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A New Family of Heavy Tailed Symmetric Distribution for Modeling Financial Data

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Abstract: In this article we study a new family of distributions in the real line. The proposed model can be seen as a suitable model for fitting symmetric and kurtotic datasets. It arises as a mixture of the Laplace and bilateral gamma densities. We study some of its analytical properties and estimate the unknown parameters using maximum likelihood method. Algorithm of simulation and applications to the real dataset of monthly interest rate data are presented. An asymmetric generalization of the new model is discussed.

Keywords: Lindley distribution, Laplace distribution, inverse scale factor, skewness.

1 Introduction

The success and accuracy of the statistical data analysis depends mainly on the assumption of the underlying probability distribution. In recent years there has been a growing interest in studying different symmetric and their asymmetric parametric families of distribution. One reason for this is that many of the existing symmetric families of distributions are not able to model the kurtotic, skewed and heavy tailed data sets arising in various real life situations. Also, we can see that many of the symmetric family of distributions are unimodal so that it is unable to model the bimodality inherent in the dataset. So there came the importance in the search of new family of symmetric distributions. One easy way to tackle this problem is by the symmetrization of distributions with positive support. In this paper, our aim is to investigate a probability distribution that can be derived from the Lindley probability distribution and is to be found a suitable model to fit many data sets.

Lindley(1958,1965) introduced a new family of continuous distributions to a random variable X with positive real line as support. A random variable X is said to follow Lindley distribution with parameter θ if its p.d.f is given by

$$f(x) = \frac{\theta^2}{(\theta+1)}(1+x)e^{-\theta x} \quad x>0, \theta>0 \tag{1.1}$$

Ghitany et.al (2008) studied about this distribution in detail and discussed its reliability properties. From (1.1) it is clear that Lindley distribution is a mixture distribution of exponential (θ) and gamma distribution with parameter $(2,\theta)$. Also the distribution is unimodal and positively skewed. They have shown that even though the Lindley distribution is similar to the exponential distribution it can be used as a better model than the exponential distribution in many situation.

Different extensions of Lindley distribution can be seen in statistical literature. Nadarajah et.al (2011) introduced a generalized form of Lindley distribution and shown that this distribution is better alternative to Gamma,Lognormal and exponetiated form of different distributions. Zakerzadeh and Dolati (2009) studied a more flexible form of Lindley distribution. Some other extensions are Power Lindley distribution of Ghitany et.al (2013), generalized Poison-Lindley distribution of Mahmoudi and Zakerzadeh (2010).

Since, Lindley distribution shares many advantages in modelling its extension into the real line as support produces a competitive model for many different class of symmetric distributions with support on $(-\infty,\infty)$. In this work, we propose the symmetric extended Lindley distribution and study the important properties. Estimation of the unknown parameters

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of the distribution are done using the method of maximum likelihood. As an application, we successfully fitted the model to the monthly interest data set which has been considered by many authors see Vandorp and Kotz(2002).

The paper is organized as follows. Section 2 is devoted to the newly introduced Double Lindley distribution and its properties like moments, skewness, kurtosis and the entropy measures. In Section 3, we obtained the Maximum likelihood estimator of the unknown parameter. Simulation procedure is discussed in Section 4. The new model is fitted to a real data set in Section 5. An asymmetric generalization of the newly introduced distribution is considered in Section 6.

2 Double Lindley Distribution

If $f(x)$ is a probability density function corresponding to a r.v X having support on $(0,\infty)$ then it can be converted into symmetric about zero by defining

$$g(x) = kf(|x|), \quad -\infty < x < +\infty \tag{2.1}$$

where k is the normalizing constant. Now we extend the Lindley distribution into the complete real line as support, by taking f(x) as the p.d.f of Lindley distribution given in (1.1). We obtain a new family of distributions which we termed it as double Lindley distribution and denote corresponding random variable as $X \rightarrow DLD(\theta)$. The probability density function (pdf) of a DLD random variable X with a scale parameter θ is given by

$$f(x) = \frac{\theta^2}{2(\theta+1)}(1+|x|)e^{-\theta|x|} \quad -\infty < x < \infty, \theta > 0 \tag{2.2}$$

Note that the probability density function of $DLD(\theta)$ random variable can be viewed as mixture of two probability densities with representation

$$f_\theta(x) = \beta f_1(x) + (1-\beta) f_2(x), \tag{2.3}$$

where $\beta = \frac{\theta}{1+\theta}$, $f_1(x) = \frac{\theta}{2}e^{-\theta|x|}$, the probability density function of a Laplace random variable with mean zero and variance $2\theta^2$ and $f_2(x) = \frac{\theta^2}{2}|x|e^{-\theta|x|}$, the probability density function of a two sided gamma random variable with shape parameter 2 and scale parameter θ.

The probability density function is unimodal for the values of $\theta \geq 1$ with mode located at the point zero and it is bimodal for the values of $\theta < 1$ with modes concentrated at the points $\pm\left(1-\frac{1}{\theta}\right)$.

The distribution function is

$$F(x) = \begin{cases} \frac{1}{2(\theta+1)}(1+\theta(1-x))e^{\theta x}, & \text{if } x \leq 0; \\ 1-\frac{1}{2(\theta+1)}(1+\theta(1+x))e^{-\theta x}, & \text{if } x > 0. \end{cases} \tag{2.4}$$

Figure 1 shows the shape of the pdf of $DLD(\theta)$ for different values of θ. From the figure it is clear that $DLD(\theta)$ distribution is symmetric and becomes more peaked for larger values of θ. Next we study the analytical properties of DLD(θ).

2.1 Moments and Related Measures

The r^{th} moment about origin of a $DLD(\theta)$ random variable X is given by

$$E(X^r) = \frac{\Gamma(r+1)}{2(\theta+1)\theta^{r-1}}\left(1+\frac{r+1}{\theta}\right)(1+(-1)^r), r = 1,2,3... \tag{2.5}$$

Since it is a symmetric distribution, note that all the odd order moments are zero for $DLD(\theta)$ distribution.

Also, in particular, we obtain $E(X) = 0$ and $V(X) = \frac{2(\theta+3)}{\theta^2(1+\theta)}$.

The kurtosis coefficient is given by,

$$\beta_2 = \frac{6(\theta+5)(\theta+1)}{(3+\theta)^2} \tag{2.6}$$

Cumulants

The characteristic function $\varphi(t) = E\left(e^{itX}\right)$ of $DLD(\theta)$ distribution can be easily derived using the mixture representation of the density function.

Using (2.3), we can write $\varphi(t) = \beta\varphi_1(t) + (1-\beta)\varphi_2(t)$, where $\varphi_1(t)$ and $\varphi_2(t)$ are the characteristic functions

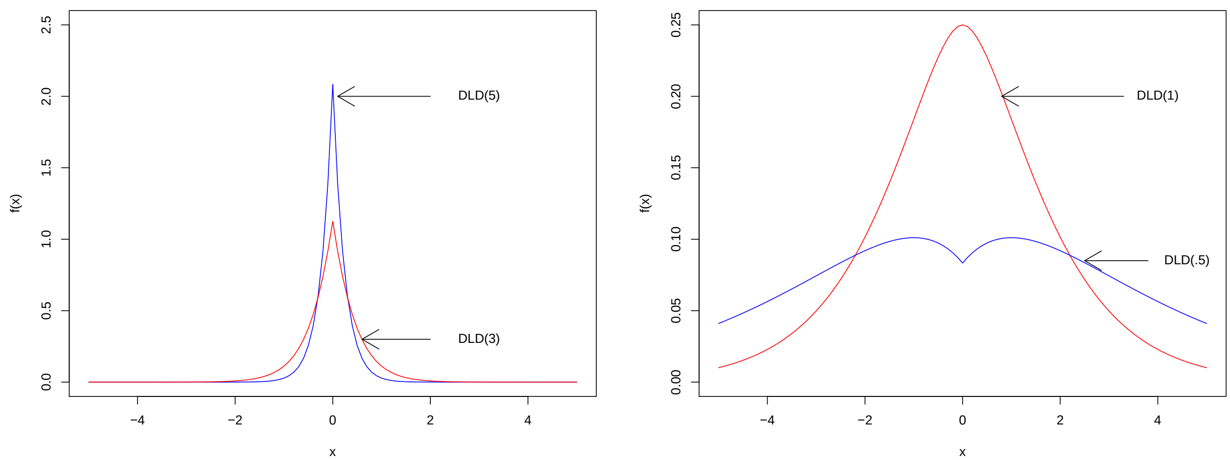

Fig. 1: Shape of the density function for different values of θ

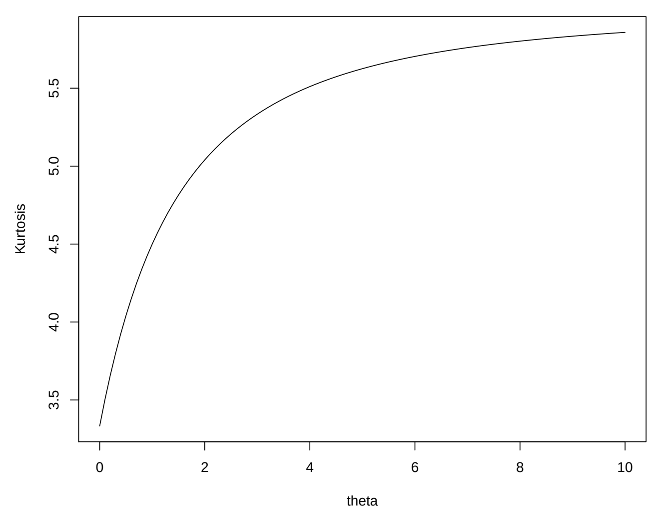

Fig. 2: Peakedness of the DLD(θ) distribution for the different values of θ

corresponding to $f_1(x)$ and $f_2(x)$ respectively.
But we have $\varphi_1(t) = \frac{\theta^2}{\theta^2+t^2}$ and $\varphi_2(t) = \frac{\theta^2(\theta^2-t^2)}{(\theta^2+t^2)^2}$, then the characteristic function of a $DLD(\theta)$ random variable as

$$\varphi(t) = \frac{\theta^2}{(1+\theta)(\theta^2+t^2)} \left(\theta + \frac{\theta^2 - t^2}{\theta^2 + t^2} \right) \tag{2.7}$$

2.2 Entropy Measures

Renyi Entropy
An entropy of a random variable X is a measure of variation of the uncertainity. Jaynes (1951) introduced one of the most

powerful techniques employed in the field of probability and statistics called the ”maximum entropy method”. Renyi entropy is defined by

$$\tau(\gamma)=\frac{1}{1-\gamma}log\left(\int f^{\gamma}(x)dx\right),\quad \gamma>0,\gamma\neq 1.$$

$$\int f^{\gamma}(x)dx=\left(\frac{\theta^2}{2(\theta+1)}\right)^{\gamma}\int_{-\infty}^{+\infty}(1+|x|)^{\gamma}e^{-\gamma\theta|x|}dx$$

$$=c^{\gamma}\int_{-\infty}^{0}(1-x)^{\gamma}e^{\gamma\theta x}dx+c^{\gamma}\int_{0}^{+\infty}(1+x)^{\gamma}e^{-\gamma\theta x}dx,\qquad \text{where } c=\frac{\theta^2}{2(\theta+1)}$$

$$=2\left(\frac{\theta^2}{2(\theta+1)}\right)^{\gamma}\frac{e^{\gamma\theta}}{(\gamma\theta)^{(\gamma+1)}}\Gamma(\gamma+1,\gamma\theta)$$

$$\tau(\gamma)=\frac{1}{1-\gamma}log\left[2\left(\frac{\theta^2}{2(\theta+1)}\right)^{\gamma}\frac{e^{\gamma\theta}}{(\gamma\theta)^{(\gamma+1)}}\Gamma(\gamma+1,\gamma\theta)\right]$$

$$=-log\theta+\frac{1}{1-\gamma}[\theta\gamma+log\Gamma(\gamma+1,\gamma\theta)-\gamma log2(\theta+1)-(\gamma+1)log\gamma+log2]$$

Shannon’s Entropy
Shannon(1948) introduced the probabilistic definition of entropy which is closely connected with the definition of entropy in statistical mechanics. Then the Shannon’s entropy is defined by $E(-log f(x))$,it is the particular case of Renyi entropy for γ increases to 1. Limiting γ increases to 1 in $\tau(\gamma)$ and using L’hospital’s rule, we obtain

$$E(-log f(x))=-log\theta-\theta+2-\frac{e^{\theta}}{\theta+1}\frac{d}{dr}\Gamma(\gamma+1,\gamma\theta) \tag{2.8}$$

where $\Gamma(.,.)$ is the incomplete gamma function defined by

$$\Gamma(\alpha,x)=\int_{x}^{\infty}t^{\alpha-1}e^{-t}dt \tag{2.9}$$

Next we compare the DLD with Laplace distribution with regard to the tail behaviour
Tail comparison
Here we compare the tail behaviour of $DLD(\theta)$ with Standard Laplace distribution $L(\theta)$. For this purpose we use the concept of limiting ratio (LR) of two probability distributions, The same idea is used by many authors see Sastry and Deepesh (2016). Consider the random variables $X_1\sim DLD(\theta)$ and $X_2\sim L(\theta)$, then the limit ratio(LR) of their density is given by

$$LR=\lim_{x\longrightarrow\infty}\frac{f_{X_1}(x)}{f_{X_2}(x)} \tag{2.10}$$

$$=\lim_{x\longrightarrow\infty}\frac{\frac{\theta^2}{2(\theta+1)}(1+|x|)e^{-\theta|x|}}{\frac{\theta}{2}e^{-\theta|x|}} \tag{2.11}$$

Here, $LR\to\infty$ as $x\longrightarrow\infty$ which means that $f_{X_1}(x)$ has thicker tail than $f_{X_2}(x)$. That is tails of the $DLD(\theta)$ probability density function is more thicker than that of the Standard Laplace probability density function. See Figure (3).

3 Estimation of the parameter

Maximum Likelihood Estimation
Let x_1, x_2, ...x_n be a random sample from the DLD(θ) distribution. Then the Likelihood function is given by

$$L(\theta)=\prod_{i=1}^{n}\frac{\theta^2}{2(\theta+1)}(1+|x_i|)e^{-\theta|x_i|} \tag{3.1}$$

Taking logarithm on both sides we obtain the log likelihood function in the form

$$logL(\theta)=2nlog(\theta)-nlog2-nlog(1+\theta)-\theta\sum_{i=1}^{n}|x_i|+\sum_{i=1}^{n}log(1+|x_i|) \tag{3.2}$$

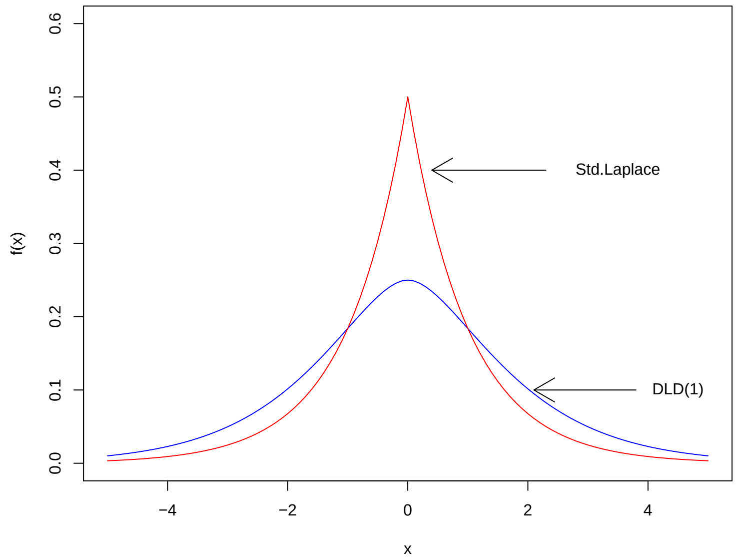

Fig. 3: Tail comparison of DLD(θ) with Standard Laplace Distribution.

On differentiation and equating the log likelihood function we get the score function as

$$\frac{\partial logL(\theta)}{\partial\theta} = \frac{2n}{\theta} - \frac{n}{1+\theta} - \sum_{i=1}^{n}|x_i| = 0 \tag{3.3}$$

which gives a quadratic equation in θ as

$$\overline{|x|}\theta^2 + (\overline{|x|}-1)\theta - 2 = 0, \text{where } \overline{|x|} = \frac{1}{n}\sum_{i=1}^{n}|x_i| \tag{3.4}$$

with roots

$$\hat{\theta} = \frac{-(\overline{|x|}-1) \pm \sqrt{(\overline{|x|}-1)^2 + 8\overline{|x|}}}{2|x|} \tag{3.5}$$

Since $\theta > 0$, only positive value of θ need to be taken.

4 Simulation

Making use of the mixture representation (2.3), random observations X_i can be generated using following algorithm.
step 1: Generate $U_i \longrightarrow U(0,1)$.
step 2: Generate $E_{ki} \longrightarrow Exp(\theta)$, i=1,2,...,n; k=1,2.
step 3: Set $Z_i = E_{1i} - E_{2i}$
step 4: Generate $G_{ki} \longrightarrow Gamma(2,\theta)$, i=1,2,...,n; k=1,2.
step 5: set $Y_i = G_{1i} - G_{2i}$, i=1,2,...,n; k=1,2.
step 6: If $U_i \leq \beta = \frac{\theta}{1+\theta}$, then set $X_i = Z_i$, otherwise set $X_i = Y_i$.
Next we estimate the unknown parameter of the proposed distribution.

Table 4.1: Parameter Estimates.

	$\theta=0.1$			$\theta=0.5$			$\theta=1.5$			$\theta=2$		
n	$\hat{\theta}$	$Bias$	MSE	$\hat{\theta}$	$Bias$	MSE	$\hat{\theta}$	$Bias$	MSE	$\hat{\theta}$	$Bias$	MSE
50	0.1317	$-.0317$	0.3045	0.6211	$-.1211$	3.104	1.7391	$-.2391$	11.0673	2.2699	-0.2699	15.195
100	0.1309	$-.0309$	1.2576	0.6133	$-.1133$	2.2919	1.7232	$-.2232$	9.5899	2.2602	-0.2600	14.700
500	0.1298	$-.0298$	0.2510	0.6091	$-.1091$	2.1236	1.7159	$-.2159$	7.9003	2.2433	$-.2433$	10.559
1000	0.1297	$-.0297$	0.2467	0.6084	$-.1084$	2.1166	1.7120	$-.2120$	7.7104	2.2431	-0.2431	10.37

5 Application

As an application we have used on monthly interest rates for 30 year treasury maturity rates over the period from 1977-2001. The same data set is used by many authors see Van Dorp and Kotz (2002). Table 4.1 displays the maximum likelihood estimates and the corresponding value of Kolmogorov- Smirnov statistic for the fitted model. Figure 4 provides the histogram and curve of the probability density function to the monthly interest data (American. can, Martin. Marieta and Value. Weighted CRSP. Index)

Table 5.1: Estimated parameter values and goodness of fit to the data.

$Data$	$m.l.e$	$K-Sstatistic$	$P-value$
$American.can$	18.93276	0.1373	0.1896
$Martin.Marieta$	13.66158	0.11052	0.4255
$Value.Weighted.CRSP.Index$	30.75854	0.12213	0.3324

An asymmetric generalized form of the proposed distributionis considered in the next section.

6 Asymmetric DLD Distribution

Since the DLD distribution is a symmetric family of distribution, it limits the applicability to real data sets which can be skewed. There are different methods of introducing skewness in to a symmetric family of distribution see for example Kozubowski and Ayebo (2003), Kotz et. al (2001) and Azzalini (1985). For an application to so formed distributions see Julia and Vives Rego (2005) and Kozubowski and Podgorski (2001). Here we introduce an asymmetric form of DLD distribution using the idea of inverse scale factors of Fernandez and Steel (1998). In this method a new parameter is added which acts as a skewing parameter in the symmetric family of distribution. The probability density function of an asymmetric Double Lindley Distribution(ADLD) distribution with parameters $\theta>0$ and $\kappa>0$ is given by

$$f(x;\theta,\kappa)=\frac{\theta^2}{(\theta+1)}\frac{\kappa}{(1+\kappa^2)}\begin{cases}(1-\frac{x}{\kappa})e^{\frac{\theta x}{\kappa}} & \text{if} \quad x\leq 0\\ (1+\kappa x)e^{-\theta\kappa x} & \text{if} \quad x>0\end{cases} \tag{6.1}$$

We denote the random variable having the above probability density function as $X\sim ADLD(\theta,\kappa)$. Note that for all values of κ other than $\kappa=1$ the above distribution is asymmetric and when $\kappa=1$ gives symmetric $DLD(\theta)$ distribution. The Figure 5 shows the shape of (6.1) for different values of θ and κ. The raw moments of $ADLD(\theta,\kappa)$ can be derived as

$$\begin{aligned}E(X^r)&=\frac{\theta^2}{(\theta+1)}\left(\int_{-\infty}^{0}x^r\left(1-\frac{x}{\kappa}\right)e^{\frac{\theta x}{\kappa}}dx+\int_{0}^{\infty}x^r(1+kx)e^{-\theta\kappa x}dx\right)\\&=\frac{\theta^{(1-r)}}{(\theta+1)}\frac{\kappa}{1+\kappa^2}\Gamma(r+1)\left(1+\left(\frac{r+1}{\theta}\right)\right)\left[(-1)^r\kappa^{r+1}+\frac{1}{\kappa^{r+1}}\right]\end{aligned} \tag{6.2}$$

In particular, when $r=1$, we obtain

$$E(X)=\frac{\theta+2}{\theta}\left(\frac{1-\kappa^2}{\kappa}\right) \tag{6.3}$$

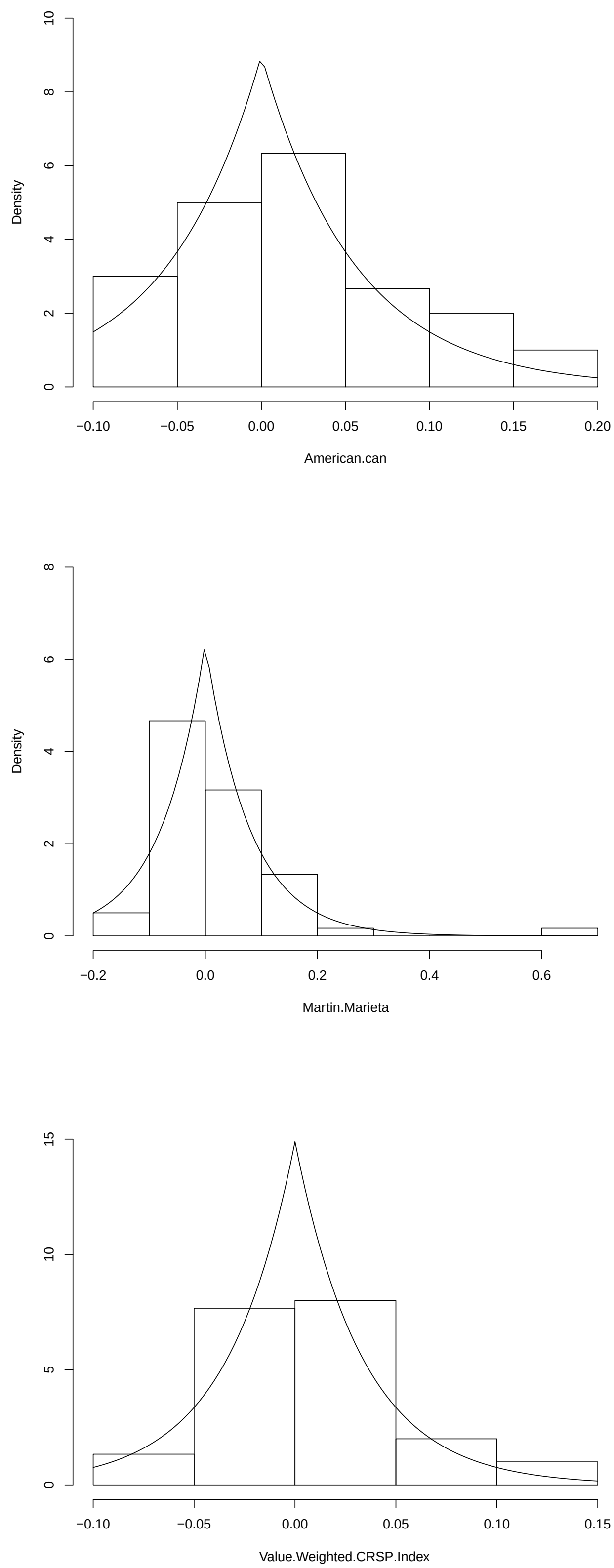

Fig. 4: Fitted model to the monthly interest data

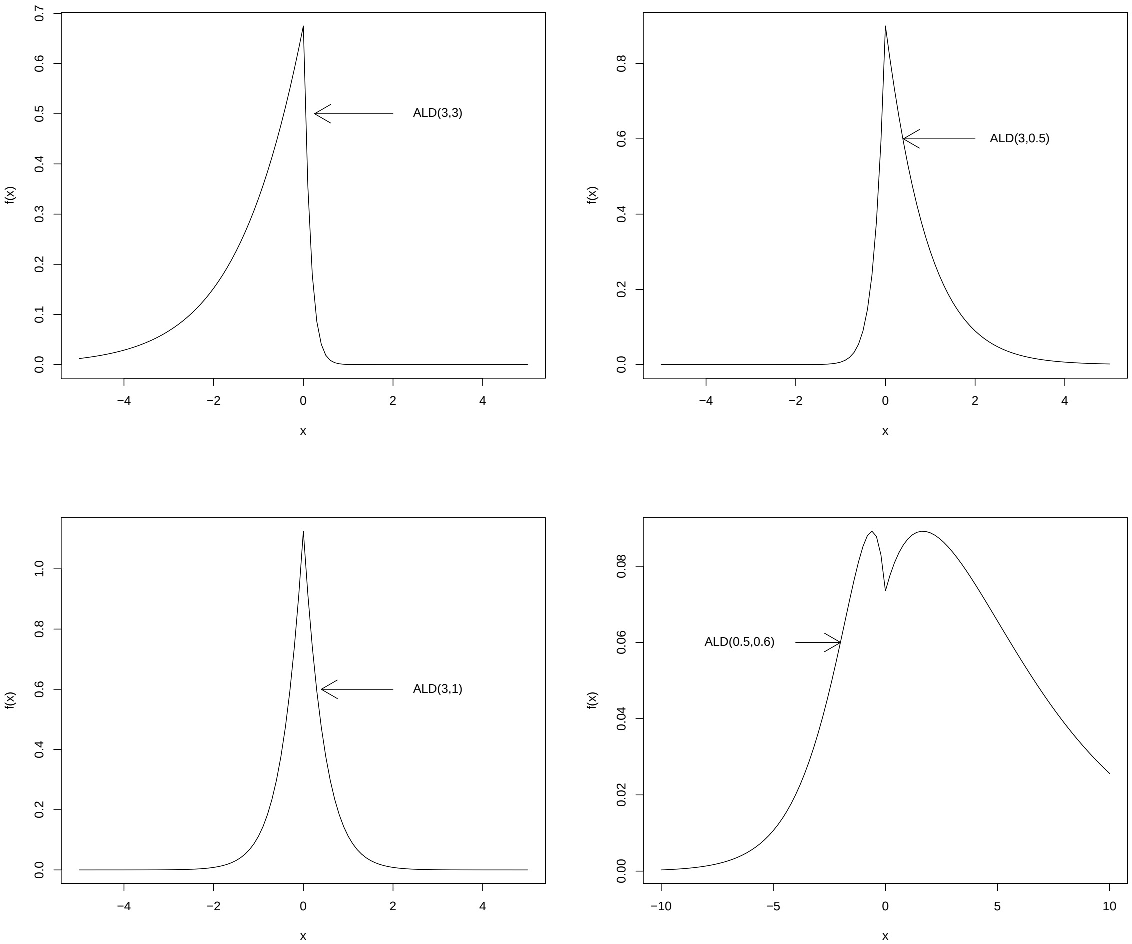

Fig. 5: Shape of the density function (6.1) for different values of θ and κ

$r = 2,$

$$E(X^2) = \frac{(\theta+3)2}{(\theta^2)(\theta+1)} \frac{\kappa}{1+\kappa^2} \left(\kappa^3 + \frac{1}{\kappa^3} \right) \tag{6.4}$$

$r = 3,$

$$E(X^3) = \frac{\theta^2}{\theta+1} \frac{\kappa}{1+\kappa^2} \left(\kappa^4 + \frac{4\kappa^5}{\theta} - \frac{1}{\kappa^4} - \frac{4}{\theta\kappa^5} \right) \tag{6.5}$$

$r = 4$

$$E(X^4) = \frac{\theta^2}{\theta+1} \frac{\kappa}{1+\kappa^2} \left(\kappa^5 + \frac{5\kappa^6}{\theta} + \frac{1}{\kappa^5} - \frac{5}{\theta\kappa^6} \right) \tag{6.6}$$

Since the central moments are in lengthy form, their expressions are omitted here. We numerically calculate the values of the first four central moments, moment measures of skewness and kurtosis for different values of θ and κ using R programming and they are given in Table 6.1. Apparently the same inference about the skewness and kurtosis can be drawn from the form of the pdf given in Figure 5.

Table 6.1: Moment measures of skewness and kurtosis for different values of θ and κ.

Parameters (θ,κ)	central moments	Skewness	Kurtosis	Inference
3,3	$\mu_2 = 1.4692$ $\mu_3 = -3.1796$ $\mu_4 = 16.6625$	-1.7856	7.7198	Negatively skewed and lepto kurtic
3,0.5	$\mu_2 = 0.6826$ $\mu_3 = 0.9363$ $\mu_4 = 3.4758$	1.6242	7.2443	positively skewed and leptokurtic
3,1	$\mu_2 = 0.333$ $\mu_3 = 0$ $\mu_4 = 0.5926$	0	5.3345	Symmetric and leptokurtic
0.5,0.6	$\mu_2 = 27.26$ $\mu_3 = 146.2038$ $\mu_4 = 3729.493$	1.0270	5.0177	positively skewed and leptokurtic

The distribution function and characteristic function of $X \rightarrow ADLD(\theta,\kappa)$ is given by

$$F(x) = \begin{cases} \frac{k^2 e^{\frac{\theta x}{k}}}{(\theta+1)(1+k^2)}[1+\theta(1-\frac{x}{k})], & \text{if } x \leq 0; \\ 1 - \frac{e^{-\theta k x}}{(\theta+1)(1+k^2)}(1+\theta(1+kx)), & \text{if } x > 0. \end{cases}$$

$$\begin{aligned} \phi(t) &= \frac{\theta^2 k}{(\theta+1)(1+k^2)}\left[\frac{1}{it+\frac{\theta}{k}}\left(1+\frac{1}{k(it+\frac{\theta}{k})}\right)+\frac{1}{it-\theta k}\left(1+\frac{k}{it-\theta k}\right)\right] \\ &= \frac{\theta^2 k}{(\theta+1)(1+k^2)}\left[\frac{\theta+1+kit}{k(it+\frac{\theta}{k})^2}+\frac{it-k(\theta-1)}{(it-\theta k)^2}\right] \end{aligned}$$

The Suvival and hazard functions are

$$S(x) = \begin{cases} 1 - \frac{k^2 e^{\frac{\theta x}{k}}}{(\theta+1)(1+k^2)}[1+\theta(1-\frac{x}{k})], & \text{if } x \leq 0; \\ \frac{e^{-\theta k x}}{(\theta+1)(1+k^2)}(1+\theta(1+kx)), & \text{if } x > 0. \end{cases}$$

and

$$H(x) = \begin{cases} \frac{\theta^2 e^{\frac{\theta x}{k}}(k-x)}{(\theta+1)(k^2)(1-e^{\frac{\theta x}{k}})+\theta(1+ke^{\frac{\theta x}{k}})+1}, & \text{if } x \leq 0; \\ \frac{\theta^2 k(1+kx)}{1+\theta(1+kx)}, & \text{if } x > 0. \end{cases}$$

The Renyi entropy takes the form

$$\begin{aligned} \int f^{\gamma}(x)dx &= \left(\frac{\theta^2}{2(\theta+1)}\right)^{\gamma}\frac{e^{\gamma\theta}}{(\gamma\theta)^{(\gamma+1)}}\Gamma(\gamma+1,\gamma\theta)\left[\kappa+\frac{1}{\kappa}\right] \\ \tau(\gamma) &= -log\theta+\frac{1}{1-\gamma}[\theta\gamma+log\Gamma(\gamma+1,\gamma\theta)-\gamma log2(\theta+1)-(\gamma+1)log\gamma+log(\kappa+\frac{1}{\kappa})] \end{aligned}$$

7 Conclusion

We have introduced a new family of symmetric distributions on real line, which is a generalization of the Lindley distribution. Properties of the newly introduced Double Lindley Distribution are studied and estimation of the parameters is done. Comparison with Laplace distribution is done regarding the behaviour of tail probability. Application of the distribution is illustrated with the help of a real data set. An asymmetric generalization is also provided for modelling skewed data sets.

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