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# Heat and Mass Transfer Flow of MHD Viscous Dissipative Fluid in a Channel with a Stretching and Porous Plate

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**Abstract:** Present analysis has been carried out to study the two dimensional steady hydromagnetic convective fluid flow between two parallel plates with heat and mass transfer in the presence of thermal radiation and viscous dissipation. Here the lower plate is stretching and porous. Governing equations are solved by forth order Runge - Kutta Method along with shooting technique. The effects of the flow parameters such as magnetic parameter, radiation parameter, suction parameter, Reynolds number, Eckert number, Prandtl number and Schmidt number on the velocity, temperature and concentration profile are examined and the effects are explained graphically.

**Keywords:** Stretching sheet, suction, viscous dissipation, MHD.

## 1 Introduction

The heat and mass transfer due to a stretching surface in a channel, used in a present work, has important application in polymer technology and metallurgy. For example, a number of technical processes concerning polymers involve the cooling of continuous strips extruded from a die by drawing them through a quiescent stretched. Other examples are continuous casting of metals, manufacture of plastic and rubber sheets, glass blowing, the extruded material issues through die and spinning of fibers. In view of these applications, Afify [1] studied Similarity solution in MHD: Effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. Ali [2] presented the thermal boundary layer on a power law stretched surface with suction or injection. Borkakoti and Bharali [4] have studied hydromagnetic flow and heat transfer between two horizontal plates. Chakrabarti and Gupta [5] reported on the hydromagnetic flow and heat transfer over a stretching sheet. The effect of heat transfer of a continuous, stretching surface with suction or blowing has been investigated by Chen and Char [6]. Crane [7] discussed the flow past a stretching plate. Cortell [8] trailed the investigation of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. Heat and mass transfer on a moving continuous flat plate with suction or injection has been studied by Erickson et al. [9]. Giterere et al. [10] analyzed MHD flow in porous media over stretching surface in a rotating system with heat and mass transfer. Guptaand Gupta [11] inspected the effect of heat and mass transfer on a stretching sheet with suction or blowing. Hazem [12] discussed the effect of suction and injection on unsteady Couette flow with variable properties.Boundary layer flow and heat transfer over an unsteady stretching vertical surface is analyzed by Ishak et al. [14]. The effects of MHD flow past a stretching permeable sheet are examined by Kumaran [15]. Kung and Srivastava [16] analyzed analytic transient solutions of a cylindrical heat equation with oscillating heat flux. Several researches used different approaches and find some applications [3,13,17,18,19,20,21,22] Mukhopadhyay [23] considered the unsteady boundary layer flow and heat transfer past a porous stretching sheet in presence of variable viscosity and thermal diffusivity. Subhas and Mahesha [24] estimated the heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. Sheikholeslami [25] inspected the rotating MHD viscous flow and heat transfer between stretching and porous surfaces using analytical method.

The purpose of present study is to examine the effects of mass transfer,viscous dissipation and suction parameter on two dimensional steady hydromagnetic viscous fluid flow between two parallel plates in the presence of

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thermal radiation, where the lower plate is stretching and porous.In this study Runge-Kutta forth order method successfully applied to solve the governing equations.

## 2 Mathematical Formulation

Consider steady two dimensional flow of an electrically conducting incompressible dissipative fluid between two vertical parallel plates when the lower plate is a permeable and stretching plate and two equal opposite forces are applied along the $x$ - axis, so that the wall is stretched ($a$ >0) keeping the origin fixed. The flow is assumed to be in the $x$ - direction which is taken along the plates and $y$ - axis is normal to it. A uniform magnetic field $B$ is acting along $y$ - axis. The lower plate at $y=0$ and upper plate at $y=h$ of the channel are kept at temperature $T_w$ and $T_h$. The lower plate is subjected to a constant flow suction with a velocity $v_0$. Under these assumptions, the Geometry and governing equations of the problem are:

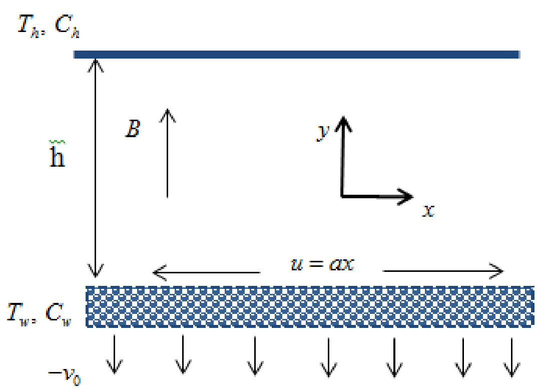


**Fig. 1:** Geometry of the Problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\upsilon}{\rho c_p}(\frac{\partial u}{\partial y})^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Here $u$ and $v$ are the velocities in the $x$, $y$ directions respectively, $T$ is the temperature, $C$ is the concentration, $\rho$ is the base fluid density, $\upsilon$ is the dynamic viscosity, $k$ is the thermal conductivity, $c_p$ is the specific heat and $D$ is the chemical molecular diffusivity. The boundary conditions applicable to the present channel flow are

$$\begin{cases} u=ax, v=-v_0, T=T_w, C=C_w \ at \ y=0 \\ u=0,\ v=0, T=T_h, C=C_h \ at \ y=h \end{cases} \tag{5}$$

By using the Rosseland approximation consider the radiative heat flux for optically thick fluid is given by

$$q_r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where is the Stefan-Boltzmann constant and $k^*$ is the mean absorption coefficient. Assuming that the difference in temperature within the flow are sufficiently small such that $T^4$ can be expressed as a linear function of the temperature, we expand $T^4$ in a Taylor's series about $T_h$ and neglecting higher order terms, thus

$$T^4=4T_h^3T-3T_h^4$$

Let us introduce the following similarity transformations

$$\begin{aligned} &\eta=\tfrac{y}{h},\ u=axf'(\eta),\ v=-ahf(\eta), \\ &\theta(\eta)=\tfrac{T-T_h}{T_w-T_h},\ \xi(\eta)=\tfrac{C-C_h}{C_w-C_h} \end{aligned} \tag{7}$$

and non-dimensional parameters

$$\begin{aligned} &M=\tfrac{\sigma B^2h^2}{\rho\upsilon},\ Re=\tfrac{ah^2}{\upsilon},\ R=\tfrac{kk^*}{4\sigma T_h^3},\ Pr=\tfrac{\mu C_p}{k}, \\ &Ec=\tfrac{(ax)^2}{C_p(T_w-T_h)},\ S=\tfrac{v_0}{ah},\ Sc=\tfrac{\upsilon}{D} \end{aligned} \tag{8}$$

where $M$ is the magnetic parameter, $Re$ is the Reynolds number, $R$ is the radiation parameter, $Pr$ is the Prandtl number, $Ec$ is the Eckert number, $S$ suction parameter $(S>0)$, $Sc$ Schmidt number and a prime denotes differentiation with respect to $\eta$.

The above equations (6)-(8) reduces the equations (1)-(4) into the following system of non-dimensional equations

$$f'''-Ref'^2+Reff''-Mf'=0 \tag{9}$$

$$(3R+4)\theta''+3RePr\left(Rf\theta'+Ecf''^2\right)=0 \tag{10}$$

$$\xi''+ReScf\xi'=0 \tag{11}$$

Non- dimensional boundary conditions are

$$\begin{cases} f'=1, f=S, \theta=1, \xi=1 \ at \ \eta=0 \\ f'=0,\ f=0, \theta=0, \xi=0 \ at \ \eta=1 \end{cases} \tag{12}$$

The local skin-friction coefficient $(\tau)$ on the lower stretching wall is given by

$$\tau = \frac{\tau_w}{\rho(ax)^2} \Rightarrow \frac{Rex}{h}\tau = f''(0)$$

where the wall shear stress may be written as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

The rate of heat transfer (Nusselt number) and rate of mass transfer (Sherwood number) on the lower stretching wall are defined below

$$Nu = h\left(\frac{\frac{\partial T}{\partial y}}{T_h - T_w}\right)_{y=0} \Rightarrow Nu = -\theta'(0)$$

$$Sh = h\left(\frac{\frac{\partial C}{\partial y}}{C_h - C_w}\right)_{y=0} \Rightarrow Sh = -\xi'(0)$$

## 3 Numerical results and discussions

Using a similarity transformation, the governing equations (1)-(4) are transformed into a system of non-dimensional ordinary differential equations (9)-(11), which are solved numerically using forth order Runge - Kutta method along with shooting technique with the help of MATLAB (R2010a) software and throughout the computations we employ $M = 0.5, Re = 2, R = 0.8, Pr = 3, Ec = 0.8, Sc = 0.3, S = 0.5$.

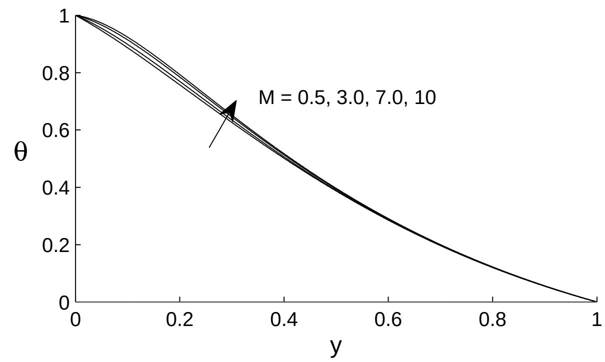


**Fig. 3:** Temperature profiles for various values of $M$ with $Re = 2$, $R = 0.8$ $Pr = 3$,$Ec = 0.8$, $S = 0.5$.

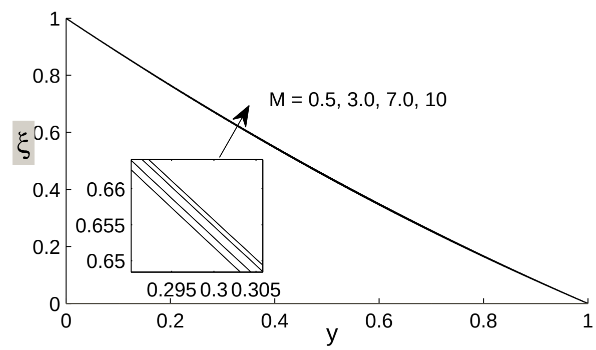


**Fig. 4:** Concentration profiles for various values of $M$ with $Re = 2$, $Sc = 0.3$, $S = 0.5$.

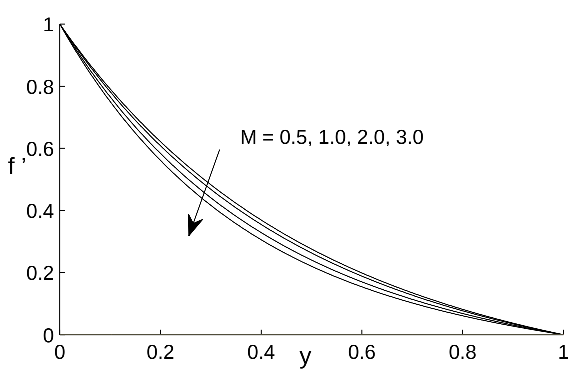


**Fig. 2:** Velocity profiles for various values of $M$ with $Re = 2$, $S = 0.5$.

The results are illustrated graphically in figures (2)-(14) for the non-dimensional parameters, namely the magnetic parameter $M$, suction parameter $S$ , Reynolds number $Re$, radiation parameter $R$, Prandtl number $Pr$, Eckert number $Ec$,Schmidt number $Sc$ on velocity, temperature and concentration profiles. Also, the skin friction coefficient, Nusselt and Sherwood numbers are discussed and given in tabular form.

For the case of different values of magnetic field parameter $M$ , the velocity, temperature and concentration profiles are shown in Figures (2)-(4). It is noticed from figure (2) that the velocity decreases with increase the magnetic parameter $M$. This is due to the fact that an increase in transverse magnetic field develops the opposite force to the flow direction (Lorentz force), which results in retarding force on the velocity field. This force has the tendency to reduce the velocity boundary layer and increase the thermal boundary layer thickness. It is clear from the Figures (3) and (4), that an increase in the magnetic field parameter enhances the temperature and concentration profile.

Influence of suction parameter $S$ over velocity, temperature and concentration profiles are displayed in Figures (5)-(7). We know that the effect of suction is to bring the fluid closer to the surface and, therefore, to

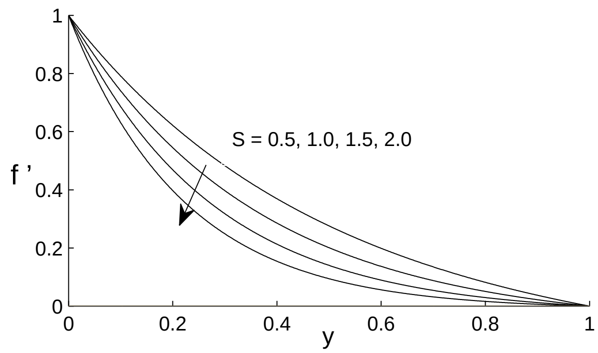


**Fig. 5:** Velocity profiles for various values of $S$ with $M = 0.5$, $Re = 2$.

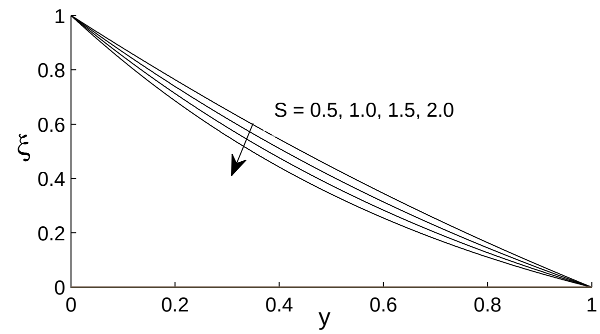


**Fig. 7:** Concentration profiles for various values of $S$ with $M = 0.5$, $Re = 2$, $Sc = 0.3$.

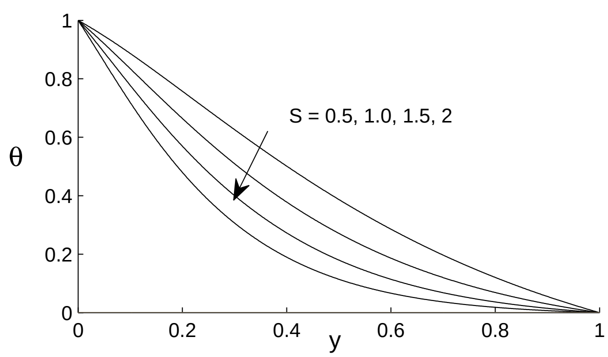


**Fig. 6:** Temperature profiles for various values of $S$ with $M = 0.5$, $Re = 2$, $R = 0.8$ $Pr = 3$,$Ec = 0.8$.

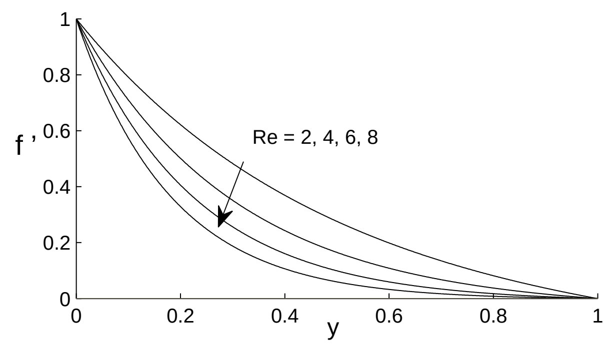


**Fig. 8:** Velocity profiles for various values of $Re$ with $M = 0.5$, $S = 0.5$.

reduce the thermal boundary layer thickness. Thus, these plots show that velocity, temperature and concentration profiles decreases significantly with increasing suction parameter.

Figures (8) - (10) depict the effects of the Reynolds number $Re$ on velocity, temperature and concentration profiles, respectively. It is observed from the figures that an increase in the Reynolds number decreases the velocity, temperature and concentration flow profile.

Figure (11) demonstrates the variations of temperature distribution for different value of Eckert number $Ec$. This figure reveals that an increase in the value of the Eckert number indicates that there is an increase in the viscous dissipation and thereby increases the temperature of the fluid as the dissipative force adds energy to the fluid.

The temperature distribution for different values of radiation parameter $R$ and Prandtl number $Pr$ are shown in Figure (12)-(13). From this figures it is noticed that temperature decreases with the increase in radiation parameter and Prandtl number. From Figure (12), it is observed that increase in the radiation parameter $R$ decreases the temperature distribution in the thermal boundary layer. This is because large values of radiation parameter correspond to an increase the conduction over radiation, thereby decreasing the thickness of the thermal boundary layer. From Figure (13), The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. Therefore, an increase in the Prandtl number means slow rate of thermal diffusion.

Figure (14) illustrates the concentration profiles for different values of the Schmidt number $Sc$. The Schmidt number defined as the ratio of viscosity and molecular diffusivity, and it is physically related to the relative thickness of the mass-transfer boundary layer. It is clear that the concentration of the fluid reduces with an increase of the Schmidt number. As can be expected, the concentration boundary layer thickness decreases as Schmidt number increases, with all other parameters fixed. All these physical behaviors are due to the increase of Schmidt number means a decrease of molecular diffusion.

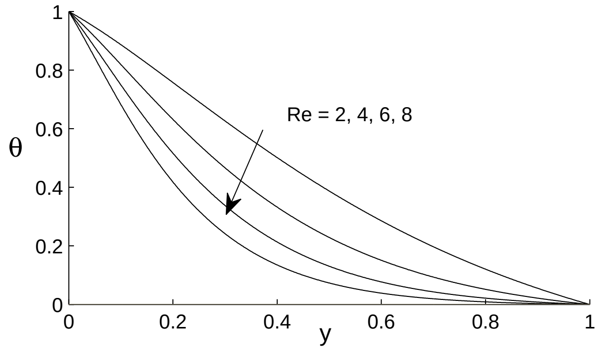


**Fig. 9:** Temperature profiles for various values of $Re$ with $M = 0.5$, $R = 0.8$, $Pr = 3$, $Ec = 0.8$, $S = 0.5$.

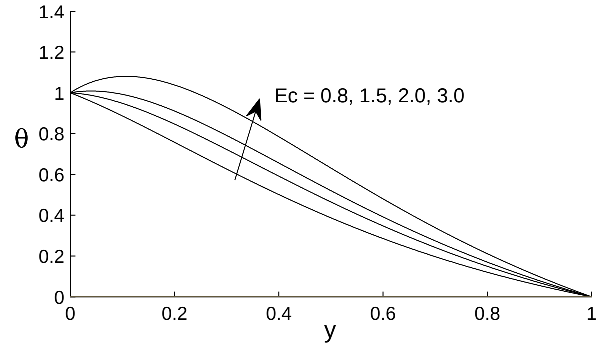


**Fig. 11:** Temperature profiles for various values of $Ec$ with $M = 0.5$, $Re = 2$, $R = 0.8$, $Pr = 3$,$S = 0.5$.

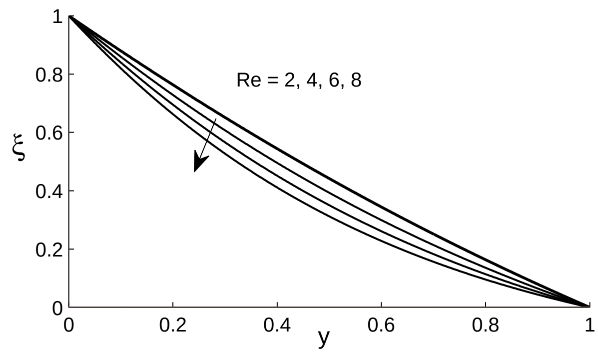


**Fig. 10:** Concentration profiles for various values of $R$ with $M = 0.5$, $Re = 2$, $S = 0.5$.

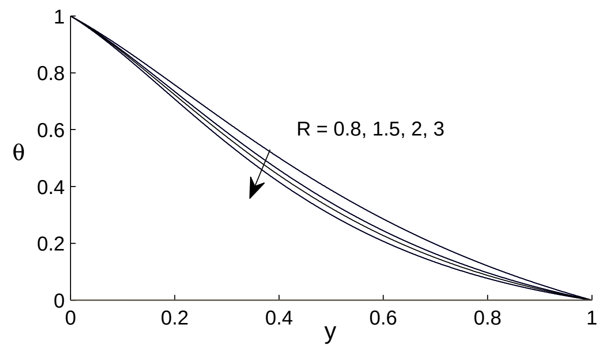


**Fig. 12:** Temperature profiles for various values of $R$ with $M = 0.5$, $Re = 2$, $Pr = 3$, $Ec = 0.8$, $S = 0.5$.

**Table 1:** Effects of various physical parameters on local skin friction coefficient $f''$ and Nusselt number $-\theta'$ at the lower plate.

| $M$ | $Re$ | $S$ | $R$ | $Pr$ | $Ec$ | $f''(0)$ | $-\theta'(0)$ |
|---|---|---|---|---|---|---|---|
| 0.5 | 2 | 0.5 | 0.8 | 3 | 0.8 | -2.310084 | 0.988451 |
| 2 | 2 | 0.5 | 0.8 | 3 | 0.8 | -2.645753 | 0.851188 |
| 3 | 2 | 0.5 | 0.8 | 3 | 0.8 | -2.850548 | 0.763521 |
| 0.5 | 4 | 0.5 | 0.8 | 3 | 0.8 | -3.410390 | 1.595065 |
| 0.5 | 6 | 0.5 | 0.8 | 3 | 0.8 | -4.486997 | 2.258052 |
| 0.5 | 2 | 1 | 0.8 | 3 | 0.8 | -2.980835 | 1.507637 |
| 0.5 | 2 | 1.5 | 0.8 | 3 | 0.8 | -3.747670 | 2.102212 |
| 0.5 | 2 | 0.5 | 1.5 | 3 | 0.8 | -2.310084 | 1.033381 |
| 0.5 | 2 | 0.5 | 2 | 3 | 0.8 | -2.310084 | 1.061181 |
| 0.5 | 2 | 0.5 | 0.8 | 2.3 | 0.8 | -2.310084 | 0.974540 |
| 0.5 | 2 | 0.5 | 0.8 | 1.5 | 0.8 | -2.310084 | 0.970477 |
| 0.5 | 2 | 0.5 | 0.8 | 3 | 1.5 | -2.310084 | 0.147843 |
| 0.5 | 2 | 0.5 | 0.8 | 3 | 2 | -2.310084 | -0.452592 |

**Table 2:** Effects of various physical parameters on Sherwood number $-\theta'$ at the lower plate.

| $M$ | $Re$ | $S$ | $Sc$ | $-\xi'(0)$ |
|---|---|---|---|---|
| 0.5 | 2 | 0.5 | 0.3 | 1.222485 |
| 2 | 2 | 0.5 | 0.3 | 1.219291 |
| 3 | 2 | 0.5 | 0.3 | 1.217432 |
| 0.5 | 4 | 0.5 | 0.3 | 1.447782 |
| 0.5 | 6 | 0.5 | 0.3 | 1.679041 |
| 0.5 | 2 | 1 | 0.3 | 1.392227 |
| 0.5 | 2 | 1.5 | 0.3 | 1.575777 |
| 0.5 | 2 | 0.5 | 1 | 1.829956 |
| 0.5 | 2 | 0.5 | 1.5 | 2.321273 |

To assess the results of the analytical method, we have tabulated our local skin friction coefficient, Nusselt number $-\theta'(0)$ and Sherwood number $-\xi\prime(0)$ for different values of involving physical parameters at the lower stretching plate in Table 1 and Table 2. It can be seen that the skin friction coefficient decreased by increasing the magnetic parameter $M$ , Reynolds number

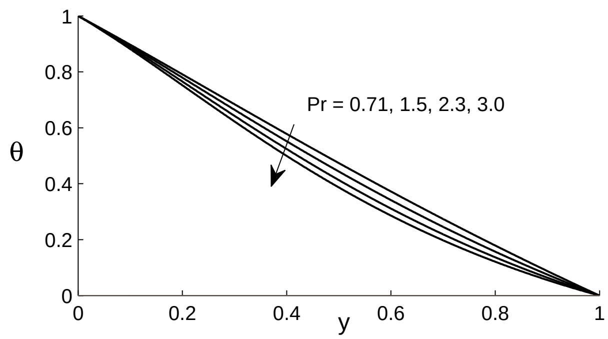


**Fig. 13:** Temperature profiles for various values of $Pr$ with $M = 0.5$, $Re = 2$, $R = 0.8$, $Ec = 0.8$, $S = 0.5$.

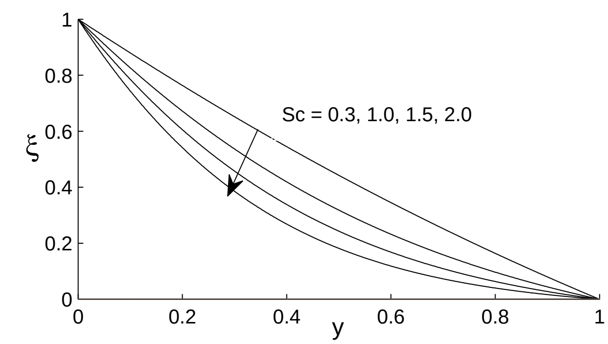


**Fig. 14:** Concentration profiles for various values of $Sc$ with $M = 0.5$, $Re = 2$, $S = 0.5$.

$Re$ and suction parameter $S$ . Also it is noticed that the Nusselt number increases by increasing the Reynolds number $Re$ , suction parameter $S$ , radiation parameter $R$ and Prandtl number $Pr$ , whereas it decreased with increasing the magnetic parameter $M$ and Eckert number $Ec$ . From Table 2, it is observed that the Sherwood number increased by increasing Reynolds number $Re$ , suction parameter $S$ and Schmidt number $Sc$ , whereas it decreases with the increasing value of the magnetic parameter $M$ .□.

## 4 Conclusion

A theoretical analysis is performed to study the heat and mass transfer flow in the presence of thermal radiation and viscous dissipation between two parallel plates, where the lower plate is stretching and porous. The Runge - Kutta Method along with shooting technique is employed to find the results of velocity, temperature and concentration. The findings of the numerical results can be summarized as follows:

- It is found that velocity, temperature and concentration decreases with the increase of variable suction parameter and Reynolds number.
- Also, the magnetic parameter has the effect of decreasing the velocity profile.
- An increase in magnetic field parameter, Eckert number and Prandtl number enhances the temperature profile. However, it decreases with the increase of radiation parameter.
- In the case of magnetic parameter and Schmidt number increases, the concentration field increases.
- A rise in the values of magnetic parameter, Reynolds number and suction parameter reduces the local skin friction for stretching wall and enhance the local skin friction for upper wall.
- Nussult number increases with decreasing values of magnetic parameter, Eckert number and increases with increasing values of Reynolds number, suction parameter, radiation parameter, Prandtl number for the stretching wall.
- Reynolds number, suction parameter, Schmidt number have the tendency to enhance the rate of mass transfer (Sherwood number) and magnetic parameter have the tendency to reduce the Sherwood number for the stretching wall.

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