

Generation of Rogue Waves by Interaction of Relativistic Electron Beam with Unmagnetized Dusty Plasma

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Abstract: Theoretical investigations for generation of rogue waves in plasma system have been done; We studied the behavior of the nonlinear rogue waves in Jupiter as result of interaction of streaming relativistic electron beam with three components dusty plasma composed of positive dust grains, as well as Maxwellian electrons and positive ions. The most typical nonlinear equation to describe the propagation of rogue wave is the nonlinear Schrodinger equation so it has been calculated. The interaction of a relativistic electron beam with the plasma system can introduce new modes and instability.

Keywords: Generation of Rogue Waves, Relativistic Electron Beam, unmagnetized dusty Plasma, Relativistic Electron with dust, rogue waves in plasma

1 Introduction

The number of theoretical publications devoted to various collective processes in electron–dust plasma interaction has increased enormously in recent years. This interest is primarily due to the fact that such plasma is typical rather than exceptional in astrophysical conditions. Recent research has demonstrated that extreme waves, waves with crest to trough heights of 20 to 30 meters [1]-[10], the nature of the rogue wave problem from the general viewpoint based on the wave process ideas, briefly discuss the generality of the physical mechanisms suggested for the rogue wave explanation; which are valid for rogue wave phenomena in other media such as solid matters, superconductors, plasmas and nonlinear optics [11]. Interaction of electron beam with plasma may be used for various purposes; it present a great interest for development of new methods in amplification and generation of electromagnetic waves, acceleration of charged particles in plasma, confinement and heating plasma so the electron beam being a source of free energy that can excite oscillations, many studies aimed at the employment of these beam to heat plasma up to thermonuclear temperatures 10^4 ev, the instability resulting from the interaction of charged particle beam with plasma is one of the very common and at present best known instabilities [12]-[15]. Lacina.J. et al. states

that the beam not only amplifies waves in the plasma but also provides for effective absorption of these waves by the plasma [16]. Results have demonstrated that random localizations of energy, induced by the linear dispersive mixing of different harmonics, can grow significantly due to modulation instability [17]. they have numerically calculated chaotic waves of the focusing nonlinear Schrodinger equation (NLSE), starting with a plane wave modulated by relatively weak random waves, they show that the peaks with highest amplitude of the resulting wave composition (rogue waves) can be described in terms of exact solutions of the NLSE in the form of the collision[18]. a method for finding the hierarchy of rational solutions of the self-focusing nonlinear Schrödinger equation and present explicit forms for these solutions from first to fourth order are presented, they explain their relation to the highest amplitude part of a field that starts with a plane wave perturbed by random small amplitude radiation waves, They conclude that the appearance of rogue waves in the deep ocean can be applied to the observation of rogue light pulse waves in optical fibers[19]. They numerically investigate dispersive perturbations of the nonlinear Schrödinger (NLS) equation [20]. It is known that the astrophysical dusty plasmas are presented in our solar system and in the interstellar environments such as in cometary tails, asteroid zones, planetary rings, interstellar medium, lower

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part of the earth’s ionosphere, and magnetosphere [21]-[23]. Indeed, rogue waves have been studied in many different systems including nonlinear fiber optics [24]. Parametrically driven capillary waves [25]. Bose-Einstein condensates [27]. The effect of ion temperature and ion streaming velocity on the modulation of ion –acoustic waves studied at [30].

In this work, it is assumed that the streaming relativistic electrons interact with the solar wind of the Jupiter magnetosphere that contains unmagnetized collisionless positive dust grains of, as well as Maxwellian distribution electron and ion. Therefore, it would be interesting to examine heating plasma and different plasma parameters; such as dust-acoustic speed and streaming densities, to test the existence region of the dust acoustic rogue waves and the rogue waves profile. For this purpose, we focus our attention on the specific balancing between the group dispersion and the nonlinear effect to understand these giant waves in more details and being able to predict their occurrence. The basic equation and system of calculation in [24]-[30] for helping in investigation of our system are used.

2 Basic Equations

The normalized basic fluid equations of the dynamics dust grains are given by,

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + \frac{\partial \phi}{\partial t} = 0, \tag{2}$$

The relativistic electron beam fluid equations are

$$\frac{\partial n_b}{\partial t} + \frac{\partial(n_b u_b)}{\partial x} = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + u_b \frac{\partial}{\partial x}\right) \gamma u_b + 3\mu_b \sigma_b \frac{\partial u_b}{\partial x} + \mu_b \frac{\partial \phi}{\partial t} = 0, \tag{4}$$

The relativistic factor $\gamma = (1 - \frac{u_b^2}{c^2})^{-1/2}$ As in ref [30], the relativistic factor can be approximated by its expansion up to the second term because of the weakly relativistic effect, i.e. $\gamma \approx 1 + \frac{u_b^2}{2c^2}$ The Maxwellian ions and electrons are expressed, respectively, as

$$n_{i,e} = \delta_{i,e} \exp(-s_{i,e} \phi), \tag{5}$$

Equations (1)-(5) are closed by Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} - n_b \delta_b + n_d + n_i \delta_i - n_e \delta_e = 0, \tag{6}$$

Where; $n_b, u_b, n_d, u_d, n_i, u_i, n_e$ and u_e are the number densities and velocity of the electron beam, dust grains,

streaming electrons and ions respectively, ϕ is the electrostatic potential. The densities n_b, n_i and n_e are normalized by $n_{d0} z_{d0}$ and n_d is normalized by n_{d0} . The space coordinate x and time t are normalized by the Debye length $\gamma_D = (\frac{T_{eff}}{4\pi n_{d0} Z_{d0} e^2})^{1/2}$ and the dusty plasma frequency $\omega = (\frac{m_d}{4\pi n_{d0} Z_{d0} e^2})^{-1/2}$, while the velocities and the electrostatic potential ϕ are normalized by the dust-acoustic speed $C_d = (\frac{Z_{d0} T_{eff}}{m_d})^{1/2}$, and $\frac{T_{eff}}{e}$, respectively. Also $\mu_b = \frac{m_d}{m_b Z_{d0}}$, $\sigma_b = T_b/T_{eff}$, $\delta_b = \frac{n_{b0}}{n_{d0} Z_{d0}}$, $\delta_i = \frac{n_{i0}}{n_{d0} Z_{d0}}$, $\delta_e = \frac{n_{e0}}{n_{d0} Z_{d0}}$, $s_{i,e} = \frac{T_{eff}}{T_{i,e}}$ where $T_{e,b}$ is the electron, electron beam temperature, T_i is the ion temperature, and $T_{eff} = n_{d0} Z_{d0} [\frac{n_{i0}}{T_i} + \frac{n_{e0}}{T_e}]$ is effective temperature, Z_{d0} is the dust grain charge number.

3 Derivative of the KDV Equation

We introduce the following stretched space-time variables in order to investigate the propagation of the dust-acoustic conoidal waves (DACWs), and we employ the reductive perturbation method. According to this method,

$$\zeta = \varepsilon^{1/2}(x - Vt) \text{ and } \tau = \varepsilon^{3/2}t \tag{7}$$

Where $\varepsilon \ll 1$ and V is the dust acoustic phase speed. The physical quantities appearing in equations (1)-(6) obey the function; $\Psi = [n_d, u_d, n_b, u_b, n_i, n_e, \phi]$ and expanded as a power series in ε about their equilibrium values as;

$$\Psi = \Psi_0 + \sum_{j=1}^{\infty} \varepsilon^j \Psi_j, \tag{8}$$

Where $\Psi_j = [n_{dj}, u_{dj}, n_{bj}, u_{bj}, n_{ij}, n_{ej}, \phi_j]^M$ and $\Psi_0 = [1, 0, \mathcal{S}_b, u_{b0j}, \mathcal{S}_i, \mathcal{S}_e, 0]^M$. The lowest-order in ε by substituting (7), (8) into equations. (1)- (6) gives;

$$\begin{aligned} n_{d1} &= \phi_1/V^2, u_{d1} = \phi_1/V, u_{b1} = \frac{-(V - u_{b0})\mu_b}{\gamma_1(V - u_{b0})^2 - 3\mathcal{S}_b^2\mu_b\sigma_b} \phi_1, \\ n_{b1} &= \frac{-\mathcal{S}_b\mu_b}{\gamma_1(V - u_{b0})^2 - 3\mathcal{S}_b^2\mu_b\sigma_b} \phi_1, n_i = -\delta_i s_i \phi_1 \text{ and} \\ n_e &= \delta_e s_e \phi_1 \end{aligned} \tag{9}$$

While as in [30], $\gamma_1 = 1 + \frac{3u_{b0}^2}{2C^2}$, and $\gamma_2 = \frac{3u_{b0}^2}{2C^2}$ At our work we suppose weakly relativistic electron beam and so neglect γ_2 , The Poisson equation gives the compatibility condition;

$$\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\mathcal{S}_b \mu_b}{\gamma_1(V - u_{b0})^2 - 3\mathcal{S}_b^2 \mu_b \sigma_b} \phi_1 \delta_b + \phi_1/V^2 - s_i \phi_1 \delta_i^2 - s_e \phi_1 \delta_e^2 = 0, \tag{10}$$

Solving (10) gives the values of V , that will be used in our numerical analysis. If we consider the next-order in ε , we

obtain a system of equation in the second-order perturbed quantities. Solving the system, we finally obtain mKDV equation;

$$\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \zeta} + B \frac{\partial^3 \phi_1}{\partial \zeta^3} = 0, \quad (11)$$

Where A and B are given by;

$$A = B \left(\frac{-3(\gamma_1(V - u_{b0})^2 \mu_b^2 \delta_b^2 + \delta_b^4 \mu_b^3 \sigma_b)}{(\sqrt{\gamma_1}(V - u_{b0}) - 3\mathcal{S}_b^2 \mu_b \sigma_b)^3} + 3/V^4 + s_i^2 \delta_i^2 - \frac{1}{2} s_e^2 \delta_e^2 \right),$$

$$B = 1/2 \left(\frac{-3(\gamma_1(V - u_{b0})^2 + \delta_b^2 \mu_b)}{(\sqrt{\gamma_1}(V - u_{b0}) - 3\delta_b^2 \mu_b \sigma_b)^2} + 1/V^3 \right),$$

The propagation of positive and negative pulse depends on the sign of the coefficient of the nonlinear term A in KDV, also the relativistic of electron beam more affect on the propagation pulse, equation (11) i.e;

$$\left\{ \begin{array}{l} \text{if } A > 0, \text{ the pulse are positive} \\ \text{if } A < 0, \text{ the pulses are negative} \\ \text{if } A = 0 \text{ the KDV equation break down} \end{array} \right\}$$

And the KDV equation breakdown if the electron concentration reaches to critical values, so $A = 0$ at $\delta_e = \delta_{ec}$

To investigate the rogue waves, transformation of the KDV equation to the nonlinear Schrodinger equation (NLSE) should be done. However, the NLSE that derived from the KDV equation may not support the existence of rogue wave. And then the general method of the reductive perturbation theory introduce the modified stretched variables defined by

$$\begin{aligned} \zeta &= \varepsilon(x - Vt) \\ \tau &= \varepsilon^3 t \end{aligned} \quad (12)$$

using the stretching (12) along with the expansion (8) into the basic equations. (1)- (6), after some algebraic manipulations, we finally obtain the modified Korteweg-de Vries (mKDV) equation as

$$\frac{\partial \phi_1}{\partial \tau} + C \phi_1^2 \frac{\partial \phi_1}{\partial \zeta} + B \frac{\partial^3 \phi_1}{\partial \zeta^3} = 0, \quad (13)$$

Where

$$C = B \left(\frac{15(\sqrt{\gamma_1}(V - u_{b0}))^4 \mu_b^3 \delta_b^2 + 90\gamma_1(V - u_{b0})^2 \mathcal{S}_b^4 \mu_b^4 \sigma_b + 27\delta_b^6 \mu_b^5 \sigma_b^2}{2(\gamma_1(V - u_{b0})^2 - 3\mathcal{S}_b^2 \mu_b \sigma_b)^5} + 15/2V^6 - \frac{1}{72} s_i^3 \delta_i^2 - \frac{1}{72} s_e^3 \delta_e^2 \right),$$

Now, it is interesting to transform the mKDV equation (13) to NLSE to describe the behavior of the weakly nonlinear wave packet that gives rise to rogue wave propagation. So we expand ϕ_1 as in ref. [28]

$$\phi_1(\zeta, \tau) = \sum_{n=1}^{\infty} \varepsilon^n \sum_{l=-n}^n \phi_{1l,n}(X, T) \exp il(k\zeta - \omega\tau), \quad (14)$$

The new spatial and temporal coordinates (X, T) are stretched as

$$\begin{aligned} X &= \varepsilon(\zeta - V_g \tau) \text{ and} \\ X &= \varepsilon^3 \tau \end{aligned} \quad (15)$$

Where k is the carrier wavenumber and ω is the frequency for the given dust- acoustic waves and V_g is the group velocity of the envelope wavepaket. Assume that all perturbed states depend on the fast scales via the phase $(\zeta - V_g \tau)$ only, while the slow scales (X, T) enter the arguments of the l th harmonic amplitude $\phi_{1l,n}$. Since $\phi_1(\zeta, \tau)$ must be real, the coefficient in equation (14) have to satisfy the condition $\phi_{1l,n} = \phi_{1l,n}^*$, where the asterisk stands for the complex conjugate.

The derivative operators appearing in equation (13) become

$$\frac{\partial}{\partial \zeta} \Rightarrow \frac{\partial}{\partial \zeta} + \varepsilon \frac{\partial}{\partial X} \text{ and } \frac{\partial}{\partial \tau} \Rightarrow \frac{\partial}{\partial \tau} - \varepsilon V_g \frac{\partial}{\partial X} + \varepsilon^2 \frac{\partial}{\partial T} \quad (16)$$

From the first order approximation ($n = 1$) with ($l = 1$) by using equations (14)-(16) into equation (13), we obtain the linear dispersion relation

$$\omega = -Ck^3, \quad (17)$$

For the first harmonic of the second- order approximation ($n = 2$) and ($l = 1$), we calculate the group velocity as

$$V_g = -3Ck^3, \quad (18)$$

And then the third- order approximation ($n = 3$) and solving for first harmonic equation ($l = 1$), and explicit compatibility condition will be found, from which we can easily obtain the nonlinear Schrodinger equation NLSE as

$$i \frac{\partial \theta}{\partial T} + P \frac{\partial^2 \theta}{\partial X^2} + Q \theta |\theta|^2 = 0, \quad (19)$$

Where $\theta = \phi_{1l,n}$, ($l = 1$), ($n = 3$). and the coefficient P and Q are given by

$$P = -3Bk \text{ and } Q = -Ck \quad (20)$$

It is interesting to mention here that since the stretching variables (7) cannot describe the wave propagation at $A = 0$. we used instead of it the stretched variables (12) So, we used equation (12) to obtain new evolution equation (13) that is valid for describing the plasma system at $A = 0$. Both equations (11), (13) have spatial and temporal coordinates ζ and τ respectively. In this frame of study, the wave can propagate with group velocity V . On the other hand when we transform equations (11) and (13) into equation (19) then the new wave packet propagate with group velocity V_g . In this case, we have two different time scales; the first one is for fast time scale ζ and τ with phase velocity V which is for the carrier wave. The second time scale is for the slow

time scale X and T with group velocity V_g which is for the envelop wave packet. Therefore, each of V and V_g has different physical meaning and they describe two different time scales. It leads to point out that equation (11) and (13) admit solution, which could be of interest if we are interesting in the central region of the envelope. It is straightforward to see that a negative sign for P/Q is required for wave amplitude modulation stability. On the other hand, a positive sign of P/Q allows for the random perturbations to grow and thus the rogue wave could be created.

The character of the dynamic wave depends on the sign of the ratio of $\frac{P}{Q} = 3/C$ this sign refers to the stability so far of the system i.e.

$$\left\{ \begin{array}{l} \text{when } \frac{P}{Q} > 0, \text{ The unstable envelope pulses propagate} \\ \text{when } \frac{P}{Q} < 0, \text{ the stable envelope pulses exist.} \\ \text{if } C < 0 \text{ the waves become stable} \\ \text{if } C > 0 \text{ the waves become unstable} \end{array} \right\}$$

The NLSE equation (19) has a rational solution that located on a nonzero background and localized both in the X and T directions [27] as

$$\theta = \sqrt{\frac{P}{Q}} \left(\frac{G_0 + i\omega G_1}{G_2} + 1 \right) \exp(i\omega), \quad (21)$$

Where

$$\begin{aligned} G_0 &= \frac{3}{8} - 3X^2 - 2X^4 - 9\omega^2 - 10\omega^4 - 12X^2\omega^2, \\ G_1 &= \frac{15}{4} + 6X^2 - 4X^4 - 2\omega^2 - 4\omega^4 - 8X^2\omega^2, \\ G_2 &= \frac{1}{8} + 9X^2 + 4X^4 + \frac{16}{3}X^6 + 33\omega^2 + 36\omega^4 \\ &\quad + \frac{16}{3}\omega^6 - 24X^2\omega^2 + 16X^4\omega^2 + 16X^2\omega^4, \\ \omega &= QT, \end{aligned} \quad (22)$$

Equation (21) represents the rogue wave solution within the unstable zone of the NLSE (19) for which the coefficient of the nonlinear term is positive. Solution (21) reveals that a significant amount of the wave energy is concentrated in a relatively small area in space. The rogue wave is usually an envelope of a carrier wave with a wavelength smaller than the central region of the envelope. On the other hand the positive sign of P/Q allows for the random perturbation grow and thus the rogue wave could be created while the negative sign of P/Q is required for wave amplitude modulation stable. Also from equations (13), (19) by the values of C, P, Q , it is appear that the relativistic of electron beam more affect on the generation of waves. On the other hand while the distribution of streaming electrons and ions considered as Maxwellian distributions and the generation waves affected by the relativistic effect then the wave generation should be rogue waves.

4 Conclusion

We have investigated the behavior of the nonlinear rogue waves as a result of interaction of relativistic streaming electron beam with it, is assumed that the streaming relativistic electrons interact with the solar wind of the Jupiter magnetosphere that contains unmagnetized collisionless positive dust grains, as well as Maxwellian distribution electron and ions. It is found that at certain parameters of dust- acoustic speed, streaming densities of electron beam, and temperature ratio, the relativistic effect of streaming electron beam, the perturbations could lead to the occurrence of rogue waves. The dust-acoustic phase speed, the temperature ratio, and the relativistic streaming practices number densities of electron beam play a significant role in deciding how much energy could concentrate in the rogue waves.

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References

- [1] Simon Birkholz, Carsten Brée, Ayhan Demircan, and Günter Steinmeyer, *Physics Review Letters* 114, 213901, 2015.
- [2] Holliday, NP, MJ Yelland, RW Pascal, VR Swail, PK Taylor, CR Griffiths, and EC Kent, *Geophysical Research Letters*, Vol. 33 L05613(2006).
- [3] P. K. Shukla, I. Kourakis, B. Eliasson, M. Marklund and L. Stenflo: nlin.CD/0608012, *Physical Review Letters* (2006)
- [4] R. Colin Johnson (December 24, 2007). *Electronic Engineering Times* (1507): 14, 16.
- [5] Kibler, B.; Fatome, J.; Finot, C.; Millot, G.; Dias, F.; Genty, G.; Akhmediev, N.; Dudley, J.M. *Nature Physics* **6** (10) (2010).
- [6] Skourup, J, Hansen, N. O., Andreassen, K. K. *Journal of Offshore Mechanics and Arctic Engineering*. doi:282906, (2016).
- [7] Holliday, N.P.; Yelland, M.Y.; Pascal, R.; Swail, V.; Taylor, P.K.;Griffiths, C.R.; Kent, E.C. (2006). *Geophysical Research Letters* (Wiley) **33**(5): L05613. Retrieved (2016).
- [8] I. Nikolkina and I. Didenkulova, *Nat. Hazards Earth Syst. Sci.*, 11, 2913–2924,(2011).
- [9] Soomere, T., *Eur. Phys. J.-Spec.Top.*, 185, 81–96, (2010).
- [10] Chien, H., Kao, C.-C., and Chuang, L. Z. H., *Coast. Eng. J.*, 44(4), 301–319, 2002.
- [11] Alexey Slunyaev, Ira Didenkulova, Efim Pelinovsky. *Contemporary Physics* 52, 571-590 (2011)
- [12] Jagher P.C., Sluizter F.W. and Hopman H.J., *Review Section of Phys.Lett.* Vol. 167, 4,177, (1988)
- [13] Cap F. *Handbook of Plasma Instabilities*, Academic Press, Vol.2, (1978).
- [14] EL-Shorbagy KH.H., *phys. Lett. A*, Vol. 287, 120, (2001)

- [15] Gupta G.P., Vijayan T. and Rohatgi V.K., The Phys. of Fluids, vol.31, 3, 606.(1988)
- [16] Lacina.J. et al., Plasma Physics, vol. 17, 197, (1975).
- [17] Will Cousins and Themistoklis P. Sapsis. J. Fluid Mech. (2016), vol. 790, pp. 368–388.
- [18] Akhmediev, N.; Soto-Crespo, J. M.; Ankiewicz, A. Physics Letters A, Volume 373, Issue 25, p. 2137-2145. **2009**.
- [19] Akhmediev, Nail; Ankiewicz, Adrian; Soto-Crespo, J. M. Physical Review E, vol. 80, Issue 2, id. 026601, **2009**.
- [20] Calini, A.; Schober, C. M. Physics Letters A, Volume 298, Issue 5-6, p. 335-349. **2002**.
- [21] F. C. Michel, Rev. Mod. Phys. **54**, 1 (1982).
- [22] M. Horanyi and D. A. Mendis, J. Astrophys. 294, 357 (1985).
- [23] A. Bouchule, Dusty Plasmas (Wiley, New York, 1999).
- [24] R. Sabry, W. M. Moslem, and P. K. Shukla, Eur. Phys. J. D **51**, 233 (2009).
- [25] E. F. El-Shamy, W. M. Moslem, and P. K. Shukla, Phys. Lett.A **374**, 290 (2009).
- [26] S. K. El-Labany, E. F. El-Shamy, W. F. El-Taibany, and P. K. Shukla, Phys. Lett. A **374**, 960 (2010).
- [27] S. Ali, W. M. Muslem, P. K. Shukla, and R. Schlick-eiser, Phys. Plasmas **14**, 082307 (2007).
- [28] R. E. Tolba, W. M. Moslem, N. A. El-Bedwehy, and S. K. El-Labany. Physics of Plasma 22, 043707 (2015)
- [29] R. Z. Sagdeev, Vopr. Teor.Plazmy, No. 4, 20 (1964).
- [30] S. K. El-Labany, J. plasma Physics, vol.54, part 3, pp. 295-308,v (1995).



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