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Compound of Discrete Weibull and Minimax Distributions as a New Count Data Model with Application in Genetics

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Abstract: In this article, we attempt to introduce a new count data model which is obtained by compounding two parameter discrete Weibull distribution with Minimax distribution. The proposed model has several properties, such as it can be nested to different compound distributions on specific parameter settings. We shall first study some basic distributional and moment properties of the new distribution. Then, certain structural properties of the distribution such as its unimodality, hazard rate behavior and index of dispersion are discussed. Finally, two real data sets are analyzed to investigate the suitability of the proposed distribution in modeling count data from medical genetics.

Keywords: Discrete Weibull Distribution, Minimax distribution, compound distribution, medical science, count data.

1 Introduction

From the last few decades researchers are busy to obtain new probability distributions by using different techniques such as compounding [20], discretization [21,22], transmutation etc. but compounding of probability distribution has received maximum attention which is an innovative and sound technique to obtain new probability distributions. The compounding of probability distributions enables us to obtain both discrete as well as continuous distribution.

Compound distribution arises when all or some parameters of a distribution known as parent distribution vary according to some probability distribution called the compounding distribution, for instance negative binomial distribution can be obtained from Poisson distribution when its parameter λ follows gamma distribution. If the parent distribution is discrete then resultant compound distribution will also be discrete and if the parent distribution is continuous then resultant compound distribution will also be continuous i,e. the support of the original (parent) distribution determines the support of compound distributions.

In several research papers, it has been found that compound distributions are very flexible and can be used efficiently to model different types of data sets. With this in mind, many compound probability distributions have been constructed. In the early 1970s, Dubey [13] derived a compound gamma, beta and F distribution by compounding a gamma distribution with another gamma distribution and reduced it to the beta Ist and 2nd kind and to the F distribution by suitable transformations. Sankaran [1] introduced a compound of Poisson distribution with that of Lindley distribution for modeling count data. Gerstenkorn [14, 15] proposed several compound distributions, he obtained compound of gamma distribution with exponential distribution by treating the parameter of gamma distribution as an exponential variate and also obtained compound of polya with beta distribution. Ghitany, Al-Mutairi and Nadarajah [2, 3] introduced zero-truncated Poisson-Lindley distribution, who used the distribution for modeling count data in the case where the distribution has to be adjusted for the count of missing zeros. Zamani and Ismail [4] constructed a new compound distribution by compounding negative binomial with one parameter Lindley distribution that provides good fit for count data where the probability at zero has a large value. Rashid and Jan [5] explored a mixture of generalized negative binomial distribution with that of generalized exponential distribution which contains several compound distributions as its sub cases and proved that this particular model is better in comparison to others when it comes to fit observed count data set.

In this paper, we propose a new count data model by compounding two parameter discrete Weibull distribution with Minimax distribution as there is a need to find more plausible discrete probability models or survival models in medical science and other fields, to fit to various discrete data sets. It is well known in general that a compound model is more flexible than the ordinary model and it is preferred by many data analysts in analyzing statistical data. Moreover, it presents beautiful mathematical exercises and broadened the scope of the concerned model being compounded.

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2 Material and Methods

A discrete analogue of the continuous Weibull distribution was introduced by Nakagawa and Osaki [6], and is defined by the probability mass function (pmf):

$$f_1(x;q,\gamma)=q^{x^\gamma}-q^{(x+1)^\gamma},\quad x=0,1,2,.....(1)$$

where $\gamma>0 and\ 0<q<1$ are its parameters. The first and the second moments of the DW random variable X are given by

$$E(X)=\sum_{x=1}^{\infty}q^{x^\gamma} \tag{2}$$

$$E(X^2)=2\sum_{x=1}^{\infty}xq^{x^\gamma}+E(X)$$

Jones [7] studied two-parameter distribution on (0,1) which he has called the Minimax distribution, Minimax (α,β) ,where its two shape parameters α and β are positive. It has many of the same properties as the beta distribution but has some advantages in terms of tractability. Its probability density function is given by

$$f_2(X;\alpha,\beta)=\alpha\beta\ x^{\alpha-1}\left(1-x^\alpha\right)^{\beta-1},\ 0<x<1(3)$$

where $\alpha,\beta>0$ are shape parameters. The raw moments of Minimax distribution are given by

$$E(X^r)=\int_0^1 x^r f_2(X;\alpha,\beta)dx$$

$$E(X^r)=\frac{\Gamma(\beta+1)\Gamma\left(1+\frac{r}{\alpha}\right)}{\Gamma\left(1+\beta+\frac{r}{\alpha}\right)} \tag{4}$$

Minimax distribution is not very popular among statisticians because researchers have not analyzed and investigated it systematically in much detail. Minimax distribution is similar to the beta distribution but unlike beta distribution, it has a closed form of cumulative distribution function, which makes it very simple to deal with. For more detailed properties one can see references [7,8]

Usually the parameters $\gamma and\ q$ in DWD are fixed constants but here we have considered a problem in which the probability parameter q is itself a random variable following MD with pmf (3).

3 Definition of Proposed Model

If $X|q\sim$ DWD(q,γ) where q is itself a random variable following Minimax distribution MD(α,β), then determining the distribution that results from marginalizing over q will be known as a compound of discrete Weibull distribution with that of Minimax distribution, which is denoted by DWMD (γ,α,β). It may be noted that proposed model will be a discrete since the parent distribution DWD is discrete.

Theorem 3.1: The probability mass function of a compound of DWD(q,γ) with MD(α,β) is given by

$$f_{DWMD}(X;\gamma,\alpha,\beta)=\beta[B(\beta,\frac{x^\gamma}{\alpha}+1)-B(\beta,\frac{(x+1)^\gamma}{\alpha}+1)]$$

where $x=0,1,2,.....$ and $\gamma,\alpha,\beta>0$

Proof: Using the definition (3), the pmf of a compound of DWD(q,γ) with MD(α,β) can be obtained as

$$f_{DWMD}(X;\gamma,\alpha,\beta)=\int_0^1 f_1(x|q)f_2(q)\,dq$$

$$f_{DWMD}(X;\gamma,\alpha,\beta)=\int_0^1(q^{x^\gamma}-q^{(x+1)^\gamma})\alpha\beta\ q^{\alpha-1}\left(1-q^\alpha\right)^{\beta-1}\,dq$$

substituting $1-q^\alpha=z$, we get

$$f_{DWMD}(X;\gamma,\alpha,\beta)=\beta(\int_0^1 z^{\beta-1}(1-z)^{\frac{x^\gamma}{\alpha}}\,dz-\int_0^1 z^{\beta-1}(1-z)^{\frac{(x+1)^\gamma}{\alpha}}dz)$$

$$f_{DWMD}(X;\gamma,\alpha,\beta)=\beta[B(\beta,\frac{x^\gamma}{\alpha}+1)-B(\beta,\frac{(x+1)^\gamma}{\alpha}+1)]$$

$$f_{DWMD}(X;r,\alpha,\beta)=\beta\left[\frac{\Gamma(\beta)\,\Gamma\left(\frac{x^\gamma}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x^\gamma}{\alpha}+1\right)}-\frac{\Gamma(\beta)\,\Gamma\left(\frac{(x+1)^\gamma}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x+1)^\gamma}{\alpha}+1\right)}\right] \tag{5}$$

Where $x=0,1,2,...$ and $\gamma,\alpha,\beta>0$. From here a random variable X following a compound of DWD with MD will be symbolized by DWMD(γ,α,β).

Fig.1(a) to fig.1(i) provides a pmf plot of the proposed model DWMD(γ,α,β) for different values of parameters. It is evident that the proposed model is right skewed with unimodel behavior.

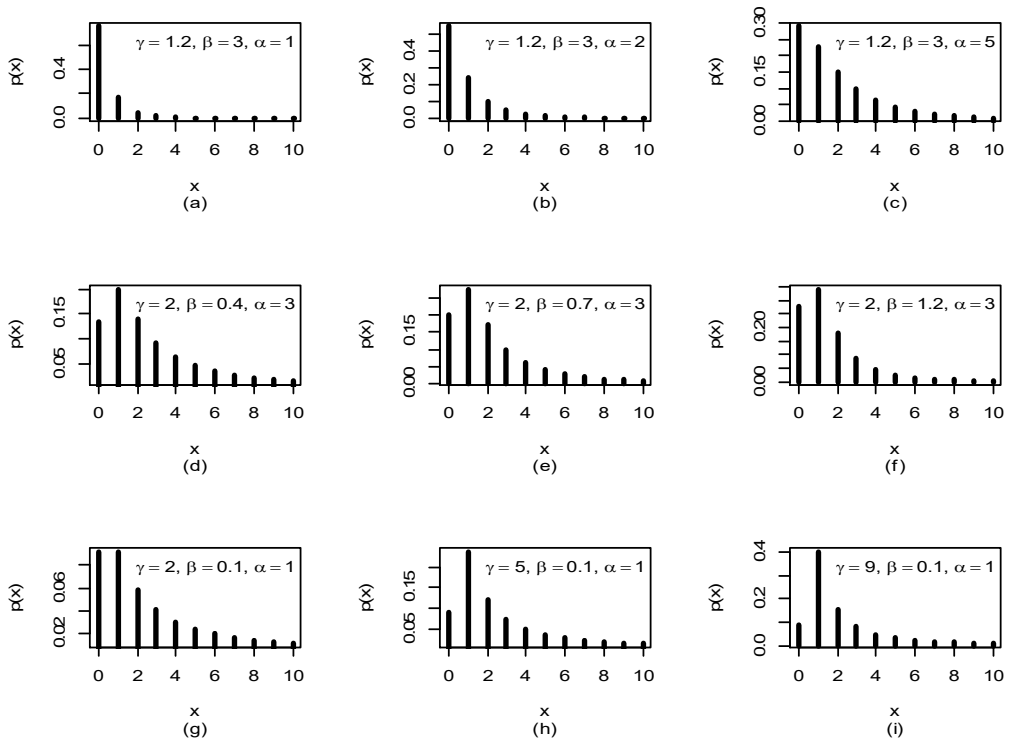

Fig 1: pmf plot of Discrete Compound Weibull minimax distribution.

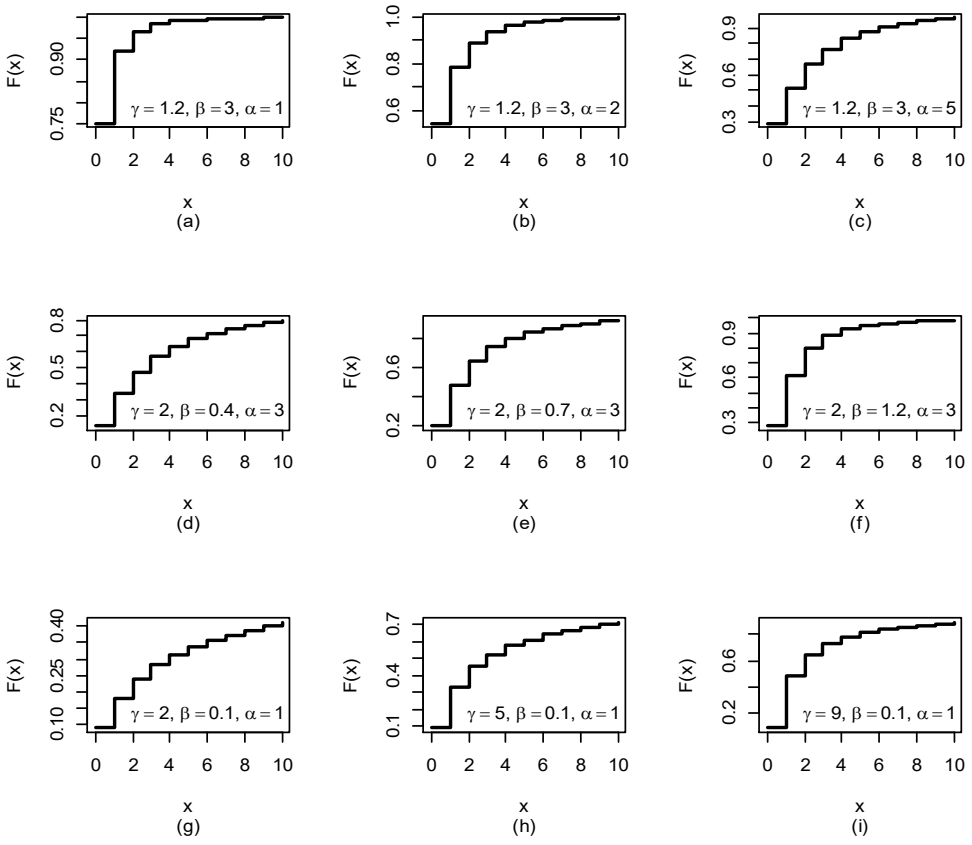

Fig 2: CDF plot of Discrete Weibull Minimax distribution.

The Cumulative distribution function of the DWMD (γ,α,β) is given by

$F(x)=1-\beta B(x;\gamma,\alpha,\beta)$ $x=0,1,2,...$ and $(\gamma>0,\alpha>0,\beta>0)$

Where $B(x;\gamma,\alpha,\beta)=\dfrac{\Gamma(\beta)\,\Gamma\left(\dfrac{(x+1)^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\dfrac{(x+1)^{\gamma}}{\alpha}+1\right)}$

Fig.2(a) to fig.2(i) provides a CDF plot of the proposed model DWMD(γ,α,β) for different values of parameters. The initial rise of the CDF plot decreases as α increases but as γ and β increases, initial rise of the CDF plot increases.

4 Nested Distributions

In this particular section, we show that the proposed model can be nested to different models under specific parameter setting.

Proposition 4.1: If $X\sim DWMD(\gamma,\alpha,\beta)$ then by setting $\gamma=1$, we get a compound of geometric distribution with Minimax distribution.

Proof: For $\gamma=1$ in (1) DWD reduces to geometric distribution (GD) hence a compound of GD with MD is followed from (5) by simply substituting $\gamma=1$ in it.

$f_{DGMD}(X;\alpha,\beta)=\beta[B(\beta,\frac{x}{\alpha}+1)-B(\beta,\frac{(x+1)}{\alpha}+1)]$ for $x=0,1,2,...,\alpha,\beta>0$

Where $B(\beta,\frac{x}{\alpha}+1)=\dfrac{\Gamma(\beta)\,\Gamma\left(\frac{x}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x}{\alpha}+1\right)}$

Which is the probability mass function of a compound of GD with MD.

Proposition 4.2: If $X\sim DWMD(\gamma,\alpha,\beta)$ then by setting, $\alpha=\beta=1$ we obtain a compound of DWD distribution with uniform distribution.

Proof: For $\alpha=\beta=1$ in MD reduces to Uniform (0,1) distribution, therefore a compound DWD with uniform distribution is followed from (5) by simply putting $\alpha=\beta=1$ in it.

$f_{DWUD}(X;\gamma)=\beta[B(1,x^{\gamma}+1)-B(1,(x+1)^{\gamma}+1)]$

$f_{DWUD}(X;\gamma)=\left[\dfrac{(x+1)^{\gamma}-x^{\gamma}}{(x^{\gamma}+1)[(x+1)^{\gamma}+1]}\right]$ For $x=0,1,2,...,\gamma>0$

Which is probability mass function of a compound of DWD with uniform distribution.

Proposition 4.3: If $X\sim DWMD(\gamma,\alpha,\beta)$ then by setting $\gamma=1$ and $\alpha=\beta=1$ we obtain a compound of geometric distribution with uniform distribution.

Proof: For $\gamma=1$ in (1), DWD reduces to geometric distribution and for $\alpha=\beta=1$, Minimax distribution reduces to U(0,1) distribution, hence a compound of geometric distribution with uniform distribution can be obtained from (5) by simply substituting $\gamma=1$ and $\alpha=\beta=1$ in it.

$f_{GUD}(X)=\dfrac{1}{(x+1)(x+2)}$ for $x=0,1,2,...$

Proposition 4.4: If $X\sim DWMD(\gamma,\alpha,\beta)$ then by setting $\gamma=2$ and $\alpha=\beta=1$ we obtain a compound of discrete Rayleigh distribution with uniform distribution.

Proof: For $\gamma=2$ in (1), DWD reduces to discrete Rayleigh distribution and for $\alpha=\beta=1$, Minimax distribution reduces to U(0,1) distribution hence a compound of geometric distribution with uniform distribution can be obtained from (5) by simply substituting $\gamma=2$ and $\alpha=\beta=1$ in it.

$f_{DRUD}(X)=\dfrac{2x+1}{(x^2+1)((x+1)^2+1)}$ for $x=0,1,2,...$

Which is the compound probability function of discrete Rayleigh distribution with U(0,1) distribution.

Proposition 4.5: If $X\sim DWMD(\gamma,\alpha,\beta)$ then by setting $\gamma=2$, we get a compound of discrete Rayleigh distribution with minimax distribution.

Proof: For $\gamma=2$ in (1) DWD reduces to discrete Rayleigh distribution (DRD), hence a compound of DRD with MD is followed from (5) by simply substituting $\gamma=2$ in it.

$f_{DGMD}(X;\alpha,\beta) = \beta[B(\beta,\frac{x^2}{\alpha}+1) - B(\beta,\frac{(x+1)^2}{\alpha}+1)]$ DRD with MD.

For $x=0,1,2,...,\alpha,\beta>0$

Where $B(\beta,\frac{x^2}{\alpha}+1)=\frac{\Gamma(\beta)\,\Gamma\left(\frac{x^2}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x^2}{\alpha}+1\right)}$

Which is the probability mass function of a compound of

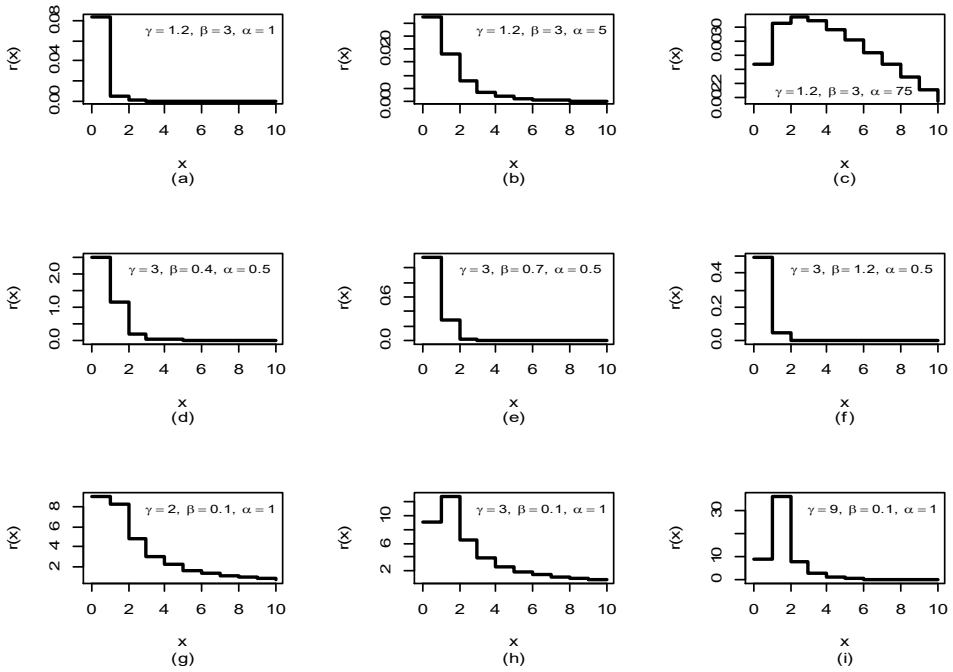

Fig 3: Hazard rate function of Discrete Weibull Minimax distribution.

5 Reliability Measures of Compound Discrete Weibull Minimax Distribution

If $X \sim DWMD(\gamma,\alpha,\beta)$, then the various reliability measures of a random variable X are given by

(a) **Survival Function.**

$$s(x) = \beta B(\beta,\frac{x^\gamma}{\alpha}+1) \quad x=0,1,2... \text{ and } \alpha>0,\beta>0$$

$$where\ B(\beta,\frac{x^\gamma}{\alpha}+1)=\frac{\Gamma(\beta)\,\Gamma\left(\frac{x^\gamma}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x^\gamma}{\alpha}+1\right)}$$

(b) **Rate of Failure Function.**

$$r(x)=\frac{p(x)}{s(x)}=\frac{B(\beta,\frac{x^\gamma}{\alpha}+1)-B(\beta,\frac{(x+1)^\gamma}{\alpha}+1)}{B(\beta,\frac{x^\gamma}{\alpha}+1)}$$

$x=0,1,2...$ and $\alpha>0,\beta>0,\gamma>0$

$$Where\ B(\beta,\frac{x}{\alpha}+1)=\frac{\Gamma(\beta)\,\Gamma\left(\frac{x}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x}{\alpha}+1\right)}$$

(c) **Second Rate of Failure Function.**

$$h(x)=\log\left(\frac{s(x)}{s(x+1)}\right)=\log\left(\frac{B(\beta,\frac{x^\gamma}{\alpha}+1)}{B(\beta,\frac{(x+1)^\gamma}{\alpha}+1)}\right)$$

$x=0,1,2...$ and $\alpha>0,\beta>0,\gamma>0$

Where, B(.) refers to the beta function defined by

$$B(a,b)=\frac{\Gamma a\,\Gamma b}{\Gamma(a+b)}$$

Fig.3(a) to fig.3(i) provides a hazard rate function plot of the proposed model DWMD(γ,α,β) for different values of parameters.

Table 1: Index of Dispersion, Mean and Variance of $DWMD(\gamma,\alpha,\beta)$ for different values of parameters

γ=1												
α	β	2.1	2.3	2.8	3.5	3.7	4.2	4.6	5.1	5.3	5.6	6.2
0.5	Mean	0.344	0.279	0.183	0.117	0.105	0.082	0.069	0.057	0.053	0.048	0.040
	Variance	9.006	2.517	0.686	0.273	0.226	0.151	0.117	0.088	0.080	0.069	0.054
	IOD	26.207	9.024	3.750	2.332	2.148	1.840	1.682	1.546	1.504	1.450	1.367
0.6	Mean	0.450	0.370	0.250	0.166	0.151	0.121	0.104	0.087	0.082	0.075	0.063
	Variance	13.022	3.670	1.022	0.418	0.349	0.238	0.186	0.143	0.131	0.115	0.091
	IOD	28.969	9.930	4.089	2.520	2.316	1.973	1.798	1.646	1.599	1.538	1.445
0.8	Mean	0.674	0.564	0.397	0.277	0.255	0.211	0.185	0.160	0.151	0.140	0.122
	Variance	23.264	6.623	1.893	0.802	0.675	0.472	0.375	0.295	0.271	0.240	0.195
	IOD	34.517	11.744	4.767	2.894	2.649	2.240	2.029	1.846	1.789	1.716	1.602
γ=4												
α	β	1.900	2.300	2.800	3.500	3.700	4.200	4.600	5.100	5.300	5.600	6.200
1.02	Mean	0.359	0.312	0.270	0.228	0.218	0.198	0.184	0.169	0.164	0.156	0.143
	Variance	0.249	0.223	0.200	0.177	0.171	0.159	0.150	0.140	0.137	0.132	0.123
	IOD	0.693	0.713	0.742	0.776	0.785	0.804	0.817	0.832	0.837	0.844	0.857
1.5	Mean	0.478	0.430	0.386	0.341	0.331	0.308	0.292	0.276	0.270	0.261	0.246
	Variance	0.286	0.263	0.245	0.228	0.224	0.214	0.208	0.200	0.197	0.193	0.186
	IOD	0.598	0.612	0.636	0.668	0.677	0.696	0.710	0.726	0.732	0.740	0.755
2.6	Mean	0.656	0.607	0.563	0.518	0.508	0.486	0.470	0.454	0.448	0.439	0.423
	Variance	0.315	0.291	0.275	0.264	0.262	0.257	0.255	0.252	0.250	0.249	0.246
	IOD	0.481	0.479	0.489	0.509	0.515	0.530	0.541	0.555	0.560	0.567	0.581

6 Moment Generating and Probability Generating Functions of $DWMD(\gamma,\alpha,\beta)$

(a) The moment generating function of the Compound discrete Weibull Minimax distribution is

$$M_x(t)=\sum_{x=0}^{\infty}e^{tx}p(x)$$

$$M_x(t)=\sum_{x=0}^{\infty}e^{tx}\beta[B(\beta,\frac{x^{\gamma}}{\alpha}+1)-B(\beta,\frac{(x+1)^{\gamma}}{\alpha}+1)]$$

$$M_x(t)=\sum_{x=0}^{\infty}e^{tx}\beta[\psi(x;\gamma,\beta,\alpha)-\psi(x+1;\gamma,\beta,\alpha)]$$

Where $\psi(x;\gamma,\beta,\alpha)=B(\beta,\frac{x^{\gamma}}{\alpha}+1)$

$$M_x(t)=\beta\begin{pmatrix}\psi(0;\gamma,\beta,\alpha)+e^{t}\psi(1;\gamma,\beta,\alpha)+e^{2t}\psi(2;\gamma,\beta,\alpha)\\+e^{3t}\psi(3;\gamma,\beta,\alpha)+....-\{\psi(1;\gamma,\beta,\alpha)\\+e^{t}\psi(2;\gamma,\beta,\alpha)+e^{2t}\psi(3;\gamma,\beta,\alpha)\\+e^{3t}\psi(4;\gamma,\beta,\alpha)+.....\}\end{pmatrix}$$

$$M_x(t)=\beta\left(\psi(0;\gamma,\beta,\alpha)+(e^{t}-1)\psi(1;\gamma,\beta,\alpha)+(e^{2t}-e^{t})\psi(2;\gamma,\beta,\alpha)+(e^{3t}-e^{2t})\psi(3;\gamma,\beta,\alpha)+...\right)$$

$$M_x(t)=1+\beta\sum_{x=1}^{\infty}(e^{xt}-e^{(x-1)t})\psi(x;\gamma,\beta,\alpha)$$

Differentiating $M_x(t)$ r times with respect to t

$$M_x^{(r)}(t)=\beta\sum_{x=1}^{\infty}(x^{r}e^{xt}-(x-1)^{r}e^{(x-1)t})\psi(x;\gamma,\beta,\alpha)$$

First four moments of the proposed model are given by

$$\mu_1^{'} = \beta \sum_{x=1}^{\infty} \psi(x;\gamma,\beta,\alpha)$$

$$\mu_2^{'} = \beta \sum_{x=1}^{\infty} (2x-1)\psi(x;\gamma,\beta,\alpha)$$

$$\mu_3^{'} = \beta \sum_{x=1}^{\infty} (3x^2-3x+1)\psi(x;\gamma,\beta,\alpha)$$

$$\mu_4^{'} = \beta \sum_{x=1}^{\infty} (4x^3-6x^2+4x-1)\psi(x;\gamma,\beta,\alpha)$$

(b) Probability generating function of the Compound discrete Weibull Minimax distribution is

$$G_{[x]}(t) = \sum_{x=0}^{\infty} t^x p(x)$$

$$G_{[x]}(t) = \sum_{x=0}^{\infty} t^x \beta[B(\beta,\frac{x^\gamma}{\alpha}+1) - B(\beta,\frac{(x+1)^\gamma}{\alpha}+1)]$$

$$G_{[x]}(t) = \sum_{x=0}^{\infty} t^x \beta[\psi(x;\gamma,\beta,\alpha) - \psi(x+1;\gamma,\beta,\alpha)]$$

Where $\psi(x;\gamma,\beta,\alpha) = B(\beta,\frac{x^\gamma}{\alpha}+1)$

$$G_{[x]}(t) = \beta\begin{pmatrix}\psi(0;\gamma,\beta,\alpha)+(t-1)\psi(1;\gamma,\beta,\alpha)\\ +t(t-1)\psi(2;\gamma,\beta,\alpha)+t^2(t-1)\psi(3;\gamma,\beta,\alpha)+\ldots\end{pmatrix}$$

$$G_{[x]}(t) = 1+\beta(t-1)\sum_{x=1}^{\infty} t^{x-1}\psi(x;\gamma,\beta,\alpha)$$

Differentiating $G_{[x]}(t)$ with respect to t

$$G_{[x]}^{'}(t) = \beta\sum_{x=1}^{\infty}((t-1)(x-1)t^{x-2}+t^{x-1})\psi(x;\gamma,\beta,\alpha)$$

$$G_{[x]}^{'}(t) = \beta\sum_{x=1}^{\infty}(t^{x-2}(xt-x+1))\psi(x;\gamma,\beta,\alpha)$$

$$G_{[x]}^{''}(t) = \beta\sum_{x=1}^{\infty}((x-1)t^{x-3}(xt-x+2))\psi(x;\gamma,\beta,\alpha)$$

At t=1, $G_{[x]}^{'}(t)$, $G_{[x]}^{''}(t)$ gives first and second factorial moments

$$E(x) = G_{[x]}^{'}(1) = \beta\sum_{x=1}^{\infty}\psi(x;\gamma,\beta,\alpha)$$

$$E(x^2) = G_{[x]}^{'}(1)+G_{[x]}^{''}(1) = \beta\sum_{x=1}^{\infty}(2x-1)\psi(x;\gamma,\beta,\alpha)$$

Table1 exhibits the index of dispersion, $IOD = \{E(X^2)-(E(X))^2\}/E(X)$, mean and variance for different values of the parameters γ,α and β for three parameter discrete Compound Weibull Minimax distribution. It can be seen that this variance to mean ratio indicates that discrete Compound Weibull Minimax model is overdispersed as well as under-dispersed.

7 Parameter Estimation

In this section the estimation of parameters of $DWMD(\gamma,\alpha,\beta)$ model will be discussed through method of moments and maximum likelihood estimation.

7.1 Moments Method of Estimation

In order estimate three unknown parameters of $DWMD(\gamma,\alpha,\beta)$ model by the method of moments, we need to equate first three sample moments with their corresponding population moments.

$m_1 = \gamma_1$; $m_2 = \gamma_2$ and $m_3 = \gamma_3$

Where γ_i is the i[th] sample moment and m_i is the i[th] corresponding population moment and the solution for $\hat{\gamma},\hat{\alpha}$ and $\hat{\beta}$ may be obtained by solving above equations simultaneously through numerical methods.

7.2 Maximum Likelihood Method of Estimation

The estimation of parameters of $DWMD(\gamma,\alpha,\beta)$ model via maximum likelihood estimation method requires the log likelihood function of $DWMD(\gamma,\alpha,\beta)$

$$\pounds(X;\gamma,\alpha,\beta) = \log L(X;\gamma,\alpha,\beta) = n\log\beta+\sum_{i=1}^{n}\log(B(\beta,\frac{x_i^\gamma}{\alpha}+1)-B(\beta,\frac{(x_i+1)^\gamma}{\alpha}+1)) \quad (9)$$

The maximum likelihood estimate of $\Theta = (\hat{\gamma},\hat{\alpha},\hat{\beta})^T$ can be obtained by differentiating (9) with respect unknown parameters γ,α and β, respectively and then equating them to zero.

$$\frac{\partial}{\partial\beta}\pounds(X;\gamma,\alpha,\beta)=\frac{n}{\beta}+\sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\beta}\left(\frac{\Gamma(\beta)\Gamma\left(\frac{x_i^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\Gamma\left(\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}\right)}{\left(\frac{\Gamma(\beta)\Gamma\left(\frac{x_i^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\Gamma\left(\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}\right)}\right) \tag{10}$$

$$\frac{\partial}{\partial\alpha}\pounds(X;\gamma,\alpha,\beta)=\sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\alpha}\left(\frac{\Gamma(\beta)\Gamma\left(\frac{x_i^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\Gamma\left(\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}\right)}{\left(\frac{\Gamma(\beta)\Gamma\left(\frac{x_i^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\Gamma\left(\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}\right)}\right) \tag{11}$$

$$\frac{\partial}{\partial\gamma}\pounds(X;\gamma,\alpha,\beta)=\sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\gamma}\left(\frac{\Gamma(\beta)\Gamma\left(\frac{x_i^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\Gamma\left(\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}\right)}{\left(\frac{\Gamma(\beta)\Gamma\left(\frac{x_i^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{x_i^{\gamma}}{\alpha}+1\right)}-\frac{\Gamma(\beta)\Gamma\left(\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{(x_i+1)^{\gamma}}{\alpha}+1\right)}\right)}\right) \tag{12}$$

These three derivative equations cannot be solved analytically, therefore $\hat{\gamma},\hat{\alpha}$ and $\hat{\beta}$ will be obtained by maximizing the log likelihood function numerically using Newton-Raphson method which is a powerful technique for solving equations iteratively and numerically. We can compute the second partial derivatives, which are useful to obtain the Fisher's information matrix as follows.

$$I_y(\gamma,\alpha,\beta)=\begin{bmatrix}-E\left(\frac{\partial^2 l}{\partial\gamma^2}\right) & -E\left(\frac{\partial^2 l}{\partial\gamma\partial\alpha}\right) & -E\left(\frac{\partial^2 l}{\partial\gamma\partial\beta}\right)\\ -E\left(\frac{\partial^2 l}{\partial\alpha\partial\beta}\right) & -E\left(\frac{\partial^2 l}{\partial\alpha^2}\right) & -E\left(\frac{\partial^2 l}{\partial\alpha\partial\gamma}\right)\\ -E\left(\frac{\partial^2 l}{\partial\beta\partial\alpha}\right) & -E\left(\frac{\partial^2 l}{\partial\beta\partial\gamma}\right) & -E\left(\frac{\partial^2 l}{\partial\beta^2}\right)\end{bmatrix} \tag{13}$$

One can show that the discrete Weibull Minimax distribution satisfies the regularity conditions (see, e.g., Ferguson, 1996, p. 121 [19]). Hence, the MLE vector $\Theta=\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)^T$ is consistent and asymptotically normal; that is, $I_y^{\frac{1}{2}}(\gamma,\alpha,\beta)=\left[\left(\hat{\gamma},\hat{\alpha},\hat{\beta}\right)^T-(\gamma,\alpha,\beta)^T\right]$ converges in distribution to a normal distribution with the (vector) mean zero and the identity covariance matrix.

Also, the Fisher's information matrix can be computed using the approximation

$$I_y(\hat{\gamma},\hat{\alpha},\hat{\beta})\approx\begin{bmatrix}-\left(\frac{\partial^2 l}{\partial\gamma^2}\right)\Big|_{(\hat{\gamma},\hat{\alpha},\hat{\beta})} & -\left(\frac{\partial^2 l}{\partial\gamma\partial\alpha}\right)\Big|_{(\hat{\gamma},\hat{\alpha},\hat{\beta})} & -\left(\frac{\partial^2 l}{\partial\gamma\partial\beta}\right)\Big|_{(\hat{\gamma},\hat{\alpha},\hat{\beta})}\\ -\left(\frac{\partial^2 l}{\partial\alpha\partial\beta}\right)\Big|_{(\hat{\gamma},\hat{\alpha},\hat{\beta})} & -\left(\frac{\partial^2 l}{\partial\alpha^2}\right)\Big|_{(\hat{\gamma},\hat{\alpha},\hat{\beta})} & -\left(\frac{\partial^2 l}{\partial\alpha\partial\gamma}\right)\Big|_{(\hat{\gamma},\hat{\alpha},\hat{\beta})}\\ -\left(\frac{\partial^2 l}{\partial\beta\partial\alpha}\right)\Big|_{(\hat{\gamma},\hat{\alpha},\hat{\beta})} & -\left(\frac{\partial^2 l}{\partial\beta\partial\gamma}\right)\Big|_{(\hat{\gamma},\hat{\alpha},\hat{\beta})} & -\left(\frac{\partial^2 l}{\partial\beta^2}\right)\Big|_{(\hat{\gamma},\hat{\alpha},\hat{\beta})}\end{bmatrix}$$

Where $\hat{\gamma},\hat{\alpha}$ and $\hat{\beta}$ are the MLEs of γ,α and β, respectively (see, e.g., G´omez-D´eniz [18]. Using this approximation, we may construct confidence intervals for parameters of the discrete Weibull Minimax model.

8 Application of Discrete CWMD in Medical Genetics

The term "Medical Genetics" has been variously defined as the science of human biological variation as it relates to health and disease. Heredity includes those traits or characteristics which are transmitted from generation to generation, and is therefore fixed for a particular individual. Variation, on the other hand, is mainly of two types, namely hereditary and environmental. Hereditary variation refers to differences in inherited traits whereas environmental variations are those which are mainly due to environment.

Much quantitative works seem to be done in genetics but so far no works has been done on fitting of discrete Log-logistic model for count data in genetics. Various mathematical/ statistical models have been used in radiation research for studying variations in the frequency of chromosome aberrations. In the analysis of data observed on chemically induced chromosome aberrations in cultures of human leukocytes, Loeschke & Kohler [9] suggested the negative binomial distribution while Janardan & Schaeffer [10] suggested modified Poisson distribution. Shanker & Hagos [11] have detailed study on the applications of Poisson Lindley distribution (PLD) to model data from genetics.

Shanker , Hagos and Teklay [12] have suggested Poisson Akasha distribution (PAD) as another model for studying variations in the frequency of chromosome aberrations. In this section an attempt has been made to fit to data relating to genetics as given in table 2 and table 6, using discrete Weibull Minimax distribution in comparison with discrete Weibull, PAD, PLD and other classical discrete models.

In order to give an impression of the typical form of our observed distribution and the deviations from the expected ones, table 6 shows the observed chromatid aberrations X per cell in a culture of human leukocytee. This culture was treated with 0 .02γ Chinon I over a period of 24 hrs. We compute the expected frequencies for fitting discrete Log-

logistic, Poisson, Poisson Akasha, Poisson Lindley and DRayleigh distributions with the help of R studio statistical software and Pearson's chi-square test is applied to check the

Table 2: Distribution of number of Chromatid aberrations in human leukocyte (0.2 g Chinon I, 24 hours).

Number of Aberrations	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7+**	**Total**
Frequency	268	87	26	9	4	2	1	3	400

Table 3: Descriptive statistics of Counts of Chromatid aberrations in human leukocyte (0.2 g Chinon I, 24 hours).

		Statistic	**Standard Error**	**Bootstrap[a]**			
				Bias	Std. Error	95% Confidence Interval	
						Lower	Upper
Chromatid Aberrations data	Mean	.5475	.05305	.0034	.0535	.4451	.6599
	Std. Deviation	1.06092		-.00168	.10258	.86966	1.26823
	Variance	1.126		.007	.218	.756	1.608
	Skewness	3.134	.122	-.077	.303	2.391	3.654
	Kurtosis	12.858	.243	-.593	2.527	7.121	17.609
	N	400		0	0	400	400
a. Noted, bootstrap results are based on 1000 bootstrap samples							

Table 4: Estimated parameters by ML method for fitted distributions for Counts of Chromatid aberrations in human leukocyte (0.2 g Chinon I, 24 hours).

Distribution	**parameter Estimates**	**Standard Error of the estimates**	**Model function**
Discrete Weibull Minimax	$\gamma = 1.2, \beta = 3.1, \alpha = 1.30$	$SE(\gamma, \beta, \alpha)$ (0.3, 2.9,0.7)	$p(x) = \beta[B(\beta, \frac{x^{\gamma}}{\alpha} + 1) - B(\beta, \frac{(x+1)^{\gamma}}{\alpha} + 1)]$ $x = 0,1,2,\ldots$ for $\gamma > 0, \beta > 0, \alpha > 0$
Poisson	$\lambda = 0.55$	$SE(\lambda) = 0.04$	$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$ $\lambda > 0;\ x = 0,1,2,\ldots..$
Poisson Akasha	$\theta = 2.66$	$SE(\theta) = 0.15$	$p(x) = \frac{\theta^3(x^2 + 3x + (\theta^2 + 2\theta + 2))}{(\theta^2 + 2)(\theta + 1)^{x+3}}$ $x = 0,1,2,\ldots. \ \theta > 0$
Poisson Lindley	$\theta = 2.38$	$SE(\theta) = 0.17$	$p(x) = \frac{\theta^2(x + \theta + 2)}{(\theta + 1)^{x+3}}$ $x = 0,1,2,\ldots. \ \theta > 0$
Discrete Rayleigh	$q = 0.63$	$SE(q) = 0.015$	$p(x) = q^{x^2} - q^{(x+1)^2}$ $0 < q < 1; x = 0,1,2,\ldots..$
Discrete Weibull	$q = 0.32, \gamma = 0.86$	$SE(q, \gamma)$ (0.2, 0.05)	$p(x) = q^{x^{\gamma}} - q^{(x+1)^{\gamma}}$ $x = 0,1,2,\ldots..$ $0 < q < 1$; $\gamma > 0$
NBD	$r = 0.62, p = 0.53$	$SE(r, p)$ (0.12,0.05)	$p(x) = \binom{x + r - 1}{x} p^{r} q^{x}$, $x = 0,1,2,\ldots$ $r > 0$ *and* $0 < p < 1$

Table 5: Table for goodness of fit for Counts of Chromatid aberrations in human leukocyte (0.2 g Chinon I, 24 hours).

X	Observed	DWMD	Poisson	PAD	PLD	DRayleigh	DWD	NBD
0	268	268.23	231.357	260.455	257.023	146.812	270.51	270.175
1	87	85.83	126.668	89.671	93.390	188.980	78.53	78.552
2	26	26.72	34.675	32.086	32.761	57.684	29.63	29.838
3	9	9.96	6.328	11.531	11.210	6.258	12.06	12.220
4	4	4.30	0.866	4.095	3.766	0.261	5.13	5.186
5	2	2.08	0.095	1.428	1.247	0.004	2.25	2.247
6	1	1.10	0.009	0.489	0.408	0.000	1.01	0.987
7+	3	1.78	0.001	0.245	0.195	0.000	0.88	0.794
χ^2	*p-values*	0.67	0.00	0.24	0.10	0.000	0.33	0.311

X: Counts of Chromatid aberrations in human leukocyte.
Observed: Observed frequency of Counts of Chromatid aberrations in human leukocyte.

Table 6: Distribution of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 µg|kg..

Class/Exposure (μ g/kg)	0	1	2	3	4	5	6+	Total
Frequency	155	83	33	14	11	3	1	300

Table 7: Estimated parameters by ML method for fitted distributions for Counts of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 µg|kg..

Distribution	parameter Estimates	Model function
Discrete Weibull Minimax	$\gamma = 1.15, \beta = 45.01, \alpha = 5.9$	$p(x) = \beta[B(\beta, \frac{x^\gamma}{\alpha} + 1) - B(\beta, \frac{(x+1)^\gamma}{\alpha} + 1)]$ $x = 0,1,2,\ldots$ for $\gamma > 0, \beta > 0, \alpha > 0$
Poisson	$\lambda = 0.85$	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $\lambda > 0;\ x = 0,1,2,\ldots..$
Poisson Akasha	$\theta = 1.96$	$p(x) = \frac{\theta^3(x^2 + 3x + (\theta^2 + 2\theta + 2))}{(\theta^2 + 2)(\theta + 1)^{x+3}}$ $x = 0,1,2,\ldots.$ $\theta > 0$
Poisson Lindley	$\theta = 1.61$	$p(x) = \frac{\theta^2(x + \theta + 2)}{(\theta + 1)^{x+3}}$ $x = 0,1,2,\ldots.$ $\theta > 0$
Discrete Rayleigh	$q = 0.73$	$p(x) = q^{x^2} - q^{(x+1)^2}$ $0 < q < 1; x = 0,1,2,\ldots..$
ZIP	$\alpha = 0.33, \lambda = 1.27$	$p(x) = \begin{cases} \alpha + (1-\alpha)\frac{e^{-\lambda}\lambda^x}{x!}, \lambda > 0; x = 0 \\ (1-\alpha)\frac{e^{-\lambda}\lambda^x}{x!}, \lambda > 0; x = 0,1,2,.. \end{cases}$ $0 < \alpha < 1$; $\lambda > 0$

Table 8: Table for goodness of fit for Counts of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 µg|kg.

X	Observed	Poisson	DRayleigh	ZIP	PLD	DWMD	PAD
0	155	127.798	81.994	155.000	158.335	154.488	160.668
1	83	109.054	134.348	71.935	77.209	82.183	74.335
2	33	46.530	66.705	45.658	35.884	36.262	35.268
3	14	13.235	15.139	19.319	16.149	15.488	16.483
4	11	2.823	1.712	6.131	7.102	6.585	7.495
5	3	0.482	0.099	1.557	3.069	2.816	3.312
6	1	0.078	0.003	0.400	2.254	2.179	2.440
χ^2	p-values	0.000	0.000	0.002	0.479	0.227	0.419

X: Counts of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 $\mu g|kg$

Observed: Observed frequency of Counts of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 $\mu g|kg$

Table 9: AIC, BIC and loglikelihood values for fitted distributions for number of Chromatid aberrations in human leukocyte (0.2 g Chinon I, 24 hours).

Criterion	Poisson	DRayleigh	PLD	PAD	DWMD	DWD	NBD
Loglikelihood Value	-439.5136	-557.5330	-403.455	-402.285	-398.532	-399.689	-399.857
AIC	881.02725	1117.06604	808.9099	806.5696	803.0649	803.3787	803.7137
BIC	881.62931	1117.6681	809.512	807.1716	804.8711	804.5828	804.9178

Table 10: AIC, BIC and loglikelihood values for fitted distributions for Counts of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 µg|kg.

Criterion	Poisson	DRayleigh	ZIP	PLD	DWMD	PAD
Loglikelihood value	-400.462	-451.535	-386.797	-383.052	-383.084	-383.311
AIC	802.9234	905.0705	777.5945	768.1049	772.1678	768.6216
BIC	806.6272	908.7743	785.0021	771.8087	783.2792	772.3254

goodness of fit of the models discussed. The calculated figures are given in the table 5.

The p-values of Pearson's Chi-square statistic are 0.67, 0.00, 0.24, 0.10, 0.00, 0.33 and 0.31 for discrete Weibull Minimax, Poisson, Poisson Akasha, Poisson Lindley, discrete Rayleigh, discrete Weibull and Negative binomial distributions, respectively (see Table 5). This reveals that Poisson and discrete Rayleigh distributions are not good fit at all, whereas discrete Weibull Minimax, Poisson Akasha Poisson Lindley, Negative binomial and discrete Weibull distributions are good fit distributions with discrete Weibull Minimax model being the best one. The null hypothesis that data come from discrete Weibull Minimax distributions is strongly accepted.

Table 6 provides us an overview of the dataset related to Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 µg|kg. Even though the Distribution of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 µg|kg. is best fitted by PLD but DWMD also fits the data significantly with p-value 0.22 (>0.05). The p-values of Pearson's Chi-square statistic and expected frequencies for Distribution of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 µg|kg is given in table 8. Poisson, DRayleigh and ZIP doesn't fit the data at all, whereas PLD, DWMD and PAD fits the data significantly.

We have compared discrete Weibull Minimax distribution

with discrete Rayleigh, discrete Poisson Lindley, Poisson Akasha, Poisson, Zero Inflated Poisson and Negative binomial distributions using the Akaike information criterion (AIC), given by Akaike [16] and the Bayesian information criterion (BIC), given by Schwarz [17] .

Generic function calculating Akaike's 'An Information Criterion' for one or several fitted model objects for which a log-likelihood value can be obtained, according to the formula -2*log-likelihood + k*npar, where npar represents the number of parameters in the fitted model, and k = 2 for the usual AIC, or k = log(n) (n being the number of observations) for the so called BIC or SBC (Schwarz's Bayesian criterion).

The AIC, BIC and Loglikelihood values for the fitted distributions to two real datasets are given in table 9 and table 10. By comparing their AIC, BIC and Loglikelihood values for the fitted models, we conclude that in case of number of Chromatid aberrations in human leukocyte (0.2 g Chinon I, 24 hours), discrete Weibull Minimax model yields best fit in contrast to Poisson, Poisson Akasha, Poisson Lindley, discrete Rayleigh, discrete Weibull and Negative binomial distributions. In case of Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 µg|kg, DWMD fits better in contrast to Poisson, DRayleigh and ZIP models.

9 Conclusion

In this paper, a new model is proposed by compounding discrete Weibull distribution (DWD) with Minimax distribution (MD) and it has been shown that proposed model can be nested to different compound distributions. Some important probabilistic properties and the problem of estimation of its parameters are studied. In addition, the discrete Weibull Minimax distribution is appropriate for modeling both over and under dispersed data since, depending on the values of the parameters, its variance can be larger or smaller than the mean, which is not the case with some known standard classical discrete distributions.

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