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Discrete Inverse Weibull Minimax Distribution: Properties and Applications

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Abstract: There are not many known distributions for modeling discrete data. In this paper, we shall introduce a new count data model, which is obtained by compounding two parameter discrete Inverse Weibull distribution with Minimax distribution. The proposed model has several properties such as it can be nested to different compound distributions on specific parameter settings. We shall first study some basic distributional and moment properties of the new distribution. Then, certain structural properties of the distribution such as its unimodality, hazard rate behavior and index of dispersion are discussed. Finally, real data set is analyzed to investigate the suitability of the proposed distribution in modeling count data from medical genetics.

Keywords: Discrete Inverse Weibull Distribution, Minimax distribution, Compound distribution, Medical science, Count data.

1 Introduction

In the last few decades some papers dealing with probability distributions, the compounding of probability distribution has received maximum attention which is an innovative and sound technique to obtain new probability distributions. Lot of new discrete models [17,18] have been introduced by researchers to handle complex data. In several research papers it has been found that compound distributions are very flexible and can be used efficiently to model different types of data sets. In the early 1970s, Dubey [13] derived a compound gamma, beta and F distribution by compounding a gamma distribution with another gamma distribution and reduced it to the beta Ist and 2nd kind and to the F distribution by suitable transformations.

Sankaran [1] introduced a compound of Poisson distribution with that of Lindley distribution for modeling count data. Gerstenkorn [14,15] proposed several compound distributions, he obtained compound of gamma distribution with exponential distribution by treating the parameter of gamma distribution as an exponential variate and also obtained compound of polya with beta distribution. Ghitany, Al-Mutairi and Nadarajah [2,3] introduced zero-truncated Poisson-Lindley distribution, who used the distribution for modeling count data in the case where the distribution has to be adjusted for the count of missing zeros. Zamani and Ismail [4] constructed a new compound distribution by compounding negative binomial with one parameter Lindley distribution that provides good fit for count data where the probability at zero has a large value. Rashid and Jan [5] explored a mixture of generalized negative binomial distribution with that of generalized exponential distribution which contains several compound distributions as its sub cases and proved that this particular model is better in comparison to others when it comes to fit observed count data set. Most recently Adil, Ahmad and Jan [6] constructed a new count data model (Compound of Negative binomial distribution with Kumaraswamy distribution) with application in genetics and ecology.

In this paper we propose a new count data model by compounding two parameter discrete Inverse Weibull distribution with Minimax distribution as there is a need to find more plausible discrete probability models or survival models in medical science and other fields, to fit to various discrete data sets. It is well known in general that a compound model is more flexible than the ordinary model and it is preferred by many data analysts in analyzing statistical data. Moreover, it presents beautiful mathematical exercises and broadened the scope of the concerned model being compounded.

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2 Materialand Methods

Discrete inverse Weibull distribution was studied by [16], which is a discrete version of the continuous inverse Weibull variable, defined as X^{-1} where X denotes the continuous Weibull random variable and the probability mass function (pmf) of the discrete inverse Weibull random variable is defined by:

$$f_1(x;q,\gamma)=q^{(x+1)^{-\gamma}}-q^{(x)^{-\gamma}}, \quad x=0,1,2,...... \tag{1}$$

where $\gamma>0$ *and* $0<q<1$ are its parameters. The first and the second moments of the DIW random variable X are given by

$$E(X)=\sum_{x=1}^{\infty}1-q^{x^{-\gamma}} \text{ for } x=0,1,2,... \tag{2}$$

$$E(X^2)=2\sum_{x=1}^{\infty}x(1-q^{x^{-\gamma}})+E(X)$$

Jones [7] studied two-parameter distribution on (0,1) which he has called the Minimax distribution, Minimax (α,β) ,where its two shape parameters α and β are positive. It has many of the same properties as the beta distribution but has some advantages in terms of tractability. Its probability density function is given by

$$f_2(X;\alpha,\beta)=\alpha\beta\, x^{\alpha-1}\left(1-x^{\alpha}\right)^{\beta-1}, \ 0<x<1 \tag{3}$$

where $\alpha,\beta>0$ are shape parameters. The raw moments of Minimax distribution are given by

$$E(X^r)=\int_0^1 x^r f_2(X;\alpha,\beta)dx$$

$$E(X^r)=\frac{\Gamma(\beta+1)\Gamma\left(1+\frac{r}{\alpha}\right)}{\Gamma\left(1+\beta+\frac{r}{\alpha}\right)} \tag{4}$$

Minimax distribution is not very popular among statisticians because researchers have not analyzed and investigated it systematically in much detail. Minimax distribution is similar to the beta distribution but unlike beta distribution it has a closed form of cumulative distribution function which makes it very simple to deal with. For more detailed properties one can see references [7,8]

Usually the parameters γ *and* q in DIWD are fixed constants but here we have considered a problem in which the probability parameter q is itself a random variable following MD with pmf (3).

3 Definition of Proposed Model

If $X|q\sim$ DIWD(q,γ) where q is itself a random variable following Minimax distribution MD(α,β), then determining the distribution that results from marginalizing over q will be known as a compound of discrete Weibull distribution with that of Minimax distribution, which is denoted by DIWMD (γ,α,β). It may be noted that proposed model will be a discrete since the parent distribution DWD is discrete.

Theorem 3.1: The probability mass function of a compound of DIWD (q,γ) with MD (α,β) is given by

$$f_{DWMD}(X;\gamma,\alpha,\beta)=\beta[B(\beta,\frac{(x+1)^{-\gamma}}{\alpha}+1)-B(\beta,\frac{x^{-\gamma}}{\alpha}+1)]$$

where $x = 0,1,2,......$ and $\gamma,\alpha,\beta > 0$

Proof: Using the definition (3), the pmf of a compound of DWD (q,γ) with MD (α,β) can be obtained as

$$f_{DIWMD}(X;\gamma,\alpha,\beta) = \int_0^1 f_1(x|q)\, f_2(q)\, dq$$

$$f_{DIWMD}(X;\gamma,\alpha,\beta) = \int_0^1 (q^{(x+1)^{-\gamma}} - q^{x^{-\gamma}})\alpha\beta\, q^{\alpha-1}(1-q^{\alpha})^{\beta-1}\, dq$$

substituting $1-q^{\alpha} = z$, we get

$$f_{DIWMD}(X;\gamma,\alpha,\beta) = \beta(\int_0^1 z^{\beta-1}(1-z)^{\frac{(x+1)^{-\gamma}}{\alpha}}\, dz - \int_0^1 z^{\beta-1}(1-z)^{\frac{x^{-\gamma}}{\alpha}}\, dz)$$

$$f_{DIWMD}(X;\gamma,\alpha,\beta) = \beta[B(\beta,\frac{(x+1)^{-\gamma}}{\alpha}+1) - B(\beta,\frac{x^{-\gamma}}{\alpha}+1)]$$

$$f_{DIWMD}(X;\gamma,\alpha,\beta) = \beta\left[\frac{\Gamma(\beta)\,\Gamma\left(\frac{(x+1)^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x+1)^{-\gamma}}{\alpha}+1\right)} - \frac{\Gamma(\beta)\,\Gamma\left(\frac{x^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x^{-\gamma}}{\alpha}+1\right)}\right] \quad (5)$$

where $x = 0,1,2,...$ and $\gamma,\alpha,\beta > 0$. From here a random variable X following a compound of DIWD with MD will be symbolized by DIWMD(γ,α,β).

Fig.1(a) to fig.1(i) provides a pmf plot of the proposed model DIWMD(γ,α,β) for different values of parameters. It is evident that the proposed model is right skewed with unimodel behavior.

The Cumulative distribution function of the DIWMD(γ,α,β) is given by

$$F(x) = \beta B(x+1;\gamma,\alpha,\beta)\ x = 0,1,2,... \text{ and } (\gamma > 0, \alpha > 0, \beta > 0)$$

Where $B(x+1;\gamma,\alpha,\beta) = \dfrac{\Gamma(\beta)\,\Gamma\left(\frac{(x+1)^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x+1)^{-\gamma}}{\alpha}+1\right)}$

Fig.2(a) to fig.2(i) provides a CDF plot of the proposed model DIWMD(γ,α,β) for different values of parameters. The initial rise of the CDF plot increases as α and γ increases but as β increases, initial rise of the CDF plot decreases.

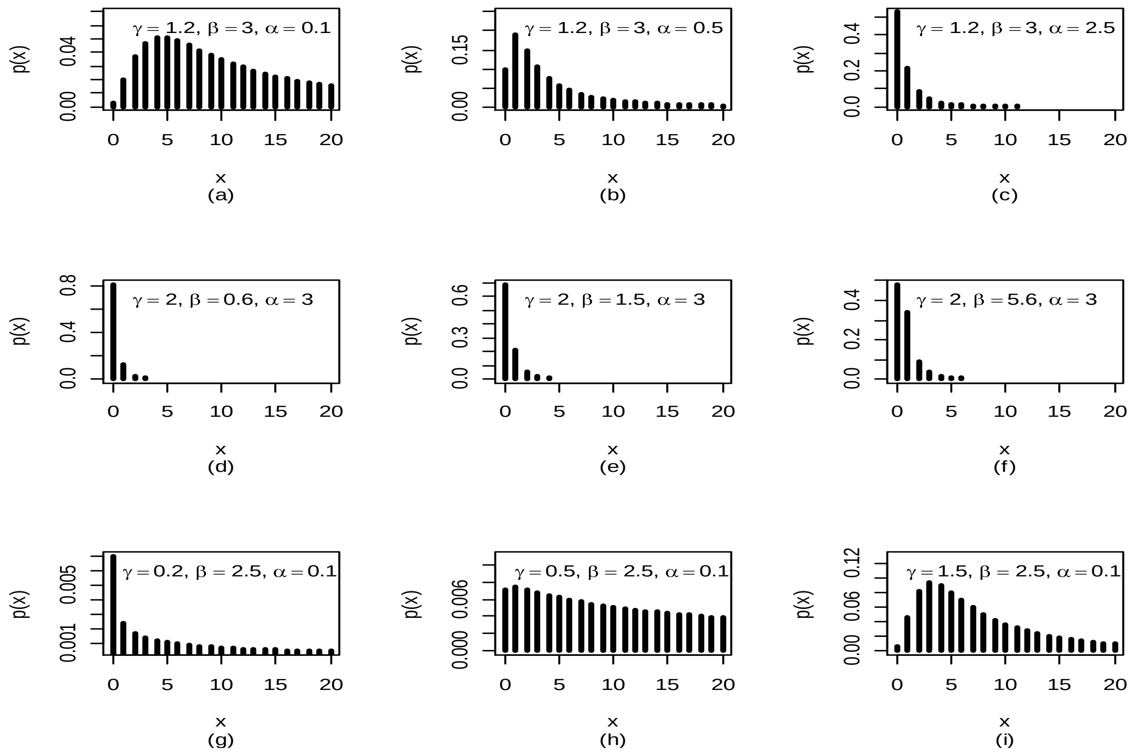

Fig 1:pmf plot of Discrete Inverse Weibull Minimax Distribution

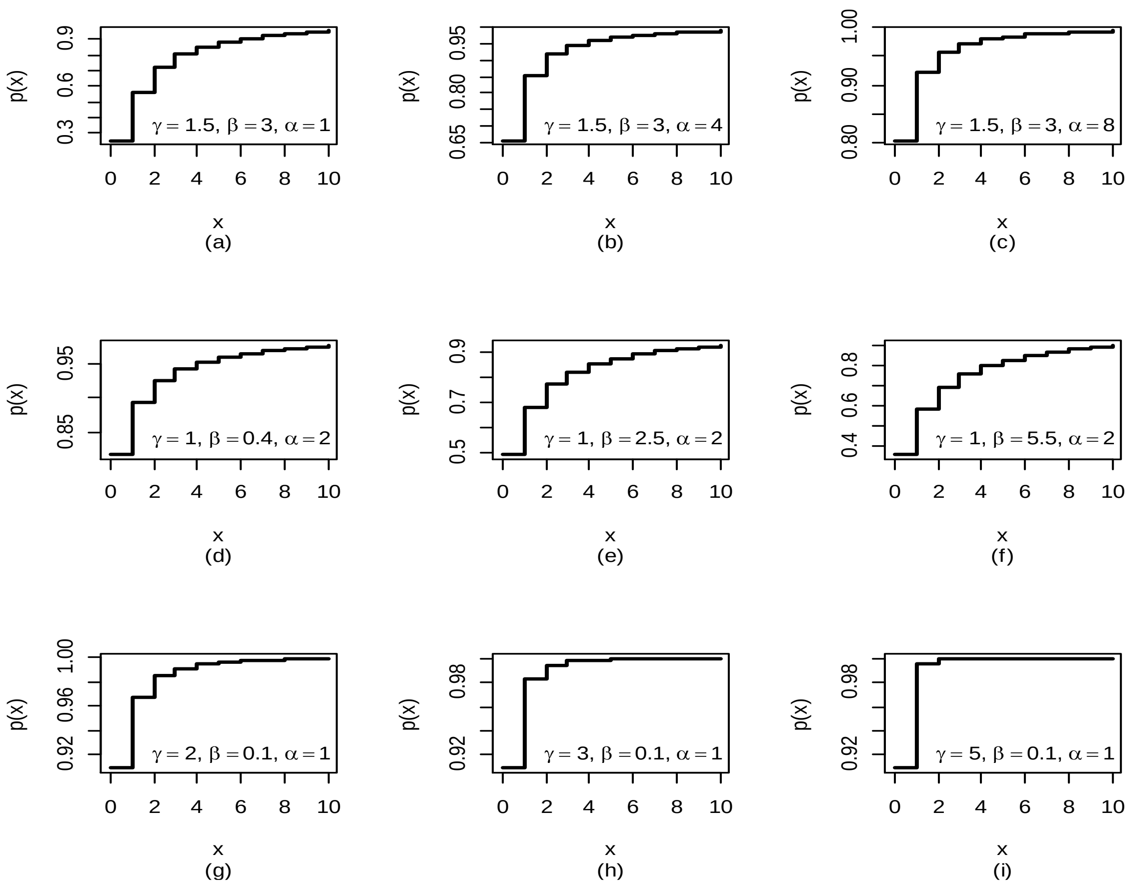

Fig 2: CDF plot of Discrete Inverse Weibull Minimax Distribution

4 Nested Distributions

In this particular section we show that the proposed model can be nested to different models under specific parameter setting.

Proposition 4.1: If $X \sim DIWMD(\gamma, \alpha, \beta)$ then by setting $\gamma = 1$, we get a compound of Inverse geometric distribution with Minimax distribution.

Proof: For $\gamma = 1$ in (1)DIWD reduces to inverse geometric distribution (IGD) hence a compound of IGD with MD is followed from (5) by simply substituting $\gamma = 1$ in it.

$$f_{DIGMD}(X;\alpha,\beta) = \beta[B(\beta, \frac{(x+1)^{-1}}{\alpha}+1) - B(\beta, \frac{x^{-1}}{\alpha}+1)], \text{ for } x = 0,1,2,...;\alpha,\beta > 0$$

$$\text{Where } B(\beta, \frac{x^{-1}}{\alpha}+1) = \frac{\Gamma(\beta)\,\Gamma\left(\frac{x^{-1}}{\alpha}+1\right)}{\Gamma\left(\beta + \frac{x^{-1}}{\alpha}+1\right)}$$

which is the probability mass function of a compound of IGD with MD.

Proposition 4.2: If $X \sim DWMD(\gamma, \alpha, \beta)$ then by setting $\alpha = \beta = 1$,we obtain a compound of DIWD distribution with uniform distribution.

Proof:For $\alpha = \beta = 1$ in MD reduces to Uniform (0,1) distribution, therefore a compound DWD with uniform distribution is followed from (5) by simply putting $\alpha = \beta = 1$ in it.

$$f_{DIWUD}(X;\gamma) = \beta[B(1,(x+1)^{-\gamma}+1) - B(1, x^{-\gamma}+1)]$$

$$f_{DIWUD}(X;\gamma) = \left[\frac{x^{-\gamma} - (x+1)^{-\gamma}}{(x^{-\gamma}+1)[(x+1)^{-\gamma}+1]}\right] \text{for } x = 0,1,2,...,\gamma > 0$$

which is probability mass function of a compound of DWD with uniform distribution.

Proposition 4.3: If $X \sim DIWMD(\gamma, \alpha, \beta)$ then by setting $\gamma = 1$ and $\alpha = \beta = 1$ we obtain a compound of inverse geometric distribution with uniform distribution.

Proof: For $\gamma = 1$ in (1), DIWD reduces to inverse geometric distribution and for $\alpha = \beta = 1$, Minimax distribution reduces to U(0,1) distribution hence a compound of inverse geometric distribution with uniform distribution can be obtained from (5) by simply substituting $\gamma = 1$ and $\alpha = \beta = 1$ in it.

$$f_{IGUD}(X) = \frac{1}{(x+1)(x+2)} \text{ for } x = 0,1,2,...$$

Proposition 4.4: If $X \sim DIWMD(\gamma, \alpha, \beta)$ then by setting $\gamma = 2$ and $\alpha = \beta = 1$ we obtain a compound of discrete inverse Rayleigh distribution with uniform distribution.

Proof: For $\gamma = 2$ in (1), DIWD reduces to discrete inverse Rayleigh distribution and for $\alpha = \beta = 1$, Minimax distribution reduces to U(0,1) distribution hence a compound of geometric distribution with uniform distribution can be obtained from (5) by simply substituting $\gamma = 2$ and $\alpha = \beta = 1$ in it.

$$f_{DIRUD}(X)=\left[\frac{x^{-2}-(x+1)^{-2}}{(x^{-2}+1)[(x+1)^{-2}+1]}\right] \text{for } x=0,1,2,...$$

which is the probability mass function of discrete inverse Rayleigh uniform distribution

Proposition 4.5: If $X \sim DIWMD(\gamma,\alpha,\beta)$ then by setting $\gamma=2$, we get a compound of discrete inverse Rayleigh distribution with Minimax distribution.

Proof: For $\gamma=2$ in (1)DIWD reduces to discrete inverse Rayleigh distribution (DIRD) hence a compound of DIRD with MD is followed from (5) by simply substituting $\gamma=2$ in it.

$$f_{DIRMD}(X;\alpha,\beta)=\beta[B(\beta,\frac{(x+1)^{-2}}{\alpha}+1)-B(\beta,\frac{x^{-2}}{\alpha}+1)] \text{ for } x=0,1,2,...,\alpha,\beta>0$$

$$\text{Where } B(\beta,\frac{x^{-2}}{\alpha}+1)=\frac{\Gamma(\beta)\,\Gamma\left(\frac{x^{-2}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x^{-2}}{\alpha}+1\right)}$$

which is the probability mass function of a compound of DIRD with MD.

5 Reliability Measures of Compound Discrete Inverse Weibull Minimax Distribution.

If $X \sim DIWMD(X;\gamma,\alpha,\beta)$, then the various reliability measures of a random variable X are given by

(a) **Survival Function.**

$$s(x)=1-\beta B(\beta,\frac{x^{-\gamma}}{\alpha}+1) \quad x=0,1,2... \text{ and } \alpha>0,\beta>0,\gamma>0$$

$$where\ B(\beta,\frac{x^{-\gamma}}{\alpha}+1)=\frac{\Gamma(\beta)\,\Gamma\left(\frac{x^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x^{-\gamma}}{\alpha}+1\right)}$$

(b) **Rate of Failure Function.**

$$r(x)=\frac{p(x)}{s(x)}=\frac{\beta[B(\beta,\frac{(x+1)^{-\gamma}}{\alpha}+1)-B(\beta,\frac{x^{-\gamma}}{\alpha}+1)]}{1-\beta B(\beta,\frac{x^{-\gamma}}{\alpha}+1)} \quad x=0,1,2... \text{ and } \alpha>0,\beta>0,\gamma>0$$

$$where\ B(\beta,\frac{x^{-\gamma}}{\alpha}+1)=\frac{\Gamma(\beta)\,\Gamma\left(\frac{x^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x^{-\gamma}}{\alpha}+1\right)}$$

(c) **Second Rate of Failure Function.**

$$h(x)=\log\left(\frac{s(x)}{s(x+1)}\right)=\log\left(\frac{1-\beta B(\beta,\dfrac{x^{-\gamma}}{\alpha}+1)}{1-\beta B(\beta,\dfrac{(x+1)^{-\gamma}}{\alpha}+1)}\right)\quad x=0,1,2\ldots \text{ and } \alpha>0,\beta>0,\gamma>0$$

Where, B(.) refers to the beta function defined by $B(a,b)=\dfrac{\Gamma a\,\Gamma b}{\Gamma(a+b)}$

Fig.3(a) to fig.3(i) provides a hazard rate function plot of the proposed model DIWMD (γ,α,β) for different values of parameters.

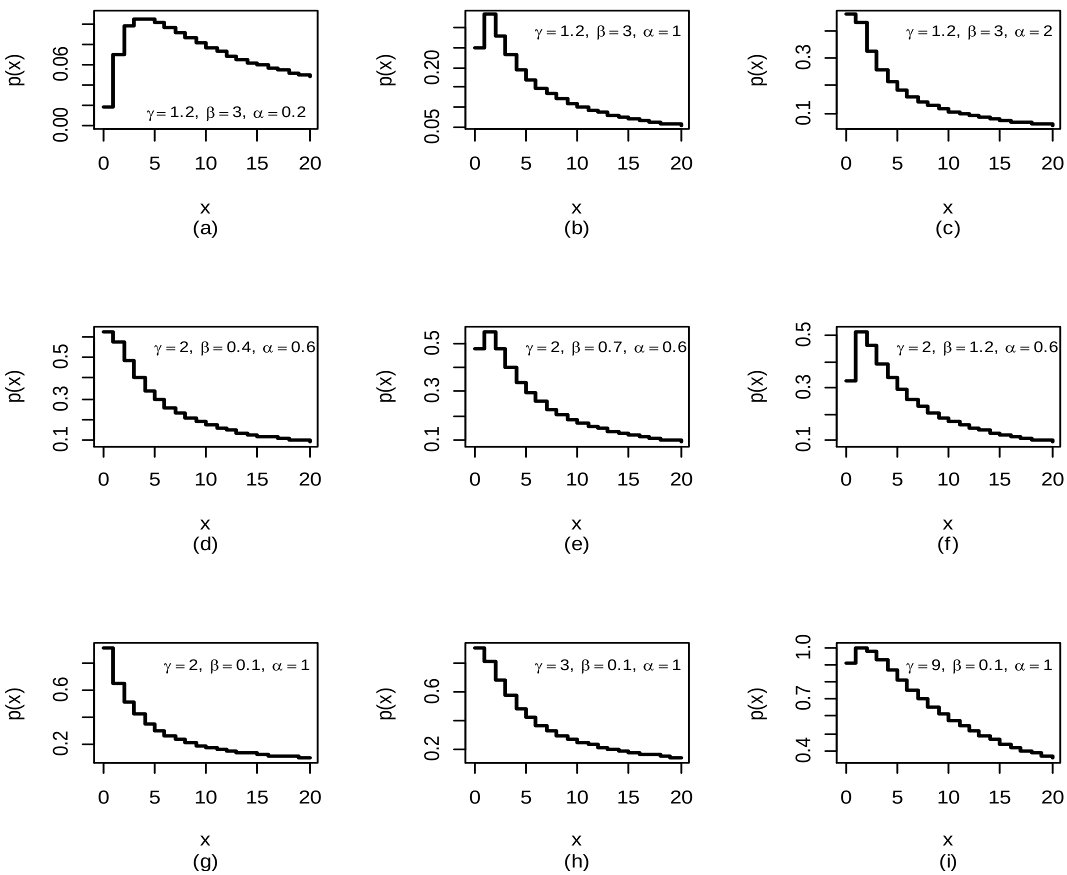

Fig 3: Hazard rate function plot of Discrete Inverse Weibull Minimax distribution

6 Moment Generating and Probability Generating Functions of $DIWMD(\gamma,\alpha,\beta)$

(a) The moment generating function of the Compound discrete Weibull Minimax distribution is

$$M_x(t)=\sum_{x=0}^{\infty}e^{tx}p(x)$$

$$M_x(t)=\sum_{x=0}^{\infty}e^{tx}\left[\left\{1-\beta B(\beta,\frac{x^{-\gamma}}{\alpha}+1)\right\}-\left\{1-\beta B(\beta,\frac{(x+1)^{-\gamma}}{\alpha}+1)\right\}\right]$$

$$M_x(t)=\sum_{x=0}^{\infty}e^{tx}[\psi(x;\gamma,\beta,\alpha)-\psi(x+1;\gamma,\beta,\alpha)]$$

Where $\psi(x;\gamma,\beta,\alpha)=1-\beta B(\beta,\frac{x^{-\gamma}}{\alpha}+1)$

$$M_x(t)=\begin{pmatrix}\psi(0;\gamma,\beta,\alpha)+e^{t}\psi(1;\gamma,\beta,\alpha)+e^{2t}\psi(2;\gamma,\beta,\alpha)+e^{3t}\psi(3;\gamma,\beta,\alpha)+....-\{\psi(1;\gamma,\beta,\alpha)\\+e^{t}\psi(2;\gamma,\beta,\alpha)+e^{2t}\psi(3;\gamma,\beta,\alpha)+e^{3t}\psi(4;\gamma,\beta,\alpha)+......\}\end{pmatrix}$$

$$M_x(t)=\psi(0;\gamma,\beta,\alpha)+(e^{t}-1)\psi(1;\gamma,\beta,\alpha)+(e^{2t}-e^{t})\psi(2;\gamma,\beta,\alpha)+(e^{3t}-e^{2t})\psi(3;\gamma,\beta,\alpha)+....$$

$$M_x(t)=1+\sum_{x=1}^{\infty}(e^{xt}-e^{(x-1)t})\psi(x;\gamma,\beta,\alpha)$$

Differentiating $M_x(t)$ r times with respect to t

$$M_x^{(r)}(t)=\sum_{x=1}^{\infty}(x^{r}e^{xt}-(x-1)^{r}e^{(x-1)t})\psi(x;\gamma,\beta,\alpha)$$

First four moments of the proposed model are given by

$$\mu_1^{'}=\sum_{x=1}^{\infty}\psi(x;\gamma,\beta,\alpha)$$

$$\mu_2^{'}=\sum_{x=1}^{\infty}(2x-1)\psi(x;\gamma,\beta,\alpha)$$

$$\mu_3^{'}=\sum_{x=1}^{\infty}(3x^{2}-3x+1)\psi(x;\gamma,\beta,\alpha)$$

$$\mu_4^{'}=\sum_{x=1}^{\infty}(4x^{3}-6x^{2}+4x-1)\psi(x;\gamma,\beta,\alpha)$$

(b) Probability generating function of the Compound discrete Weibull Minimax distribution is

$$G_{[x]}(t)=\sum_{x=0}^{\infty}t^{x}p(x)$$

$$G_{[x]}(t)=\sum_{x=0}^{\infty}t^{x}\left[\left\{1-\beta B(\beta,\frac{x^{-\gamma}}{\alpha}+1)\right\}-\left\{1-\beta B(\beta,\frac{(x+1)^{-\gamma}}{\alpha}+1)\right\}\right]$$

$$G_{[x]}(t)=\sum_{x=0}^{\infty}t^{x}[\psi(x;\gamma,\beta,\alpha)-\psi(x+1;\gamma,\beta,\alpha)]$$

Where $\psi(x;\gamma,\beta,\alpha)=\left\{1-\beta B(\beta,\frac{x^{-\gamma}}{\alpha}+1)\right\}$

$$G_{[x]}(t)=\psi(0;\gamma,\beta,\alpha)+(t-1)\psi(1;\gamma,\beta,\alpha)+t(t-1)\psi(2;\gamma,\beta,\alpha)+t^{2}(t-1)\psi(3;\gamma,\beta,\alpha)+....$$

$$G_{[x]}(t) = 1 + (t-1)\sum_{x=1}^{\infty} t^{x-1}\psi(x;\gamma,\beta,\alpha)$$

Differentiating $G_{[x]}(t)$ with respect to t

$$G'_{[x]}(t) = \sum_{x=1}^{\infty}((t-1)(x-1)t^{x-2} + t^{x-1})\psi(x;\gamma,\beta,\alpha)$$

$$G'_{[x]}(t) = \sum_{x=1}^{\infty}(t^{x-2}(xt - x + 1))\psi(x;\gamma,\beta,\alpha)$$

$$G''_{[x]}(t) = \sum_{x=1}^{\infty}(x-1)t^{x-3}\{(t-1)(x-2)+2t\}\psi(x;\gamma,\beta,\alpha)$$

At t=1, $G'_{[x]}(t)$, $G''_{[x]}(t)$ gives first and second factorial moments

$$E(x) = G'_{[x]}(1) = \sum_{x=1}^{\infty} \psi(x;\gamma,\beta,\alpha)$$

$$E(x^2) = G'_{[x]}(1) + G''_{[x]}(1) = \beta\sum_{x=1}^{\infty}(2x-1)\psi(x;\gamma,\beta,\alpha)$$

Table1 exhibits the index of dispersion, $IOD = \{E(X^2) - (E(X))^2\} / E(X)$, mean and variance for different values of the parameters γ,α and β for three parameter discrete Compound Weibull Minimax distribution. It can be seen that this variance to mean ratio indicates that discrete Compound Weibull Minimax model is overdispersed as well as under-dispersed.

Table 1: Index of Dispersion, Mean and Variance of $DIWMD(\gamma,\alpha,\beta)$ for different values of parameters

γ =4												
α	β	0.3	0.4	0.5	0.7	0.9	1.2	1.5	1.8	2.5	2.7	3.5
0.01	Mean	1.789	2.095	2.332	2.676	2.917	3.171	3.354	3.494	3.726	3.777	3.941
	Variance	2.960	3.114	3.202	3.317	3.410	3.538	3.652	3.755	3.955	4.004	4.175
	IOD	1.654	1.486	1.373	1.239	1.169	1.116	1.089	1.075	1.061	1.060	1.060
0.03	Mean	1.247	1.474	1.652	1.913	2.096	2.289	2.428	2.535	2.712	2.751	2.876
	Variance	1.715	1.823	1.882	1.952	2.005	2.079	2.146	2.206	2.320	2.347	2.441
	IOD	1.375	1.237	1.140	1.021	0.957	0.908	0.884	0.870	0.855	0.853	0.849
1.02	Mean	0.260	0.322	0.377	0.468	0.541	0.626	0.693	0.746	0.837	0.857	0.922
	Variance	0.285	0.336	0.375	0.428	0.460	0.488	0.503	0.510	0.515	0.515	0.514
	IOD	1.099	1.042	0.994	0.914	0.851	0.779	0.725	0.684	0.616	0.601	0.557
1.2	Mean	0.234	0.291	0.340	0.424	0.491	0.571	0.634	0.684	0.772	0.792	0.855
	Variance	0.259	0.307	0.344	0.397	0.431	0.462	0.480	0.490	0.500	0.501	0.501
	IOD	1.107	1.055	1.010	0.936	0.877	0.809	0.757	0.716	0.647	0.632	0.586

7 Parameter Estimation

In this section the estimation of parameters of $DIWMD(\gamma,\alpha,\beta)$ model will be discussed through method of moments and maximum likelihood estimation.

7.1 Moments Method of Estimation

In order estimate three unknown parameters of $DIWMD(\gamma,\alpha,\beta)$ model by the method of moments , we need to equate first three sample moments with their corresponding population moments.

$m_1=\gamma_1$; $m_2=\gamma_2$ and $m_3=\gamma_3$

Where γ_i is the i[th] sample moment and m_i is the i[th] corresponding population moment and the solution for $\hat{\gamma},\hat{\alpha}$ and $\hat{\beta}$ may be obtained by solving above equations simultaneously through numerical methods.

7.2 Maximum Likelihood Method of Estimation

The estimation of parameters of $DIWMD(\gamma,\alpha,\beta)$ model via maximum likelihood estimation method requires the log likelihood function of $DIWMD(\gamma,\alpha,\beta)$

$$£(X;\gamma,\alpha,\beta)=\log L(X;\gamma,\alpha,\beta)=n\log\beta+\sum_{i=1}^{n}\log([B(\beta,\frac{(x+1)^{-\gamma}}{\alpha}+1)-B(\beta,\frac{x^{-\gamma}}{\alpha}+1)]) \quad (9)$$

The maximum likelihood estimate of $\Theta=\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)^T$ can be obtained by differentiating (9) with respect unknown parameters γ,α and β respectively and then equating them to zero.

$$\frac{\partial}{\partial\beta}£(X;\gamma,\alpha,\beta)=\frac{n}{\beta}+\sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\beta}\left(\frac{\Gamma(\beta)\,\Gamma\left(\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\,\Gamma\left(\frac{x_i^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{-\gamma}}{\alpha}+1\right)}\right)}{\left(\frac{\Gamma(\beta)\,\Gamma\left(\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\,\Gamma\left(\frac{x_i^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{-\gamma}}{\alpha}+1\right)}\right)}\right) \quad (10)$$

$$\frac{\partial}{\partial\alpha}£(X;\gamma,\alpha,\beta)=\sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\alpha}\left(\frac{\Gamma(\beta)\,\Gamma\left(\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\,\Gamma\left(\frac{x_i^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{-\gamma}}{\alpha}+1\right)}\right)}{\left(\frac{\Gamma(\beta)\,\Gamma\left(\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\,\Gamma\left(\frac{x_i^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{-\gamma}}{\alpha}+1\right)}\right)}\right) \quad (11)$$

$$\frac{\partial}{\partial\gamma}\pounds(X;\gamma,\alpha,\beta)=\sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\gamma}\left(\frac{\Gamma(\beta)\,\Gamma\left(\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\,\Gamma\left(\frac{x_i^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{-\gamma}}{\alpha}+1\right)}\right)}{\left(\frac{\Gamma(\beta)\,\Gamma\left(\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{-\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\,\Gamma\left(\frac{x_i^{-\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{-\gamma}}{\alpha}+1\right)}\right)}\right) \tag{12}$$

These three derivative equations cannot be solved analytically, therefore $\hat{\gamma},\hat{\alpha}$ and $\hat{\beta}$ will be obtained by maximizing the log likelihood function numerically using Newton-Raphson method which is a powerful technique for solving equations iteratively and numerically. We can compute the second partial derivatives, which are useful to obtain the Fisher's information matrix as follows.

$$I_y(\gamma,\alpha,\beta)=\begin{bmatrix} -E\left(\frac{\partial^2 l}{\partial\gamma^2}\right) & -E\left(\frac{\partial^2 l}{\partial\gamma\partial\alpha}\right) & -E\left(\frac{\partial^2 l}{\partial\gamma\partial\beta}\right) \\ -E\left(\frac{\partial^2 l}{\partial\alpha\partial\beta}\right) & -E\left(\frac{\partial^2 l}{\partial\alpha^2}\right) & -E\left(\frac{\partial^2 l}{\partial\alpha\partial\gamma}\right) \\ -E\left(\frac{\partial^2 l}{\partial\beta\partial\alpha}\right) & -E\left(\frac{\partial^2 l}{\partial\beta\partial\gamma}\right) & -E\left(\frac{\partial^2 l}{\partial\beta^2}\right) \end{bmatrix} \tag{13}$$

One can show that the discrete Weibull Minimax distribution satisfies the regularity conditions (see, e.g., [10]). Hence, the MLE vector $\Theta=\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)^T$ is consistent and asymptotically normal; that is, $I_y^{\frac{1}{2}}(\gamma,\alpha,\beta)=\left[\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)^T-\left(\gamma,\alpha,\beta\right)^T\right]$ converges in distribution to a normal distribution with the (vector) mean zero and the identity covariance matrix. Also, the Fisher's information matrix can be computed using the approximation

$$I_y\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)\approx\begin{bmatrix} -\left(\frac{\partial^2 l}{\partial\gamma^2}\right)\Big|_{\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)} & -\left(\frac{\partial^2 l}{\partial\gamma\partial\alpha}\right)\Big|_{\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)} & -\left(\frac{\partial^2 l}{\partial\gamma\partial\beta}\right)\Big|_{\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)} \\ -\left(\frac{\partial^2 l}{\partial\alpha\partial\beta}\right)\Big|_{\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)} & -\left(\frac{\partial^2 l}{\partial\alpha^2}\right)\Big|_{\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)} & -\left(\frac{\partial^2 l}{\partial\alpha\partial\gamma}\right)\Big|_{\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)} \\ -\left(\frac{\partial^2 l}{\partial\beta\partial\alpha}\right)\Big|_{\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)} & -\left(\frac{\partial^2 l}{\partial\beta\partial\gamma}\right)\Big|_{\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)} & -\left(\frac{\partial^2 l}{\partial\beta^2}\right)\Big|_{\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)} \end{bmatrix}$$

Where $\hat{\gamma},\hat{\alpha}$ and $\hat{\beta}$ are the MLEs of γ,α and β, respectively (see, e.g.,[9]). Using this approximation, we may construct confidence intervals for parameters of the discrete Weibull Minimax model.

8 Application of Discrete CIWMD in medical science.

Here we analyse the dataset related to Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 μg|kg. Much quantitative works seem to be done in fitting various probability models to the dataset of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 μg|kg but so far no works has been done on fitting of discrete inverse Weibull Minimax (DIWMD) model for such dataset. Shanker&Hagos [11] have detailed study on the applications of Poisson Lindley distribution (PLD) to model count data of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 µg|kg. Shanker ,Hagos and Teklay [12] have suggested Poisson Akasha distribution (PAD) as another model for studying Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 µg|kg. In this section an attempt has been made to fit to data relating to genetics as given in table 2, using discrete inverse Weibull Minimax distribution (DIWMD) in comparison with discrete Weibull (DW), Poisson Akasha distribution (PAD), Poisson Lindley distribution (PLD), Poisson Sujatha distribution (PSD) and other classical discrete models.

Table 2: Distribution of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 µg|kg.

Class/Exposure (μ g/kg)	**0**	**1**	**2**	**3**	**4**	**5**	**6+**	**Total**
Frequency	200	57	30	7	4	0	2	300

The ML estimates provided by the fitdistr procedure in R studio are given in the table 3.

Table 3 .Estimated parameters by ML method for fitted distributions for Counts of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 µg|kg..

Distributions	**parameter Estimates**	**Model function**
Discrete Inverse Weibull Minimax	$\gamma = 4.27, \beta = 0.09, \alpha = 0.02$ $SE(\gamma,\beta,\alpha) = (0.8, 0.03, 0.02)$	$p(x) = \beta[B(\beta, \frac{(x+1)^{-\gamma}}{\alpha}+1) - B(\beta, \frac{x^{-\gamma}}{\alpha}+1)]$ $x = 0,1,2,...$ for $\gamma > 0, \beta > 0, \alpha > 0$
Poisson	$\lambda = 0.55$, $SE(\lambda) = 0.04$	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $\lambda > 0;\ x = 0,1,2,.....$
Poisson Akasha	$\theta = 2.62$, $SE(\theta) = 0.17$	$p(x) = \frac{\theta^3(x^2+3x+(\theta^2+2\theta+2))}{(\theta^2+2)(\theta+1)^{x+3}}$ $x = 0,1,2,....\ \theta > 0$
Discrete Weibull	$q = 0.34, \gamma = 0.92$ $SE(q,\gamma) = (0.03, 0.07)$	$p(x) = q^{x^\gamma} - q^{(x+1)^\gamma}$ $0 < q < 1; \gamma > 0; x = 0,1,2,.....$
Poisson Lindley	$\theta = 2.35$, $SE(\theta) = 0.20$	$p(x) = \frac{\theta^2(x+\theta+2)}{(\theta+1)^{x+3}}$ $x = 0,1,2,....\ \theta > 0$
Geometric	$p = 0.64$ $SE(p) = 0.02$	$p(x) = q^x p$ $0 < q < 1; q = 1\text{-}p; x = 0,1,2,.....$
Poisson Sujatha	$\theta = 2.79$, $SE(\theta) = 0.19$	$p(x) = \frac{\theta^3}{\theta^2+\theta+2} \frac{x^2+(\theta+4)x+(\theta^2+3\theta+4)}{(\theta+1)^{x+3}}$ $x = 0,1,2,....\ \theta > 0$
NBD	$r = 0.72, p = 0.57$ $SE(r, p) = (0.07, 0.18)$	$p(x) = \binom{x+r-1}{x} p^r q^x, \quad x = 0,1,2,...$ $r > 0\ and\ 0 < p < 1$
Discrete Rayleigh	$q = 0.61$, $SE(q) = 0.02$	$p(x) = q^{x^2} - q^{(x+1)^2}$ $0 < q < 1; x = 0,1,2,.....$

Table 4: Table for goodness of fit for Counts of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 µg|kg.

X	Observed	Poisson	DRayleigh	Geometric	PLD	NBD	PAD	DWD	DIWMD	PSD
0.00	200.00	172.51	117.49	193.13	192.03	198.98	194.07	198.82	199.89	192.03
1.00	57.00	95.46	141.42	68.80	70.11	62.27	67.63	62.55	58.25	70.11
2.00	30.00	26.41	37.67	24.51	24.93	23.27	24.49	23.25	27.47	24.93
3.00	7.00	4.87	3.32	8.73	8.62	9.17	8.90	9.10	9.32	8.62
4.00	4.00	0.67	0.10	3.11	2.90	3.70	3.19	3.67	3.00	2.90
5.00	0.00	0.07	0.00	1.11	0.96	1.52	1.13	1.51	1.10	0.96
6.00	2.00	0.01	0.00	0.61	0.31	1.08	0.59	1.11	0.97	0.45
χ^2	p-values	0.000	0.000	0.172	0.140	0.232	0.208	0.228	0.315	0.149

X: Counts of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 $\mu g|kg$

Observed: Observed frequency of Counts of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 $\mu g|kg$

The p-values of Pearson's Chi-square statistic are 0.00, 0.00, 0.172, 0.140, 0.232, 0.208, 0.228, 0.315 and 0.149 for Poisson, discrete Rayleigh, Geometric, Poisson Lindley, Negative binomial, Poisson Akasha, discrete Weibull , discrete inverse Weibull Minimax and Poisson Sujatha distributions, respectively (see Table 4). This reveals that Poisson and discrete Rayleigh distributions are not good fit at all, whereas Geometric, Poisson Lindley, Negative binomial, Poisson Akasha, discrete Weibull, discrete inverse Weibull Minimax and Poisson Sujatha distributions are good fit distributions with discrete inverse Weibull Minimax model being the best one. The null hypothesis that data come from discrete inverse Weibull Minimax distribution is strongly accepted.

9 Conclusion

In this paper, a new model is proposed by compounding discrete inverse Weibull distribution (DIWD) with Minimax distribution (MD) and it has been shown that proposed model can be nested to different compound distributions. Some important probabilistic properties and the problem of estimation of its parameters are studied. In addition, the discrete Weibull Minimax distribution is appropriate for modeling both over and under dispersed data since, depending on the values of the parameters, its variance can be larger or smaller than the mean, which is not the case with some known standard classical discrete distributions.

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