving the Schrodinger equation

$$i\frac{dU^{'}(t)}{dt} = H_I^{'}U^{'}(t), \quad (22)$$

we obtain

$$\begin{aligned} C(t) &= \frac{1}{2}\int_0^t \Gamma dt^{'} = \Gamma t, \\ B(t) &= g\int_0^t e^{i\delta t^{'}} dt^{'} = \frac{g}{i\delta}(e^{i\delta t} - 1), \\ A(t) &= ig\int_0^t B(t^{'}) e^{-i\delta t^{'}} dt^{'} = \frac{g^2}{\delta}\left[t + \frac{1}{i\delta}(e^{-i\delta t} - 1)\right]. \end{aligned} \quad (23)$$

Setting $t = 2\pi/|\delta|$ and $\delta t = 2\pi$, we have $B(\tau) = 0$ and $A(t) = g^2\tau/\delta$. Then, the evolution operator $U'(t)$ become

$$U^{'}(t) = e^{-2i\lambda S_x^2 t} \prod_{\substack{i,j=1\\ i\neq j}}^{2} e^{-i\Gamma t\sigma_{zi}\sigma_{zj}}, \quad (24)$$

where $\lambda = \frac{g^2}{2\delta}$. Then, we obtain the evolution operator of the system as

$$\begin{aligned} U(t) &= e^{-iH_0 t}U'(t) = e^{-2i\Omega t S_x} e^{-2i\lambda t S_x^2} e^{-4i\Gamma t\sigma_{z_1}\sigma_{z_2}}, \\ &= e^{-ihbS_x} e^{-ibS_x^2} e^{-i2a\sigma_{z_1}\sigma_{z_2}}, \end{aligned} \quad (25)$$

with $b = 2\lambda t$, $h = \frac{\Omega}{\lambda}$ and $a = 2\Gamma t$. In the subspace spanned by $(|e_1\rangle|e_2\rangle, |e_1\rangle|g_2\rangle, |g_1\rangle|e_2\rangle, |g_1\rangle|g_2\rangle)$, we define the two-qubit Hadamard gate as

$$H^{\otimes 2} = \prod_{i=1}^{2} H_i = \frac{1}{2}\begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \quad (26)$$

where H_i is the Hadamard gate acting on the i qubit, transforming states as $|g_i\rangle \to \frac{1}{\sqrt{2}}(|g_i\rangle + |e_i\rangle)$, $|e_i\rangle \to \frac{1}{\sqrt{2}}(|g_i\rangle - |e_i\rangle)$. Then the evolution operator of the system $U_I(t)$ can be expressed in the same basis as

$$U_I(t) = e^{-ia}\begin{bmatrix} \frac{1}{2}A & \frac{-i}{2}Ce^{2ia} & \frac{-i}{2}Ce^{2ia} & \frac{-1}{2}B \\ \frac{-i}{2}C & \frac{1}{2}e^{2ia}A & -\frac{1}{2}e^{2ia}B & \frac{-i}{2}C \\ \frac{-i}{2}C & -\frac{1}{2}e^{2ia}B & \frac{1}{2}e^{2ia}A & \frac{-i}{2}C \\ \frac{-1}{2}B & \frac{-i}{2}Ce^{2ia} & \frac{-i}{2}Ce^{2ia} & \frac{1}{2}A \end{bmatrix}, \quad (27)$$

with $A = 1+\cos(bh)e^{-ib}$, $B = 1-\cos(bh)e^{-ib}$ and $C = \sin(bh)e^{-ib}$. The diffusion transform D is given by [21,24]

$$D = -I + 2|\Psi_0\rangle\langle\Psi_0| = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}, \quad (28)$$

where the average state $|\Psi_0\rangle = \frac{1}{2}(|e_1\rangle|e_2\rangle + |e_1\rangle|g_2\rangle + |g_1\rangle|e_2\rangle + |g_1\rangle|g_2\rangle)$ and I is the 4×4 identity matrix. If we choose $a = 2k\pi$ (k is integer), $b = \frac{\pi}{2}$, and $bh = \frac{\pi}{2} + 2m\pi$ (m is integer), we can obtain

$$U_I(t) = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = -D. \quad (29)$$

Then, by choosing an appropriate value of Ω, we can generate a two-qubit diffusion transform D (different by -1 prefactor). The two-qubit conditional phase gate to label different target states will also be generated in a natrural way.

In the following, we show how we can use the evolution operator $U_I(t)$ $(Eq.(27))$ and the two-qubit Hadamard gate $(Eq.(26))$ to implement the two-superconducting-qubit Grover quantum search in cavity QED. On the other hand, the two-qubit conditional phase gate to label different target states will also be generated in a natrural way.

It's easy to find

$$H^{\otimes 2}U_I(t)H^{\otimes 2} = \frac{1}{2}e^{-ia}\begin{bmatrix} \xi e^{-ib(1-h)} & 0 & 0 & \Lambda e^{-ib(1-h)} \\ 0 & \xi & \Lambda & 0 \\ 0 & \Lambda & \xi & 0 \\ \Lambda e^{-ib(1+h)} & 0 & 0 & \xi e^{-ib(1+h)} \end{bmatrix}, \quad (30)$$

with $\xi = (1+e^{2ia})$ and $\Lambda = (1-e^{2ia})$. We choose $a = 2k\pi$, $b = \frac{\pi}{2}$, $h = 4m+1$, $\lambda t_1 = \frac{\pi}{4}$, $t_1 = \frac{\pi}{4\lambda}$, and $\frac{\Omega}{\lambda} = 4m+1$, we have[9]

$$H^{\otimes 2}U_I(t_1)H^{\otimes 2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = I_{g_1g_2}. \quad (31)$$

Thus, the $I_{g_1g_2}$ phase operation is obtained and the target state $|g_1\rangle|g_2\rangle$ is labeled. Similarily, target state $|e_1\rangle|e_2\rangle$ can be labeled by setting $a = 2k\pi$, $b = \frac{\pi}{2}$, $h = 4m+3$, $\lambda t_2 = \frac{\pi}{4}$, $t_2 = \frac{\pi}{4\lambda}$, and $\frac{\Omega}{\lambda} = 4m+3$, we have

$$H^{\otimes 2}U_I(t_2)H^{\otimes 2} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_{e_1e_2}. \quad (32)$$

The slight modification of the operations $I_{e_1e_2}$ or $I_{g_1g_2}$ by the NOT gate $\sigma_{x,2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ acting on qubit 2, allows us to find the states $|e_1\rangle|e_2\rangle$ or $|g_1\rangle|g_2\rangle$ [25] as

$$\sigma_{x,2} I_{g_1g_2} \sigma_{x,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_{g_1e_2}, \tag{33}$$

$$\sigma_{x,2} I_{e_1e_2} \sigma_{x,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_{e_1g_2}. \tag{34}$$

In this way, we can implement the operations I_α easily up to a global phase ($|\alpha\rangle = |e_1\rangle|e_2\rangle, |e_1\rangle|g_2\rangle, |g_1\rangle|e_2\rangle, |g_1\rangle|g_2\rangle$, respectively). Therefore, we can achieve all the two-qubit operations necessary in the two-qubit Grover quantum search algorithm.

To carry out our scheme, in a cavity, we consider two qubits prepared in state denoted by $|g_1\rangle|g_2\rangle$, after $H^{\otimes 2}$ operation, are in the initial average state. Then, they undergo the operations in $Fig.$1 from the left to the right. For searching $|g_1\rangle|g_2\rangle$ or $|e_1\rangle|e_2\rangle$, our implementation is straightforward because the qubits interact with the cavity and the classical field simultaneously. While to search $|e_1\rangle|g_2\rangle$ or $|g_1\rangle|e_2\rangle$, since NOT gates are only performed on qubit 2, we have to employ an inhomogeneous field to distinguish the two qubits. The next operation $U_I(t)$ is for the diffusion transform D (see Fig. 1).

3 Fidelity

Let us now study the fidelity of the gate operations. In order to check the validity of our proposal, we define the following fidelity to characterize the deviation of how much the output states $|\Psi(t)\rangle$ deviate in amplitude and phase from the ideal logical gate transformation for the different input states[26,27]:

$$F = \left|\cos\left(\frac{\pi}{5}\left(\frac{g}{15\Gamma}\right)\right)\right|^4 \tag{35}$$

In the obtaining of $Eq.$20, we have discarded the fast oscillating terms, which induce Stark shifts on the states $|+_j\rangle$ and $|-_j\rangle$ (where $|+_j\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle + |g_j\rangle)$ and $|-_j\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle - |g_j\rangle)$). Here, our numerical calculation shows that a high fidelity $\sim 100\%$ can be achieved when $15\Gamma \gg g$ (Fig. 6), from which we know that large detuning is not required in our scheme.

4 Discussion

We briefly discuss the experimental feasibility of our proposal. Although the evolution operator is independent of the cavity field as decided by the condition $\delta t = 2\pi$, the two-superconducting-qubit system is entangled with the cavity during the qubit-cavity interaction. We have to neglect the cavity decay during this interaction time. After the interaction, the qubits are disentangled with the cavity, that is, the operation will not be affected by the cavity decay during the interaction time. The decoherence time $T_1 = 1.87\mu s$ [28] and the coupling strength is $g = 2\pi \times 100MHz$ [16], $\delta = g$ and $\Omega \approx 10\delta$ ($\Omega \gg \delta$, g, Γ). Then, the implementation time is $t_{imp} = \frac{\pi}{2g}$. So, the direct calculation shows that the implementation time will be $t_{imp} = 2.5$ ns, which is much shorter than the decoherence T_1. The dephasing time $T_2 = 2.22\mu s$ [28] is much longer than the operation time t_{imp}. Thus, the implementation time t_{imp} which is much shorter than the cavity-mode lifetime $\kappa^{-1} = Q/\omega_c \sim 159ns$ for a cavity with the quality factor of the cavity $Q = 10^4$ has been demonstrated by cavity QED experiments with superconducting charge qubits [29]. Furthermore, it should be pointed out that the Rabi frequency Ω during the two-qubit gate is about $2\pi \times 1GHz$ and should be slightly adjusted to satisfy the condition $\frac{\Omega}{\lambda} = 4m+1$ or $\frac{\Omega}{\lambda} = 4m+3$ mentioned above. The proposed scheme is realizable with the present cavity QED techniques.

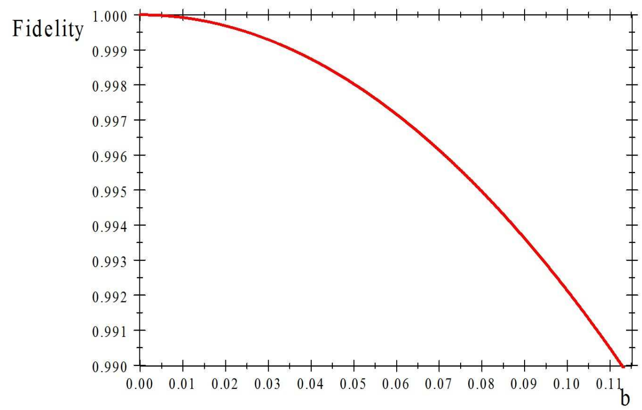

Fig. 2: Numerical results for fidelity of the gate operations versus the ratio $b = \frac{g}{15\Gamma}$.

In principle, our scheme may offer a viable way to realize a scalable quantum algorithm. Based on the qubit-qubit interaction and the effective interaction between two superconducting qubits with single mode cavity, our scheme can be also extended to multi-superconducting-qubit Grover algorithm. Furthermore, it should be pointed out that the single superconducting-qubit sources are required in our scheme.

5 Implementation of many-qubit Grover's search algorithm

Now, we generalize our scheme to the case of multi-qubits. It is well known that a one-qubit unitary gate and a two-qubit conditional phase gate are universal for building quantum computers. Thus, the multi-qubit conditional phase operation required for implementing Grover quantum search in our scheme can be realized by combining the two-qubit conditional phase operation described by $Eq.$((31), (32), (33) and (34)) and the one-bit unitary operation that is easily realized by using classical microwave field and qubit-resonator interaction. For example, the three-qubit conditional phase operation is:

$$I_{g_1g_2g_3} = \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&-1 \end{bmatrix} \quad (36)$$

A single qubit operation not can be performed via a C-NOT gate if the control qubit is set to $|1\rangle$ and viewed as an auxiliary qubit. The C-NOT gate is much more difficult to build than a single qubit not. Our two elementary gates also allow us to construct a very useful gate called the controlled-controlled-NOT gate (c^2-NOT) or the Toffoli gate [30]. The construction is given by the following network:

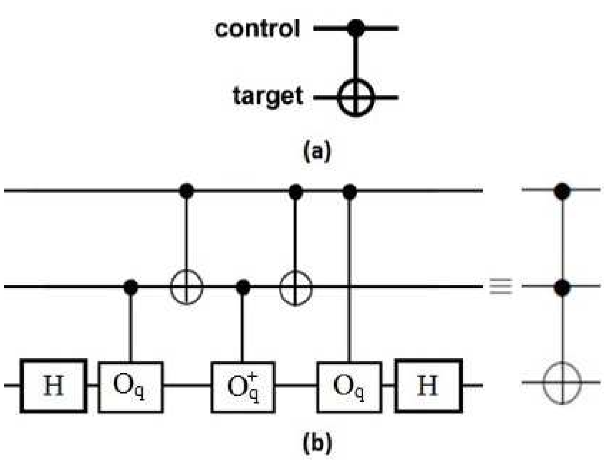

Fig. 3: (a) The symbol of a controlled-NOT gate. **(b)** Proposed quantum circuit for implementing three-qubit conditional phase operation. Here, O_q is a one-qubit unitary operation.

This gate has two control qubits (the top two wires on the diagram) and one target qubit which is negated only when the two controls are in the state $|11\rangle$. The c^2-NOT gate gives us the logical connectives we need for arithmetic. If the target is initially set to $|0\rangle$ the gate acts as a reversible AND gate-after the gate operation the target becomes the logical AND of the two control qubits.

Furthermore, the two-qubit controlled-NOT operation can be obtained by combining the two-qubit conditional phase operation ($Eq.$((31), (32), (33) and (34))) with two single-qubit H transforms, which are performed on the target qubit before and after the two-qubit conditional phase operation ($Eq.$((31), (32), (33) and (34))), respectively (see Fig. $3(b)$). Similarly, the multi-qubit conditional phase operation can be realized by using the same method. Then, the multi-qubit Grover quantum search can be implemented successfully.

6 Conclusion

In conclusion, we have proposed a simple scheme for implementing two-superconducting-qubit Grover search algorithm in cavity QED by introducing the qubit-qubit interaction. We have implemented the proposed quantum search algorithm via cavity QED driven by a strong microwave field. After $U_I(t)$ operation and $H^{\otimes 2}U_I(t_2)H^{\otimes 2}$ operation, the tow-qubit are in the initial average state $|g_1g_2\rangle$. Then, they undergo the operations in Fig.1 from the left to the right, the implementation is straightforward in the case to searching $|g_1\rangle|g_2\rangle$ or $|e_1\rangle|e_2\rangle$, and to search $|e_1\rangle|g_2\rangle$ or $|g_1\rangle|e_2\rangle$ the two NOT gates $\sigma_{x,2}$ are only performed on qubit 2. Finally, we have measured the final state of the two-qubits in output of the system. Moreover, the implementation time calculated is much shorter than the decoherence time and the dephasing time. The numerical simulation shows that our implementation is good enough to demonstrate a two-qubit Grover search with high fidelity. Thus, we have also generalized our Implementation of two-qubit Grover search to the case of multi-qubits Grover search.

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