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Estimation in Constant-Partially Accelerated Life Test Plans for Linear Exponential Distribution with Progressive Type-II Censoring

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Abstract: In this paper, we consider the estimation problem in the case of constant-partially accelerated life tests using progressively Type-II censored samples. The lifetime of items under use condition follows the two-parameters linear exponential distribution. The maximum likelihood estimates of the parameters are obtained numerically. Approximate confidence intervals for the parameters, based on normal approximation to the asymptotic distribution of maximum- likelihood estimators, studentized-t and percentile bootstrap confidence intervals are derived. A Monte Carlo simulation study is carried out to investigate the precision of the maximum likelihood estimators and to compare the performance of the confidence intervals considered. Finally, two examples presented to illustrate our results are followed by conclusion.

Keywords: Constant-stress partially accelerated life test, progressive Type-II censored, linear exponential distribution, maximum likelihood method, bootstrap method, Monte Carlo simulation.

1 Introduction

In some situations, the usual life testing methods dose not obtain enough failure data necessary to make the desired inference. So, one can use accelerated life tests (ALT) to quickly obtain information about the life time distribution of products. Such ALT or partially ALT $(PALT)$ results in shorter lives than that would be observed under normal conditions. Test units are run only at accelerated use conditions in ALT, while they run at both normal and accelerated use conditions in $PALT$.

According to Nelson [28], the stress can be applied in various ways. One way to accelerate failure is the constant stress where as each item runs at either use or accelerated conditions only, see Bai and Chung [13]. Another way is step-stress, where as one increase the stress applied to test product in a specified discrete sequence. In other words, as indicated by Xiong and Ji [11], a test unit starts at a specified low stress. If the unit does not fail at a specified time, stress on it is raised and held a specified time. Stress is repeatedly increased and held, until the test unit fails or a censoring time is reached.

Recently, the estimation of parameters from different lifetime distributions based on progressive Type-I and Type-II censored samples are studied by several authors including Childs and Balakrishnan [6], Balakrishnan and Kannan [21], Ali Mousa and Jaheen [17], Balakrishnan, et al. [22] and Soliman ([5]). In the step stress $PALT$, a test unit is first run at use condition and if it does not fail in a specified time, then it is run at accelerated condition until failures occurs or the observation censored see, for example, Abdel-Ghani [19], Ismail [3] and Abd-Elfattah et al. [8]. Bayes and maximum likelihood methods of estimation are applied on constant $ALTs$ to finite mixtures of distributions by AL-Hussaini and Abdel-Hamid ([14]-[15]). Based on progressive Type II censored samples, Abdel-Hamid [7] considered constant stress $PALT$ when the lifetime of units under use condition follows Burr XII distribution. Also, Ismail and Sarhan

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[4] study optimal design of step-stress life test with progressively Type- II censored exponential data, Srivastava and Neha [13] consider optimum step-stress *PALT* for the truncated logistic distribution with censoring. Ismail [1] estimating the parameters of Weibull distribution and the acceleration factor from hybrid *PALT*, Ismail [2] study to inference in the generalized exponential distribution under *PALT* with progressive Type-II censoring.

The linear exponential (LE) distribution with the parameters $a > 0$ and $b > 0$, will be denoted by $LE(a,b)$ distribution. Tow special cases of $LE(a,b)$ distribution are exponential distribution (when $b = 0$) and Rayleigh distributions (when $a = 0$). The corresponding cumulative distribution (CDF), probability density function (PDF), survival function (SF) and the hazard rate function (HRF) are given for $t > 0,\ a,\ b > 0$, respectively, by

$$\left.\begin{aligned} f_1(t) &= (a+bt)\exp\left\{-(at+\tfrac{b}{2}t^2)\right\}, \\ F_1(t) &= 1-\exp\left\{-(at+\tfrac{b}{2}t^2)\right\}, \\ \overline{F}_1(t) &= \exp\left\{-(at+\tfrac{b}{2}t^2)\right\}, \\ h_1(t) &= a+bt. \end{aligned}\right\} \tag{1.1}$$

The $LE(a,b)$ distribution has many applications in applied statistics and reliability analysis. Broadbent [27], uses the $LE(a,b)$ distribution to describe the service of milk bottles that are filled in a dairy, circulated to customers, and returned empty to the dairy. The $LE(a,b)$ distribution was also used by Carbone et al. [23] to study the survival pattern of patients with plasmacytic myeloma. The Type-II censored data is used by Bain [16] to discuss the least square estimates of the parameters a and b and by Pandey et al. [9] to study the Bayes estimators of (a,b). There are many situations in life-testing and reliability studies in which the experimenter may be unable to obtain complete information on failure times for all experimental items. There are also situations wherein the removal of items prior to failure is pre-planned in order to reduce the cost and time associated with testing. The conventional Type-I, and Type-II censoring schemes do not have the flexibility of allowing removal of items at points other than the terminal point of the experiment. We consider here a more general censoring scheme, known as progressively Type-II censoring, details of the scheme are discussed in Balakrishnan and Aggarwala [20].

This paper considers the constant stress *PALT* applied to items whose lifetimes under design condition are assumed to follow $LE(a,b)$ distribution under a progressive Type-II censoring scheme. The rest of the paper is organized as follows, in Section 2 the model description and *MLEs* of the involved parameters are derived using progressive Type II censoring scheme. Also, approximate and parametric bootstrap confidence intervals are derived. In Section 3, simulation studies are made using Monte Carlo method. Finally, some concluding remarks are presented in Section 4.

2 Model description and maximum likelihood estimation

According to constant *PALT*, group 1 consists of n_1 items randomly chosen among n test items is allocated to use condition and group 2 consists of $n_2 = n - n_1$ remaining items are subjected to an accelerated condition. Progressive Type-II censoring is applied as follows: In group $j,\ j = 1,2,$ at the time of the first failure, R_{j1} items are randomly withdrawn from the remaining n_{j1} surviving items. At the second failure, R_{j2} items from the remaining $n_j - 2 - R_{j1}$ items are randomly withdrawn. The test continues until the m_j^{th} failure at which time, all remaining $R_{jm_j} = n_j - m_j - R_{j1} - R_{j2} - R_{j(m_j-1)}$ items are withdrawn. The R_{ji} are fixed prior to the study, $m_j \le n_j$ and

$$m_j = np_j - \sum_{i=j}^{n-1} R_{ji}, \tag{2.2}$$

where $p_1(0 < p_1 < 1)$ and $p_2 = 1 - p_1$, are the proportions of sample units allocated to accelerated condition. It is clear that the complete sample and Type-II censored samples are special cases of this scheme.

2.1 Basic assumptions

1. The lifetime of an item tested at use condition follows $LE(a,b)$ distribution.

2. The HRF of an item tested at accelerated condition is given by $h_2(t) = \rho h_1(t)$, where ρ is an acceleration factor satisfying $\rho > 1$. So, the HRF, CDF, PDF and SF under accelerated condition are given, for $t > 0, a, b > 0$ and $\rho > 1$, respectively, by

$$\left.\begin{aligned} f_2(t) &= \rho(a+bt)\exp\left\{-\rho(at+\tfrac{b}{2}t^2)\right\}, \\ F_2(t) &= 1-\exp\left\{-\rho(at+\tfrac{b}{2}t^2)\right\}, \\ \overline{F}_2(t) &= \exp\left\{-\rho(at+\tfrac{b}{2}t^2)\right\}, \\ h_2(t) &= \rho(a+bt). \end{aligned}\right\} \tag{2.3}$$

3. The identically distributed lifetimes $T_{ji},\ j=1,2,\ i=1,...,n_j$ of items allocated to use condition $(j=1)$ and accelerated condition $(j=2)$ are mutually independent.

2.2 Maximum likelihood estimation

The maximum likelihood method is used to obtain the estimates of the parameter of the population distribution. Let $T_{j1:m_j:n_j}^{(R_{j1},...,R_{jmj})} < T_{j2:m_j:n_j}^{(R_{j1},...,R_{jmj})} < ... < T_{jm_j:m_j:n_j}^{(R_{j1},...,R_{jmj})},\ j=1,2$, are the progressive Type-II censored data from two populations whose SFs and $PDFs$ are given by (1.1) and (2.3), with $(R_{j1},...,R_{jm_j})$ being the two progressive censoring schemes. In the sequel, we will use $T_{jm_j:m_j:n_j}$ instead of $T_{jm_j:m_j:n_j}^{(R_{j1},...,R_{jm_j})}$. Also, let $t_{j1:m_j:n_j} < t_{j2:m_j:n_j} < ... < t_{jm_j:m_j:n_j}$ be the corresponding observed values. The likelihood function is given by

$$\begin{aligned} L(\theta;\mathbf{t}) &= \prod_{j=1}^{2}\left[A_j\prod_{i=1}^{m_j}(f_j(t_{ji:m_j:n_j}))[\bar{F}(t_{ji:m_j:n_j})]^{R_{ji}}\right] \\ &= \prod_{j=1}^{2}\left[A_j\prod_{i=1}^{m_j}\rho^{j-1}(a+bt_{ji:m_j:n_j})\left[\exp\left\{-\rho^{j-1}\left(at_{ji:m_j:n_j}+\frac{b}{2}t_{ji:m_j:n_j}^2\right)\right\}\right]^{R_{ji}+1}\right] \end{aligned} \tag{2.4}$$

where $\theta = (a,b,\rho),\ t = (t_1,t_2),\ t_j = (t_{j1},,t_{jm_j})$, and $A_j = n_j(n_j-1-R_{j1})(n_j-2-R_{j1}-R_{j2})(n_j-m_j+1-R_{j1}-R_{j2}-...-R_{j(m_j-1)})$.

Hence, the log-likelihood function, $\ell(\theta;t)$ is

$$\ell(\theta;t) = A + m_2\ln\rho + \sum_{j=1}^{2}\sum_{i=1}^{m_j}\left\{\ln(a+bt_{ji:m_j:n_j}) - \rho^{j-1}\eta_{ji}(a,b)\right\} \tag{2.5}$$

where $\eta_{ji}(a,b) = (R_{ji}+1)\left(at_{ji:m_j:n_j}+\frac{b}{2}t_{ji:m_j:n_j}^2\right)$ and $A = \ln A_1 + \ln A_2$. The components of the score vector $U(\theta) = (U_a,U_b,U_\rho)^T$ are

$$U_a = \sum_{j=1}^{2}\sum_{i=1}^{m_j}\left\{\frac{1}{a+bt_{ji:m_j:n_j}} - \rho^{j-1}(R_{ji}+1)t_{ji:m_j:n_j}\right\}, \tag{2.6}$$

$$U_b = \sum_{j=1}^{2}\sum_{i=1}^{m_j}\left\{\frac{t_{ji:m_j:n_j}}{a+bt_{ji:m_j:n_j}} - \frac{1}{2}\rho^{j-1}(R_{ji}+1)t_{ji:m_j:n_j}^2\right\}, \tag{2.7}$$

and

$$U_\rho = \frac{m_2}{\rho} - S(a,b), \tag{2.8}$$

where

$$S(a,b) = \sum_{i=1}^{m_2}\eta_{2i}(a,b).$$

Making use of $U(\theta) = 0$, we obtain three nonlinear equations. The system of these nonlinear equations cannot be solved analytically. So, we can apply numerical solution via iterative techniques such as Newton Raphson method, to get the $MLEs$, $\hat{\theta} = (\hat{a},\ \hat{b},\ \hat{\rho}\ )^T$ of θ.

On the other hand, setting $U_\rho = 0$ in (2.8), we can obtain

$$\hat{\rho} = \frac{m_2}{S(\hat{a},\hat{b})}. \tag{2.9}$$

substituting from (2.9) about $\hat{\rho}$ in $U_a = 0$ and $U_b = 0$ in (2.6) and (2.7), we can solve them numerically to obtain $\hat{a},\ \hat{b}$ and then substituting in (2.9), we get $\hat{\rho}$.

2.3 Observed Fisher information matrix

The observed Fisher information matrix (FI) for the $MLEs$ of the parameters $a,\ b$ and ρ is the 3×3 observed information matrix $I(\theta) = -\partial^2\ell/\partial\theta\partial\theta^T$, which are given in (2.11). The elements of the multivariate normal $N_3(0, I(\hat{\theta})^{-1})$ distribution can be used to construct asymptotic confidence intervals for the parameters. The observed FI matrix is

$$I(\hat{\theta}) = \begin{pmatrix} U_{aa} & U_{ab} & U_{a\rho} \\ U_{ba} & U_{bb} & U_{b\rho} \\ U_{\rho a} & U_{\rho b} & U_{\rho\rho} \end{pmatrix} \downarrow \theta = \hat{\theta} \tag{2.10}$$

where the elements of the matrix are given, from (2.6), (2.7) and (2.8), by

$$\begin{aligned} U_{aa} &= -\sum_{j=1}^{2}\sum_{i=1}^{m_1}\frac{1}{(a+bt_{ji:m_j:n_j})^2}, \\ U_{ab} &= -\sum_{j=1}^{2}\sum_{i=1}^{m_1}\frac{t_{ji:m_j:n_j}}{(a+bt_{ji:m_j:n_j})^2}, \\ U_{a\rho} &= -\sum_{i=1}^{m_2}(R_{2i}+1)t_{2i:m_2:n_2}, \\ U_{bb} &= -\sum_{j=1}^{2}\sum_{i=1}^{m_1}\frac{t^2_{ji:m_j:n_j}}{(a+bt_{ji:m_j:n_j})^2}, \\ U_{b\rho} &= -\frac{1}{2}\sum_{i=1}^{m_2}(R_{2i}+1)t^2_{2i:m_2:n_2}, \\ U_{\rho\rho} &= \frac{-m_2}{\rho^2}. \end{aligned} \tag{2.11}$$

3 Simulation studies

In order to obtain the $MLEs$ of (a,b,ρ) and study the properties of their estimates through the mean squared errors (MSE) and relative absolute biases (RAB), a simulation study is performed, for $j = 1,2$, according to the following steps:

1. Based on the values of n_j and m_j $(1 \le m_j \le n_j)$, generate two independent random samples of sizes m_1 and m_2 from Uniform $(0,1)$ distribution, $(U_{j1}, U_{j2}, ..., U_{jm_j})$.
2. Determine the values of the censored scheme $R_{ji},\ i = 1,2,...,m_j$.
3. Set $E_{ji} = 1/(i + \sum_{d=m_j-i+1}^{m_j} R_{jd})$.
4. Set $V_{ji} = U_{ji}^{E_{ji}}$.
5. Construct the two progressive Type-II censored samples, $(U^*_{j1}, U^*_{j2}, ..., U^*_{jm_j})$, from the Uniform $(0,1)$ distribution, where $U^*_{ji} = 1 - \prod_{d=m_j-i+1}^{m_j} V_{jd}$.
6. Using Step 5, generate two random samples $(t_{j1}, t_{j2}, ..., t_{jm_j})$, from $CDFs$ and $F_1(t)$ and $F_2(t)$ given in (1.1) and (2.3) respectively, as follows $t_{ji} = [(-a\rho^{j-1} + \sqrt{a^2\rho^{2(j-1)} - 2b\,log(1-U^*_i)})/b\rho^{j-1}]$ and hence obtain the two ordered samples $(t_{j1:m_j:n_j}, t_{j2:m_j:n_j}, ..., t_{jm_j:m_j:n_j})$ which represent two progressive Type-II censored samples from $LE(a,b)$ distribution under constant $PALT$.

7.From the ordered observations, obtained in Step 6, compute the $MLE(\hat{a})$ of the parameter a by solving nonlinear $U_a = 0$ and hence compute the $MLE(\hat{b})$ and $MLE(\hat{\rho})$ from $U_b = 0$ and $U_\rho = 0$ respectively.
8.Repeat Steps 1-7, r times representing r $MLEs$ of (a,b,ρ) based on r different samples.
9.If $\hat{\Psi}_{kd}$ is a MLE of Ψ_k, $k = 1,2,3$ (where $\Psi_1 \equiv a$, $\Psi_2 \equiv b$, $\Psi_3 \equiv \rho$) based on sample d, $d = 1,\ldots,r$, then the average of MLE, MSE and RAB, of $\hat{\Psi}_k$ over the r samples are given, respectively, by

$$\overline{\hat{\Psi}}_k = \frac{1}{r}\sum_{d=1}^{r}\hat{\Psi}_{kd}, \qquad MSE(\hat{\Psi}_k) = \frac{1}{r}\sum_{d=1}^{r}(\hat{\Psi}_{kd} - \Psi_k)^2, \qquad RAB(\hat{\Psi}_k) = \sum_{d=1}^{r}\left|\frac{\overline{\hat{\Psi}}_k - \Psi_k}{\Psi_k}\right|.$$

10.From Step 9, compute $\overline{\hat{\Psi}}_k$, $MSE(\hat{\Psi}_k)$ and $RAB(\hat{\Psi}_k)$.

3.1 Approximate confidence intervals

For the general asymptotic theory of MLE $\hat{\theta}$ for θ, the sampling distribution of $((\hat{\theta}-\theta)/\sqrt{var(\hat{\theta})})$, can be approximated by a standard normal distribution, where $\sqrt{var(\hat{\theta}_k)}$ is calculated from $I(\hat{\theta})^{-1}$. The asymptotic $100(1-\alpha)\%$ confidence intervalsof $\hat{\theta}_k$ are $(\overline{\hat{\theta}}_k \pm z_{\frac{\alpha}{2}}\ SE(\overline{\hat{\theta}}_k))$, where $\theta_1 = a$, $\theta_2 = b$, $\theta_3 = \rho$ and $z_{\frac{\alpha}{2}}$ is the quantile $(1-\frac{\alpha}{2})$ of the standard normal distribution and $SE(.)$ is the square root of the diagonal element of $I(\hat{\theta})^{-1}$ corresponding to each parameter. A two-sided $100(1-\alpha)\%$ normal approximation CI for the parameter θ_k can be constructed as $(\overline{\hat{\theta}}_k \pm z_{\frac{\alpha}{2}}\sqrt{var(\overline{\hat{\theta}}_k)},\ k = 1,2,3)$.

3.2 Bootstrap CIs

For comparison purposes, two confidence intervals based on the parametric bootstrap methods are proposed. It was observed by Kundu et al.[12] that the nonparametric bootstrap method does not work well. So, two parametric bootstrap methods are are:

- Studentized-t (Stud-t) bootstrap CI $(boot-t)$ suggested by Hall [24].
- Percentile (Percen) bootstrap CI $(boot-p)$ suggested by Efron ([10]).

Hall [24] showed that the Stud-t CI is better than the Percen bootstrap CI from an asymptotic point of view, although the finite sample properties are not yet known. In order to obtain the Stud-t and Percen CIs bootstrap methods of (a,b,ρ), a simulation study is performed, for $j = 1,2$, according to the following steps:

1.Follow the same Steps 1-5 of the algorithm described above to generate two progressive Type-II right censored samples from the Uniform$(0,1)$ distribution of the form $(V^*_{j1}, V^*_{j2}, \ldots, V^*_{jm^*_j})$,
$j = 1,2$, as shown in Step 5.
2.Use (a) and Step 7 in the above algorithm to generate two random samples $(t^*_{j1},\ldots,t^*_{jm^*_j})$,
$j = 1,2$ from $CDFs$ $F1(t)$ and $F2(t)$ given in (1.1) and (2.3), respectively, as follows
$t^*_{ji} = [(-\hat{a}\hat{\rho}^{j-1} + \sqrt{\hat{a}^2\hat{\rho}^{2(j-1)} - 2blog(1-V^*_{ji})})/\hat{\rho}^{j-1}\hat{b}]$, where $i = 1,2,\ldots,m^*_j$,where $\hat{a}$, $\hat{b}$ and $\hat{\rho}$ are as obtained in Step 7 in the above algorithm, and hence obtain the two ordered samples $(t^*_{j1:m^*_j:n^*_j},\cdots,t^*_{jm^*_j:m^*_j:n^*_j})$, which represent two progressive Type-II right-censored bootstrap samples from $LE(a,b)$ distributionunder constant $PALT$. The values of n^*_j and m^*_j , have been taken to be equal n_j and m_j , respectively.
3.From the ordered observations obtained in (b), we can get the bootstrap estimate $\hat{a}^*$ of the parameter a, by solving $U_a = 0$ and hence the bootstrap estimates $\hat{b}^*$ and $\hat{\rho}^*$ can be obtained from $U_b = 0$ and $U_\rho = 0$.
4.Repeat Steps $(a)-(c)$, r^* times representing r^* bootstrap $MLEs$ of (a,b,ρ).
5.Arrange all the values of $\hat{a}^*$'s , $\hat{b}^*$'s and $\hat{\rho}^*$'s in an ascending order to obtainthe bootstrap samples $(\hat{\psi}_l^{*[1]}, \hat{\psi}_l^{*[2]}, \ldots, \hat{\psi}_l^{*[r^*]})$, $l = 1,\ 2,\ 3$, (where $\psi^*_1 \equiv a^*, \psi^*_2 \equiv b^*, \psi^*_3 \equiv \rho^*$).

3.2.1 Stud-t bootstrap CIs

The Stud-t bootstrap CIs can be constructed as follows:

1. Find the order statistics $\delta_l^{*[1]},...,\delta_l^{*[r^*]}$, where

$$\delta_l^{*[d]} = \frac{\hat{\psi}_l^{*[d]} - \hat{\psi}_l}{\sqrt{Var(\hat{\psi}_l^{*[d]})}}, \qquad d = 1,2,...,r^*.$$

2. Consider all possible $100(1-\alpha)\%$ CIs of the form $(\delta_l^{*[h]}, \delta_l^{*[(1-\alpha)r^*+h]})$, where $h = 1,...,\alpha r^*$, $l = 1,2,3$ and choose the interval for which the width is minimum, say $(\delta_{lL}^*, \delta_{lU}^*)$.
3. A two-sided $100(1-\alpha)\%$ Stud-t bootstrap CI for ψ_l is either
$\left(\hat{\psi}_l - \delta_{lU}^*\sqrt{var(\hat{\psi}_l)},\ \hat{\psi}_l - \delta_{lL}^*\sqrt{var(\hat{\psi}_l)}\right)$.
or
$\left(\hat{\psi}_l - \delta^{*[(1-\alpha/2)r^*]}\sqrt{var(\hat{\psi}_l)},\ \hat{\psi}_l - \delta^{*[\alpha r^*/2]}\sqrt{var(\hat{\psi}_l)}\right)$.
where $var(\hat{\psi}_l)$ can be estimated by the asymptotic variance from $N_3(0, I(\hat{\theta})^{-1})$.

3.2.2 Percen bootstrap CIs

The Percen bootstrap CIs can be constructed as follows:

1. Based on the order statistics $(\hat{\psi}_l^{*[1]} < \hat{\psi}_l^{*[2]} < ... < \hat{\psi}_l^{*[r^*]})$, consider all possible $100(1-\alpha)\%$ of the form $\left(\hat{\psi}_l^{*[h]},\ \hat{\psi}_l^{*[(1-\alpha)r^*+h]}\right)$, $h = 1,...,\alpha r^*$, and choose the interval with minimum width, say $(\hat{\psi}_{lL}^*,\ \hat{\psi}_{lU}^*)$.
2. A two-sided $100(1-\alpha)\%$ Percen bootstrap CI for ψ_l is either $(\hat{\psi}_{lL}^*,\ \hat{\psi}_{lU}^*)$ or $\left(\hat{\psi}_l^{*[\alpha r^*/2]},\ \hat{\psi}_l^{*[(1-\alpha/2)r^*]}\right)$.

3.3 Simulation procedure

A Monte Carlo simulation study is carried out in order to calculate the $MLEs$, $MSEs$, $RABs$ and 95% approximate (Approx) CIs of the model parameters, based on $r = 1000$ Monte Carlo simulations. Based on $r^\star = 1000$ bootstrap replications, lower and upper bounds of 95% Stud-t and Percen bootstrap CIs are calculated and compared with Approx CIs through their lengths. The lower and upper values of the bootstrap CIs are taken to represent the average lower and upper values over 1000 bootstrap CIs that correspond to 1000 Monte Carlo simulations.

1. Table 1 lists the different censoring schemes, used in the simulation study, for different choices of sample sizes, $n_1 = n_2 = n$, and observed failure times $m_1 = m_2 = m$ which represent 40%, 70% and 100% of the sample size. The $MLEs$, $MSEs$ and $RABs$ for the parameters with $(a,\ b,\ \rho) = (0.5,\ 1.5,\ 1.2)$ and $(a,b,\rho) = (0.2,\ 1,\ 2)$ under progressive censoring schemes listed in Table 1 are shown in Table 3. The comparisons of lengths and the coverage percentages (CP) of the 95% CIs are shown in Table 4.

2. Table 2 lists the censoring schemes used in the simulation study for another example based on different values of n_1, n_2, m_1 and m_2. Based on censoring schemes listed in Table 2 and similarly as in Tables 3 and 4, Tables 5 and 6 show the results of the second example (using the same generating values $(a,\ b,\ \rho) = (0.5,\ 1.5,\ 1.2)$ and $(a,b,\rho) = (0.2,\ 1,\ 2)$). The number of items to be tested in group 2, should be selected to be greater than those in group 1, since they are subjected to an accelerated condition (as in the data of Table 2).

Now, we shall consider the following two tables, Table 1 and Table 2:

Table 1 : Progressive censoring schemes used in the Monte Carlo simulation study at $n_1 = n_2 = n$ and $m_1 = m_2 = m$.

n / m	Sc	$(R_1,\ldots,R_m)$	Sc	$(R_1,\ldots,R_m)$	Sc	$(R_1,\ldots,R_m)$
20 / 8	[1]	R_1=12, R_i=0, $i\neq1$	[2]	R_5=12, R_i=0, $i\neq5$	[3]	R_8=12, R_i=0, $i\neq8$
20 / 14	[1]	R_1=6, R_i=0, $i\neq1$	[2]	R_8=6, R_i=0, $i\neq8$	[3]	R_{14}=6, R_i=0, $i\neq14$
40 / 16	[1]	R_1=24, R_i=0, $i\neq1$	[2]	R_9=24, R_i=0, $i\neq9$	[3]	R_{16}=24, R_i=0, $i\neq16$
40 / 28	[1]	R_1=12, R_i=0, $i\neq1$	[2]	R_{15}=12, R_i=0, $i\neq15$	[3]	R_{28}=12, R_i=0, $i\neq28$
70 / 28	[1]	R_1=42, R_i=0, $i\neq1$	[2]	R_{15}=42, R_i=0, $i\neq15$	[3]	R_{28}=42, R_i=0, $i\neq28$
70 / 49	[1]	R_1=21, R_i=0, $i\neq1$	[2]	R_{25}=21, R_i=0, $i\neq25$	[3]	R_{49}=21, R_i=0, $i\neq49$

Table 2 : Progressive censoring schemes used in the Monte Carlo simulation study at $n_1 \neq n_2 = n$ and $m_1 \neq m_2 = m$.

n_1 / n_2	m_1 / m_2	scheme	$(R_1,\ldots,R_{m_1})$ / $(R_1,\ldots,R_{m_2})$	scheme	$(R_1,\ldots,R_{m_1})$ / $(R_1,\ldots,R_{m_2})$	scheme	$(R_1,\ldots,R_{m_1})$ / $(R_1,\ldots,R_{m_2})$
10 / 20	4 / 8	[1]	{R_1=6, R_i=0, $i\neq1$} {R_1=12, R_j=0, $j\neq1$}	[2]	{R_3=6, R_i=0, $i\neq3$} {R_5=12, R_j=0, $j\neq5$}	[3]	{R_4=6, R_i=0, $i\neq4$} {R_8=12, R_j=0, $j\neq8$}
	7 / 14	[1]	{R_1=3, R_i=0, $i\neq1$} {R_1=6, R_j=0, $j\neq1$}	[2]	{R_4=3, R_i=0, $i\neq4$} {R_8=6, R_j=0, $j\neq8$}	[3]	{R_7=3, R_i=0, $i\neq7$} {R_{14}=6, R_j=0, $j\neq14$}
30 / 40	12 / 16	[1]	{R_1=18, R_i=0, $i\neq1$} {R_1=24, R_j=0, $j\neq1$}	[2]	{R_7=18, R_i=0, $i\neq7$} {R_9=24, R_j=0, $j\neq9$}	[3]	{R_{12}=18, R_i=0, $i\neq12$} {R_{16}=24, R_j=0, $j\neq16$}
	21 / 28	[1]	{R_1=9, R_i=0, $i\neq1$} {R_1=12, R_j=0, $j\neq1$}	[2]	{R_{11}=9, R_i=0, $i\neq11$} {R_{15}=12, R_j=0, $j\neq15$}	[3]	{R_{21}=9, R_i=0, $i\neq21$} {R_{28}=12, R_j=0, $j\neq28$}
60 / 70	24 / 28	[1]	{R_1=36, R_i=0, $i\neq1$} {R_1=42, R_j=0, $j\neq1$}	[2]	{R_{13}=36, R_i=0, $i\neq13$} {R_{15}=42, R_j=0, $j\neq15$}	[3]	{R_{24}=36, R_i=0, $i\neq24$} {R_{28}=42, R_j=0, $j\neq28$}
	42 / 49	[1]	{R_1=18, R_i=0, $i\neq1$} {R_1=21, R_j=0, $j\neq1$}	[2]	{R_{22}=18, R_i=0, $i\neq22$} {R_{25}=21, R_j=0, $j\neq25$}	[3]	{R_{42}=18, R_i=0, $i\neq42$} {R_{49}=21, R_j=0, $j\neq49$}

4 Concluding Remarks

The subject of progressive censoring has received considerable attention in the past few years, due in part to the availability of high speed computing resources, which mak it both a feasible topic for simulation studies for researchers and a feasible method of gathering lifetime data for practitioners. It has been illustrated by Viveros and Balakrishnan [26] that the inference is feasible, and practical when the sample data are gathered according to a Type-II progressively censored experimental scheme. In this paper a constant stress *PALT* model when the observed failure times come form $LE(a,b)$ distribution under progressively Type-II censored data have been considered. The *MLEs* of the considered parameters are obtained and the performance of this estimates are studied through their *MSEs* and *RABs*. Also, We have constructed approximate and bootstrap *CIs* for the parameters. A simulation study, based on two different examples (according to choices of nj and $m_j,\ j=1,2$), has been made to examine the performance of the $MLEs$ and compare three different methods for the CIs. Three cases of censoring schemes are applied at fixed samples and failure time sizes. The first and the third assume that censoring occurs only at the first and the last observed failure, while the second scheme assumes censoring occurs only at the middle observed failure. It can be notice, from the simulation study, that:

1.The MSE and RAB of the $MLEs$ are computed over different combinations of the censored schemes.
2.For fixed values of the sample size, by increasing the failure times the $MSEs$ and $RABs$ of the considered parameters decrease.
3.For fixed values of the sample and failure time sizes, the schemes in which the censorin occurs after the first observed failure gives more accurate results through the MSE and $RABs$ than the other two schemes, and this coincides with Theorem [2.2] by Burkschat et al. [18].
4.The bootstrap CIs give more accurate results than the approximate CIs since the lengths of the former are less than the lengths of latter, for different sample sizes, observed failures and schemes.
5.For small sample sizes, the Stud-t bootstrap CIs are better than the Percen bootstrap CIs in the sense of having smaller widths. However, the differences between the lengths of CIs using both methods decrease when sample sizes increase.

Table 3 : $MLEs$, $MSEs$ and $(RABs)$ for the parameters with $(a,b,\rho)=(0.5,\ 1.5,\ 1.2)$ and $(a,b,\rho)=(0.2,\ 1,\ 2)$.

		$(0.5,\ 1.5,\ 1.2)$			$(0.2,\ 1,\ 2)$		
(n,m)	SC	a	b	ρ	a	b	ρ
(20,8)	[1]	0.491	1.113	1.474	0.2108	0.986	2.248
		0.309 (0.016)	0.913 (0.392)	0.921 (0.228)	0.128 (0.038)	0.477 (0.338)	1.114 (0.124)
	[2]	0.474	1.227	1.466	0.197	1.145	2.456
		0.288 (0.050)	1.051 (0.534)	0.878 (0.221)	0.168 (0.097)	1.051 (0.765)	0.878 (0.314)
	[3]	0.389	1.522	1.595	0.188	0.897	2.644
		0.266 (0.220)	1.470 (0.803)	1.182 (0.329)	0.234 (0.178)	1.470 (0.971)	1.182 (0.543)
(20,14)	[1]	0.465	0.973	1.348	0.205	1.098	2.085
		0.260 (0.068)	0.513 (0.216)	0.573 (0.124)	0.165 (0.025)	0.357 (0.0981)	0.786 (0.042)
	[2]	0.472	1.013	1.326	0.212	1.007	2.100
		0.244 (0.054)	0.563 (0.266)	0.549 (0.105)	0.228 (0.041)	0.386 (0.007)	0.590 (0.050)
	[3]	0.456	1.061	1.349	0.214	1.131	2.145
		0.246 (0.087)	0.693 (0.327)	0.635 (0.124)	0.191 (0.074)	0.412 (0.131)	0.819 (0.072)
(20,20)		0.467	0.959	1.288	0.311	0.826	2.123
		0.225 (0.065)	0.438 (0.199)	0.449 (0.074)	0.388 (0.156)	0.287 (0.1732)	0.441 (0.061)
(40,16)	[1]	0.491	0.9643	1.3087	0.192	1.019	2.369
		0.253 (0.016)	0.495 (0.205)	0.523 (0.090)	0.135 (0.037)	0.269 (0.019)	0.862 (0.204)
	[2]	0.482	1.851	1.333	0.1994	1.227	2.501
		0.248 (0.034)	1.072 (0.234)	0.555 (0.111)	0.027 (0.003)	0.984 (0.227)	0.917 (0.311)
	[3]	0.427	1.220	1.360	0.126	1.236	2.5429
		0.221 (0.145)	0.862 (0.525)	0.607 (0.133)	0.106 (0.365)	0.578 (0.236)	0.931 (0.371)
(40,28)	[1]	0.484	1.641	1.236	0.178	1.140	2.2498
		0.221 (0.031)	0.551 (0.094)	0.332 (0.030)	0.230 (0.079)	0.374 (0.080)	0.882 (0.124)
	[2]	0.476	1.639	1.277	0.1823	0.866	2.511
		0.201 (0.046)	0.546 (0.092)	0.352 (0.064)	0.0413 (0.088)	0.235 (0.134)	0.913 (0.351)
	[3]	0.481	1.654	1.252	0.219	1.157	1.883
		0.219 (0.038)	0.636 (0.102)	0.35819 (0.044)	0.126 (0.095)	0.396 (0.157)	0.542 (0.158)
(40,40)		0.473	1.634	1.248	0.237	1.171	1.796
		0.180 (0.024)	0.413 (0.052)	0.296 (0.038)	0.037 (0.186)	0.164 (0.081)	0.403 (0.131)
(70,28)	[1]	0.493	1.642	1.248	0.179	1.009	2.591
		0.215 (0.013)	0.526 (0.094)	0.349 (0.040)	0.202 (0.0894)	0.238 (0.009)	0.639, (0.295)
	[2]	0.489	1.687	1.250	0.207	0.861	2.143
		0.182 (0.022)	0.641 (0.125)	0.353 (0.042)	0.184 (0.073)	0.303 (0.138)	0.528 (0.071)
	[3]	0.473	1.746	1.261	0.198	0.974	2.736
		0.201 (0.054)	0.825 (0.164)	0.362 (0.051)	(0.106) (0.007)	0.252 (0.025)	0.895 (0.3681)
(70,49)	[1]	0.485	1.584	1.228	0.198	1.079	1.970
		0.172 (0.029)	0.393 (0.057)	0.257 (0.023)	0.121 (0.005)	0.205 (0.079)	0.133 (0.014)
	[2]	0.477	1.596	1.246	0.182	1.081	1.908
		0.157 (0.044)	0.416 (0.064)	0.276 (0.038)	0.161 (0.087)	0.222 (0.081)	0.147 (0.045)
	[3]	0.486	1.578	1.245	0.189	1.085	1.899
		0.169 (0.028)	0.451 (0.052)	0.262 (0.037)	0.170 (0.065)	0.204 (0.085)	0.469 (0.050)
(70,70)		0.488	1.589	1.204	0.205	1.034	2.160
		0.150 (0.022)	0.332 (0.059)	0.184 (0.004)	0.059 (0.028)	0.115 (0.034)	0.309 (0.080)

Table 4 : Comparisons of lengths and (CP) of 95% CIs for the parameters with $(a,b,\rho) = (0.5,\ 1.5,\ 1.2)$ and $(a,b,\rho) = (0.2,\ 1,\ 2)$.

		(0.5, 1.5, 1.2)			(0.2, 1, 2)		
		MLE	$boot-t$	$boot-p$	MLE	$boot-t$	$boot-p$
(n,m)	SC	a / b / ρ	a / b / ρ	a / b / ρ	a / b / ρ	a / b / ρ	a / b / ρ
(20,8)	[1]	1.326 (0.943)	1.357 (0.964)	1.220 (0.967)	0.842 (0.933)	0.690 (0.926)	0.815 (0.937)
		2.678 (0.947)	2.324(0.986)	3.403 (0.948)	2.061 (0.943)	1.927 (0.953)	2.245 (0.977)
		2.989 (0.957)	3.007 (0.927)	3.735 (0.938)	4.171 (0.944)	4.046 (0.926)	4.276 (0.965)
	[2]	1.184 (0.954)	1.206 (0.960)	1.123 (0.965)	0.858 (0.958)	0.610 (0.943)	0.816 (0.964)
		3.277 (0.957)	2.870 (0.979)	4.202 (0.934)	2.708 (0.964)	3.101 (0.976)	4.190 (0.924)
		2.984 (0.894)	2.975 (0.906)	3.822 (0.919)	4.163 (0.925)	3.764 (0.927)	4.417 (0.918)
	[3]	1.226 (0.938)	1.121 (0.949)	0.919 (0.938)	0.789 (0.957)	0.510 (0.984)	0.671 (0.976)
		4.755 (0.938)	3.441 (0.956)	4.888 (0.977)	3.466 (0.975)	2.826 (0.984)	3.870 (0.946)
		3.351 (0.929)	3.228 (0.918)	4.045 (0.926)	4.366 (0.925)	3.946 (0.914)	4.476 (0.903)
(20,14)	[1]	1.055 (0.936)	1.070 (0.935)	0.964 (0.964)	0.694 (0.926)	0.545 (0.908)	0.641 (0.938)
		1.818 (0.964)	1.663 (0.944)	2.121 (0.955)	1.551 (0.976)	1.282 (0.977)	1.710 (0.967)
		2.043 (0.894)	2.056 (0.904)	2.412 (0.934)	3.179 (0.904)	3.149 (0.946)	3.345 (0.926)
	[2]	1.002 (0.939)	1.025 (0.963)	0.939 (0.952)	0.774 (0.942)	0.550 (0.981)	0.710 (0.976)
		2.004 (0.929)	1.840 (0.927)	2.389 (0.943)	1.673 (0.905)	1.577 (0.926)	1.794 (0.966)
		2.004 (0.952)	2.026 (0.938)	2.419 (0.957)	3.254 (0.958)	3.202 (0.946)	3.453 (0.986)
	[3]	1.063 (0.924)	1.050 (0.959)	0.903 (0.933)	0.754 (0.944)	0.575 (0.988)	0.683 (0.967)
		2.398 (0.915)	2.008 (0.929)	2.669 (0.949)	1.628 (0.954)	1.532 (0.964)	1.779 (0.986)
		2.061 (0.939)	2.070 (0.954)	2.512 (0.904)	3.268 (0.938)	3.215 (0.965)	3.652 (0.978)
(20,20)		0.931 (0.927)	0.943 (0.943)	0.864 (0.966)	0.701 (0.934)	0.561 (0.945)	0.700 (0.976)
		1.534 (0.924)	1.437 (0.909)	1.704 (0.937)	1.152 (0.925)	1.324 (0.957)	1.642 (0.969)
		1.628 (0.939)	1.648 (0.979)	1.838 (0.954)	2.707 (0.970)	2.112 (0.975)	3.108 (0.948)
(40,16)	[1]	0.972 (0.943)	1.044 (0.967)	0.917 (0.976)	0.586 (0.947)	0.401 (0.977)	0.570 (0.954)
		1.706 (0.961)	1.595 (0.958)	2.002 (0.946)	1.319 (0.968)	1.240 (0.971)	1.440 (0.957)
		1.849 (0.942)	1.873 (0.975)	2.145 (0.944)	3.566 (0.948)	3.710 (0.959)	3.877 (0.974)
	[2]	1.326 (0.911)	1.357 (0.907)	1.220 (0.909)	0.498 (0.917)	0.380 (0.919)	0.503 (0.923)
		2.678 (0.952)	2.324 (0.939)	3.403 (0.956)	1.633 (0.946)	1.562 (0.936)	1.788 (0.949)
		2.985 (0.908)	3.007 (0.919)	3.735 (0.929)	3.203 (0.904)	3.063 (0.897)	4.037 (0.926)
	[3]	0.898 (0.963)	0.917 (0.974)	0.753 (0.961)	0.572 (0.946)	0.340 (0.963)	0.475 (0.953)
		3.108 (0.920)	2.519 (0.938)	3.344 (0.944)	2.116 (0.934)	1.880 (0.927)	2.645 (0.924)
		1.954 (0.963)	1.976 (0.991)	2.421 (0.984)	3.069 (0.969)	2.742 (0.978)	3.701 (0.988)
(40,28)	[1]	0.888 (0.944)	0.903 (0.946)	0.841 (0.953)	0.451 (0.958)	0.275 (0.962)	0.415 (0.947)
		2.010 (0.971)	1.942 (0.976)	2.165 (0.945)	1.708 (0.972)	1.903 (0.965)	2.216 (0.975)
		1.307 (0.918)	1.342 (0.924)	1.418 (0.921)	2.745 (0.919)	2.685 (0.904)	2.805 (0.895)
	[2]	0.810 (0.939)	0.831 (0.962)	0.780 (0.953)	0.424 (0.953)	0.345 (0.946)	0.422 (0.938)
		2.093 (0.974)	2.031 (0.983)	2.292 (0.990)	0.910 (0.979)	1.271 (0.976)	1.361 (0.986)
		1.352 (0.929)	1.388 (0.972)	1.480 (0.982)	2.771 (0.953)	2.863 (0.964)	2.930 (0.942)
	[3]	0.868 (0.944)	0.878 (0.977)	0.807 (0.935)	0.537 (0.962)	0.423 (0.942)	0.523 (0.982)
		2.418 (0.949)	2.251 (0.955)	2.593 (0.937)	1.321 (0.939)	1.218 (0.947)	1.499 (0.938)
		1.330 (0.934)	1.362 (0.946)	1.463 (0.951)	1.804 (0.959)	1.768 (0.922)	1.961 (0.948)

Table4: (continued)

		(0.5, 1.5, 1.2)			(0.2, 1, 2)		
		MLE	*boot* − *t*	*boot* − *p*	*MLE*	*boot* − *t*	*boot* − *p*
		a	a	a	a	a	a
		b	b	b	b	b	b
(n,m)	*SC*	ρ	ρ	ρ	ρ	ρ	ρ
(40,40)		0.767 (0.923)	0.786 (0.951)	0.742 (0.956)	0.577 (0.946)	0.463 (0.928)	0.587 (0.922)
		1.654 (0.921)	1.616 (0.932)	1.759 (0.938)	1.112 (0.949)	1.107 (0.939)	1.301 (0.945)
		1.101 (0.947)	1.136(0.935)	1.173 (0.934)	1.508 (0.929)	1.311 (0.941)	1.615 (0.934)
(70,28)	[1]	0.855 (0.902)	0.898 (0.896)	0.822 (0.919)	0.389 (0.920)	0.310 (0.889)	0.422 (0.910)
		1.990 (0.950)	1.938 (0.982)	2.169 (0.971)	0.952 (0.960)	0.927 (0.966)	0.977 (0.977)
		1.320 (0.965)	1.354 (0.963)	1.429 (0.947)	2.729 (0.958)	2.777 (0.962)	2.823 (0.968)
	[2]	0.694 (0.977)	0.732 (0.958)	0.697 (0.986)	0.407 (0.971)	0.381 (0.963)	0.433 (0.973)
		2.240 (0.936)	2.207 (0.947)	2.528 (0.919)	0.966 (0.917)	0.954 (0.903)	0.989 (0.933)
		1.324 (0.957)	1.365 (0.957)	1.454 (0.963)	2.352 (0.956)	2.171 (0.954)	2.622 (0.966)
	[3]	0.787 (0.966)	0.808 (0.936)	0.713 (0.938)	0.436 (0.949)	0.345 (0.943)	0.417 (0.955)
		3.178 (0.933)	2.838 (0.937)	3.344 (0.943)	1.325 (0.941)	1.198 (0.949)	1.411 (0.951)
		1.344 (0.905)	1.384 (0.918)	1.520 (0.918)	3.141(0.901)	3.049 (0.885)	3.499 (0.892)
(70,49)	[1]	0.677 (0.955)	0.703 (0.959)	0.662 (0.948)	0.404 (0.960)	0.358 (0.963)	0.415 (0.947)
		1.482 (0.946)	1.463 (0.956)	1.564 (0.949)	0.825 (0.955)	0.744 (0.977)	0.861 (0.966)
		0.979 (0.929)	1.009 (0.937)	1.031 (0.958)	1.599 (0.955)	1.334 (0.968)	1.601 (0.959)
	[2]	0.612 (0.977)	0.639 (0.972)	0.607 (0.983)	0.358 (0.974)	0.324 (0.973)	0.368 (0.985)
		1.552 (0.936)	1.540 (0.939)	1.649 (0.929)	0.855 (0.940)	0.774 (0.945)	0.815 (0.923)
		0.993 (0.949)	1.025 (0.967)	1.053 (0.959)	1.541 (0.968)	1.442 (0.972)	1.601 (0.980)
	[3]	0.658 (0.934)	0.685 (0.946)	0.643 (0.949)	0.424 (0.966)	0.415 (0.957)	0.471 (0.939)
		1.782 (0.921)	1.737 (0.916)	1.890 (0.910)	0.999 (0.928)	0.961(0.922)	1.033 (0.920)
		0.995 (0.922)	1.026 (0.934)	1.061 (0.902)	1.562 (0.918)	1.449 (0.923)	1.520 (0.919)
(70,70)		0.583 (0.946)	0.605 (0.959)	0.579 (0.947)	0.331(0.953)	0.329 (0.937)	0.341 (0.944)
		1.239 (0.934)	1.233 (0.938)	1.293 (0.945)	0.657 (0.947)	0.651 (0.951)	0.746 (0.948)
		0.813 (0.924)	0.837 (0.912)	0.849 (0.922)	1.474 (0.919)	1.385 (0.896)	1.563 (0.922)

Table 5 : $MLEs$, $MSEs$ and $(RABs)$ for the parameters with $(a,b,\rho) = (0.5,\ 1.5,\ 1.2)$ and $(a,b,\rho) = (0.2,\ 1,\ 2)$.

			(0.5, 1.5, 1.2)			(0.2, 1, 2)		
(n_1,n_2)	(m_1,m_2)	SC	a	b	ρ	a	b	ρ
(10,20)	(4,8)	[1]	0.517	1.349	1.492	0.180	1.168	2.417
			0.430 (0.035)	1.486 (0.687)	1.071 (0.243)	0.117 (0.097)	0.645 (0.168)	1.275 (0.208)
		[2]	0.498	1.437	1.599	0.211	1.038	2.808
			0.416 (0.002)	1.482 (0.796)	1.223 (0.333)	0.629 (0.137)	0.725 (0.038)	1.818 (0.404)
		[3]	0.436	1.811	1.811	0.202	1.057	2.213
			0.352 (0.126)	1.966 (1.264)	1.966 (1.264)	0.759 (0.091)	0.553 (0.057)	0.757 (0.1067)
	(7,14)	[1]	0.493	1.106	1.348	0.214	1.236	2.192
			0.333 (0.013)	0.861 (0.382)	0.685 (0.124)	0.576 (0.124)	0.510 (0.236)	0.968 (0.096)
		[2]	0.491	1.909	1.390	0.191	1.049	2.352
			0.344 (0.017)	1.294 (0.273)	0.724 (0.158)	0.130 (0.040)	0.483 (0.049)	1.079 (0.176)
		[3]	0.453	1.210	1.383	0.220	0.999	1.934
			0.299 (0.093)	1.002 (0.512)	0.742 (0.153)	0.641 (0.170)	0.392 (0.001)	0.515 (0.032)
	(10,20)		0.487	0.998	1.292	0.221	1.311	1.942
			0.290 (0.026)	0.584 (0.248)	0.523 (0.077)	0.578 (0.201)	0.754 (0.311)	1.018 (0.028)
(30,40)	(12,16)	[1]	0.503	1.776	1.301	0.187	1.057	2.565
			0.293 (0.007)	0.905 (0.184)	0.558 (0.083)	0.232 (0.062)	0.496 (0.057)	1.067 (0.282)
		[2]	0.482	1.851	1.333	0.204	0.936	2.297
			0.248 (0.034)	1.072 (0.234)	0.555 (0.112)	0.782 (0.032)	0.442 (0.063)	0.817 (0.148)
		[3]	0.458	2.081	1.326	0.179	1.0879, ,	2.270
			0.276 (0.083)	1.401 (0.387)	0.599 (0.105)	0.307 (0.090)	0.261 (0.087)	0.638 (0.135)
	(21,28)	[1]	0.490	1.658	1.254	0.205	1.056	1.747
			0.235 (0.019)	0.590 (0.105)	0.371 (0.045)	0.696 (0.042)	0.252 (0.056)	0.494 (0.1263)
		[2]	0.483	1.681	1.263	0.188	1.026	2.313
			0.224 (0.033)	0.671 (0.121)	0.383 (0.052)	0.136 (0.055)	0.401 (0.026)	0.721 (0.156)
		[3]	0.477	1.689	1.291	0.188	0.911	2.432
			0.255 (0.044)	0.702 (0.126)	0.425 (0.076)	0.112 (0.062)	0.283 (0.0882)	0.814 (0.215)
	(30,40)		0.487	1.620	1.239	0.188	1.073	2.063
			0.213 (0.025)	0.503 (0.080)	0.309 (0.032)	0.141 (0.059)	0.238 (0.0733)	0.518 (0.032)
(60,70)	(24,28)	[1]	0.498	1.613	1.266	0.219	0.926	2.131
			0.219 (0.003)	0.558 (0.075)	0.390 (0.055)	0.132 (0.097)	0.195 (0.073)	0.343 (0.065)
		[2]	0.477	1.692	1.296	0.184	1.1578	1.991
			0.177 (0.046)	0.638 (0.128)	0.403 (0.081)	0.086 (0.062)	0.346 (0.157)	0.872 (0.005)
		[3]	0.454	1.808	1.273	0.211	1.106	2.060
			0.201 (0.091)	0.868 (0.205)	0.411 (0.061)	0.211 (0.085)	0.432 (0.106)	0.566 (0.030)
	(42,49)	[1]	0.490	1.580	1.224	0.216	1.051	1.827
			0.178 (0.018)	0.418 (0.053)	0.255 (0.021)	0.184 (0.082)	0.187 (0.051)	0.273 (0.086)
		[2]	0.494	1.589	1.222	0.193	1.135	2.008
			0.162 (0.012)	0.429 (0.059)	0.279 (0.018)	0.069 (0.033)	0.222 (0.135)	0.338 (0.004)
		[3]	0.484	1.617	1.233	0.203	0.984	2.095
			0.168 (0.031)	0.498 (0.078)	0.278 (0.028)	0.084 (0.018)	0.263 (0.015)	0.504 (0.047)
	(60,70)		0.488	1.551	1.233	0.194	1.019	2.340
			0.156 (0.023)	0.331 (0.033)	0.238 (0.028)	0.197 (0.038)	0.109 (0.019)	0.746 (0.170)

Table 6 : Comparisons of lengths and (CP) of 95% CIs for the parameters with $(a,b,\rho)=(0.5,\ 1.5,\ 1.2)$ and $(a,b,\rho)=(0.2,\ 1,\ 2)$.

			(0.5, 1.5, 1.2)			(0.2, 1, 2)		
			MLE	$boot-t$	$boot-p$	MLE	$boot-t$	$boot-p$
			a	a	a	a	a	a
			b	b	b	b	b	b
(n_1,n_2)	(m_1,m_2)	SC	ρ	ρ	ρ	ρ	ρ	ρ
(10,20)	(4,8)	[1]	1.674 (0.9573)	1.713 (0.947)	1.744 (0.963)	1.426 (0.987)	1.296 (0.973)	1.515 (0.974)
			4.227 (0.933)	3.563 (0.941)	5.075 (0.929)	4.650 (0.968)	4.218 (0.971)	5.106 (0.990)
			3.707 (0.932)	4.332 (0.930)	4.224 (0.922)	4.279 (0.932)	4.449 (0.941)	4.601 (0.922)
		[2]	2.175 (0.973)	2.571 (0.952)	1.601 (0.968)	0.970 (0.921)	0.860 (0.919)	1.045 (0.923)
			5.667 (0.947)	4.385 (0.964)	5.705 (0.944)	2.975 (0.961)	2.522 (0.944)	3.958 (0.943)
			4.235 (0.966)	4.820 (0.975)	4.553 (0.964)	7.012 (0.954)	6.900 (0.961)	7.123 (0.972)
		[3]	1.603 (0.927)	1.549 (0.919)	1.334 (0.923)	0.807 (0.941)	0.795 (0.939)	0.958 (0.926)
			7.143 (0.929)	4.873 (0.948)	6.311 (0.934)	3.550 (0.921)	3.177 (0.941)	4.234 (0.972)
			3.974 (0.984)	4.219 (0.979)	4.541 (0.988)	5.411 (0.919)	5.655 (0.902)	5.996 (0.892)
	(7,14)	[1]	1.336 (0.959)	1.370 (0.966)	1.282 (0.944)	0.815 (0.962)	0.699 (0.952)	0.838 (0.944)
			2.634 (0.963)	2.334 (0.954)	3.349 (0.976)	2.339 (0.975)	2.124 (0.972)	2.661 (0.961)
			2.497 (0.982)	2.704 (0.978)	2.925 (0.979)	4.061 (0.916)	4.175 (0.931)	4.213 (0.922)
		[2]	1.440 (0.917)	1.393 (0.933)	1.355 (0.921)	0.736 (0.971)	0.630 (0.977)	0.705 (0.983)
			4.326 (0.945)	3.862 (0.971)	4.870 (0.956)	1.983 (0.929)	1.810 (0.891)	2.295 (0.895)
			2.581 (0.973)	2.723 (0.983)	2.966 (0.989)	4.467 (0.931)	4.638 (0.920)	4.701 (0.914)
		[3]	1.301 (0.924)	1.302 (0.928)	1.146 (0.962)	0.993 (0.942)	0.890 (0.951)	0.931 (0.929)
			3.245 (0.925)	2.629 (0.958)	3.913 (0.942)	2.020 (0.942)	1.895 (0.964)	2.404 (0.976)
			2.587 (0.912)	2.779 (0.902)	3.077 (0.893)	3.599 (0.963)	3.732 (0.951)	3.949 (0.944)
	(10,20)		1.150 (0.948)	1.182 (0.962)	1.093 (0.953)	0.868 (0.961)	0.735 (0.972)	0.775 (0.966)
			2.019 (0.978)	1.838 (0.986)	2.479 (0.984)	2.082 (0.926)	1.938 (0.949)	2.297 (0.959)
			1.993 (0.963)	2.101 (0.949)	2.252 (0.943)	3.083 (0.904)	2.938 (0.931)	3.152 (0.922)
(30,40)	(12,16)	[1]	1.195 (0.932)	1.222 (0.918)	1.133 (0.928)	0.556 (0.978)	0.458 (0.981)	0.553 (0.983)
			3.111 (0.958)	2.904 (0.963)	3.640 (0.956)	1.476 (0.943)	1.417 (0.934)	1.384 (0.926)
			1.975 (0.971)	2.022 (0.983)	2.238 (0.981)	3.964 (0.881)	3.820 (0.892)	4.253 (0.911)
		[2]	1.011 (0.968)	1.052 (0.961)	0.989 (0.953)	0.574 (0.945)	0.543 (0.958)	0.624 (0.952)
			3.526 (0.926)	3.302 (0.942)	4.172 (0.911)	1.586 (0.967)	1.521 (0.982)	1.783 (0.973)
			2.033 (0.954)	2.068 (0.972)	2.385 (0.977)	3.548 (0.908)	3.499 (0.921)	4.006 (0.930)
		[3]	1.133 (0.918)	1.114 (0.929)	0.936 (0.961)	0.567 (0.961)	0.483 (0.972)	0.464 (0.944)
			5.122 (0.946)	4.027 (0.957)	5.163 (0.942)	1.917 (0.982)	1.872 (0.944)	2.171 (0.962)
			2.046 (0.961)	2.056 (0.972)	2.445 (0.934)	3.613 (0.928)	3.448 (0.905)	3.847 (0.913)
	(21,28)	[1]	0.121 (0.958)	0.980 (0.944)	0.915 (0.923)	0.664 (0.964)	0.611 (0.943)	0.652 (0.952)
			0.026 (0.961)	2.155 (0.971)	2.494 (0.988)	1.327 (0.991)	1.224 (0.987)	1.402 (0.984)
			1.776 (0.942)	1.470 (0.959)	1.550 (0.952)	2.087 (0.946)	2.022 (0.971)	2.151 (0.962)
		[2]	0.894 (0.918)	0.918 (0.929)	0.858 (0.922)	0.486 (0.942)	0.368 (0.959)	0.463 (0.957)
			2.369 (0.940)	2.281 (0.951)	2.648 (0.927)	1.174 (0.938)	1.031 (0.928)	1.282 (0.922)
			1.446 (0.987)	1.483 (0.991)	1.580 (0.989)	2.722 (0.941)	2.714 (0.970)	2.891 (0.885)
		[3]	1.018 (0.918)	1.063 (0.892)	0.955 (0.902)	0.491 (0.962)	0.458 (0.977)	0.459 (0.928)
			2.342 (0.961)	2.201 (0.949)	2.562 (0.938)	1.135 (0.963)	1.083 (0.972)	1.262 (0.939)
			1.540 (0.920)	1.562 (0.938)	1.695 (0.927)	2.906 (0.981)	3.784 (0.992)	3.072 (0.966)

Table 6 : (continued)

			(0.5, 1.5, 1.2)			(0.2, 1, 2)		
			MLE	*boot − t*	*boot − p*	*MLE*	*boot − t*	*boot − p*
			a b ρ	a b ρ	a b ρ	a b ρ	a b ρ	a b ρ
(n_1,n_2)	(m_1,m_2)	*SC*						
	(30,40)		0.834 (0.972)	0.851 (0.961)	0.804 (0.982)	0.465 (0.962)	0.361 (0.958)	0.446 (0.953)
			1.860 (0.971)	1.806 (0.962)	2.009 (0.955)	1.011 (0.937)	1.499 (0.949)	1.714 (0.945)
			1.182 (0.904)	1.214 (0.901)	1.250 (0.889)	2.005 (0.961)	2.841 (0.957)	3.370 (0.952)
(60,70)	(24,28)	[1]	0.882 (0.958)	0.923 (0.961)	0.839 (0.954)	0.478 (0.972)	0.395 (0.951)	0.312 (0.942)
			2.012 (0.971)	1.955 (0.952)	2.183 (0.941)	0.993 (0.983)	0.926 (0.989)	1.131 (0.994)
			1.396 (0.959)	1.426 (0.945)	1.537 (0.922)	2.375 (0.920)	2.526 (0.912)	2.781 (0.902)
		[2]	0.703 (0.932)	0.739 (0.931)	0.703 (0.912)	0.412 (0.966)	0.338 (0.957)	0.383 (0.941)
			2.275 (0.946)	2.238 (0.929)	2.238 (0.904)	1.288 (0.972)	1.308 (0.973)	1.459 (0.957)
			1.432 (0.952)	1.470 (0.947)	1.598 (0.911)	2.269 (0.915)	2.244 (0.920)	2.454 (0.931)
		[3]	0.798 (0.949)	0.807 (0.947)	0.714 (0.951)	0.474 (0.925)	0.384 (0.906)	0.479 (0.912)
			3.263 (0.938)	2.920 (0.932)	3.412 (0.927)	1.532 (0.962)	1.469 (0.979)	1.420 (0.978)
			1.417 (0.971)	1.454 (0.992)	1.624 (0.982)	2.391 (0.958)	2.371 (0.984)	2.661 (0.994)
	(42,49)	[1]	0.701 (0.958)	0.726 (0.992)	0.681 (0.993)	0.452 (0.958)	0.411 (0.951)	0.404 (0.962)
			1.518 (0.941)	1.498 (0.958)	1.598 (0.955)	0.894 (0.961)	0.857 (0.959)	0.911 (0.931)
			1.016 (0.910)	1.048 (0.904)	1.081 (0.902)	1.537 (0.931)	1.462 (0.901)	1.642 (0.944)
		[2]	0.639 (0.962)	0.667 (0.943)	0.635 (0.951)	0.365 (0.991)	0.339 (0.986)	0.402 (0.981)
			1.602 (0.949)	1.586 (0.937)	1.708 (0.932)	0.917 (0.965)	0.853 (0.973)	1.033 (0.947)
			1.014 (0.957)	1.049 (0.942)	1.086 (0.911)	1.695 (0.894)	1.707 (0.910)	1.822 (0.887)
		[3]	0.684 (0.946)	0.708 (0.954)	0.661 (0.933)	0.379 (0.983)	0.381 (0.971)	0.418 (0.979)
			1.864 (0.920)	1.810 (0.895)	1.965 (0.914)	0.885 (0.962)	0.824 (0.958)	0.958 (0.948)
			1.027 (0.919)	1.059 (0.972)	1.104 (0.944)	1.793 (0.942)	1.804 (0.956)	1.937 (0.951)
	(60,70)		0.597 (0.972)	0.619 (0.966)	0.592 (0.942)	0.286 (0.962)	0.263 (0.957)	0.304 (0.941)
			1.261 (0.941)	1.254 (0.959)	1.318 (0.953)	0.645 (0.974)	0.606 (0.969)	0.696 (0.963)
			0.856 (0.902)	0.876 (0.926)	0.894 (0.917)	1.652(0.948)	1.601 (0.931)	1.701 (0.922)

References

[1] A.A. Ismail . Estimating the parameters of Weibull distribution and the acceleration factor from hybrid partially accelerated life test. Applied Mathematical Modelling **36**, 2920-2925 (2012).

[2] A.A. Ismail . Inference in the generalized exponential distribution under partially accelerated tests with progressive Type-II censoring. Theoretical and Applied Fracture Mechanics. **59**, 49-56 (2012).

[3] A.A. Ismail . The test design and parameter estimation of Pareto lifetime distribution under partially accelerated life tests, Ph.D. thesis, Department of Statistics, Faculty of Economics and Political Science, Cairo University, Egypt (2004).

[4] A.A. Ismail and A.M. Sarhan . Optimal Design of Step-Stress Life Test with Progressively Type-II Censored Exponential Data. International Mathematical Forum. **40**, 1963 - 1976 (2009).

[5] A.A. Soliman . Estimation of parameters of life from progressively censored data using Burr-XII model. IEEE Trans. Reliab. **54**, 34-42 (2005).

[6] A. Childs and N. Balakrishnanm . Conditional inference procedures for the Laplace distribution when the observed samples are progressively censored, Metrika, **52**, 253-265 (2000).

[7] A.H. Abdel-Hamid . Constant-partially accelerated life tests for Burr Type-XII distribution with progressive Type-II censoring. Computational Statistics and Data Analysis **53**, 2511-2523 (2009).

[8] A.M. Abd-Elfattah, A.S. Hassan and S.G. Nassr . Estimation in step-stress partially accelerated life tests for the Burr Type XII distribution using Type I censoring. Statistical Methodology **5**, 502.514 (2008).

[9] A. Pandey, A. Singh and W.J. Zimmer . Bayes estimation of the linear hazard rate model, IEEE Trans. Reliab., **42**, 636-640 (1993).

[10] B. Efron . The Jackknife, the Bootstrap and Other Re-sampling Plans. In:CBMS/NSF; Regional Conference Series in Applied Mathematics, vol. 38. SIAM, Philadelphia, PA (1982).

[11] C. Xiong and M. Ji . Analysis of Grouped and Censored Data from Step-stress Life Testing. IEEE Trans. Reliab., 53,1,22-28 (2004).

[12] D. Kundu, N. Kannan and N. Balakrishnan . Analysis of progressively censored competing risks data. In: Balakrishnan, N., Rao, C.R. (Eds.), Handbook of Statistics, vol. **23**. Elsevier, New York (2004).

[13] D.S. Bai and S.W. Chung . Optimal design of partially accelerated life tests for the exponential distribution under Type-I censoring. IEEE Trans. Reliab. **41**, 400-406 (1992).

[14] E.K. AL-Hussaini and A.H. Abdel-Hamid . Bayesian estimation of the parameters, reliability and hazard rate functions of mixtures under accelerated life tests. Commun. Stat. - Simul. Comput. 33 (4), 963-982 (2004).
[15] E.K. AL-Hussaini and A.H. Abdel-Hamid . Accelerated life tests under finite mixture models. J. Stat. Comput. Simul. **76** (8), 673-690 (2006).
[16] L.J. Bain . Statistial Analysis of Reliability and Life Testing Models, Macel Dekker (1978).
[17] M.A.M. Ali Mousa and Z.F. Jaheen . Statistical inference for the Burr model based on progressively censored data. Comput. Math. Appl. **43**, 1441-1449 (2002a).
[18] M. Burkschat, E. Cramer and U. Kamps . On optimal schemes in progressive censoring. Statist. Probab. Lett. **76**, 1032-1036 (2006).
[19] M.M. Abdel-Ghani . Investigation of some lifetime models under partially accelerated life tests, Ph.D. thesis, Department of Statistics, Faculty of Economics and Political Science, Cairo University,Egypt (1998).
[20] N. Balakrishnan and R. Aggarwala . Progressive Censoring: Theory, Methods and Applications. Birkhauser Publishers, Boston (2000).
[21] N. Balakrishnan and N. Kannan . Point and interval estimation for parameters of the logistic distribution based on progressively Type-II censored samples, in Handbook of Statistics N. Balakrishnsn and C.R. Rao, Eds. Amsterdam: North-Holand, **20,** 431-456 (2001).
[22] N. Balakrishnan, N. Kannan, C.T. Lin and H. Ng. Point and interval estimation for gaussian distribution based on progressively Type-II censored samples, IEEE Trans. Reliab. **52**, 90-95 (2003).
[23] P. Carbone, L. Kellerthouse and E. Gehan . Plasmacytic Myeloma: Astudy of the relationship of survival to various clinical manifestations and anomalous protein Type in 112 patients, American Journal of Medicine, **42**, 937-948 (1967).
[24] P. Hall . Theoretical comparison of bootstrap con dence intervals. Ann. Statist.**16**, 927-953 (1988).
[25] P.W. Srivastava, and N. Mittal . Optimum step-stress partially accelerated life tests for the truncated logistic distribution with censoring. Applied Mathematical Modelling. **34**, 31663178 (2010).
[26] R.Viveros and N. Balakrishnan . Interval estimation of parameters of life from progressively censored data. Technometrics **36**, 84-91 (1994).
[27] S. Broadbent . Simple mortality rates, Journal of Applied Statistics, **7**, 86 (1958).
[28] W. Nelson . Accelerated Testing: Statistical Models, Test Plans and Data Analysis. Wiley, NewYork (1990).

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