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The Growth of Vapour Bubble between two-Phase Peristaltic Bubbly Flow inside a Vertical Cylindrical Tube

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Abstract: The behavior of vapour bubble in superheated liquid in a vertical cylindrical tube between two-phase flow densities is discussed under the effect of peristaltic motion of long wavelength and low Reynolds number. The mathematical model is formulated by mass, momentum, and heat equations. The problem solved analytically to estimate the growth of vapour bubbles, temperature and velocity distributions. The growth of vapour bubbles, temperature and velocity distribution proportional with the amplitude ratio, Grashof number, heat source parameter, volume rate, and inversely with density fraction. The present results of bubble growth performed lower values than that obtained by Mohammadein model (2001).

Keywords: Heat transfer, two-phase flow, vapours bubble growth, superheated liquid, Newtonian fluid, peristaltic flow.

1 Introduction

Peristalsis is the phenomenon in which a circumferential progressive wave of contraction or expansion propagates along a tube. If the tube is long enough, one might see several identical waves moving along the tube simultaneously. Peristalsis is now well known to physiologist to be one of the major mechanisms for fluid transport in many biological systems. Peristalsis is an important mechanism reported in many organisms and in a variety of a living body. Peristaltic flows also provide efficient means for sanitary fluid transport and are thus exploited in industrial peristaltic pumping and medical devices [1]. The problem of the mechanism of peristaltic transport has attracted the attention of many investigators.

A good number of analytical, numerical and experimental studies has been conducted to understand peristaltic action under different conditions with reference to physiological and mechanical situations. Srinivas and Pushparaj have investigated the peristaltic transport of magneto hydrodynamics (MHD) flow of a viscous incompressible fluid in a two dimensional asymmetric inclined channel [2].The study of temperature field and spherical vapour bubble dynamics between two-phase flow [3-5, 6-10, and 11] are most important physical phenomena because of its necessity in many technical processes. The mixed lubrication, chemical metallurgic, oil and gas processes are applications of heat exchange. It plays a major role in the industry of refrigerators, boilers and nuclear reactors which are used for generation of electrical current. The vapour bubble is considered as a finite sink growing inside a mixture (vapor and superheated liquid).There are three stages for bubble growth, inertial, thermal, and diffusion. In the inertial stage, the bubble nucleus depends strongly on the interfacial mechanical interactions such as acceleration, pressure force, and surface tension forces.

The inertial stage takes a few milliseconds and thermal phenomena are negligible, therefore, this stage is called isothermal. The peristaltic transport through tubes channels have attracted considerable attention due to their wide applications in medical and engineering sciences, such as, in physiology, roller and finger pumps, sanitary fluid transport, transport of corrosive fluids etc. Several review articles have been reported by authors [12-17]. The effect of heat transfer on the peristaltic flow of a Newtonian fluid through asymmetric vertical cylindrical tube is studied by Rao et. al. [4]. The closed form solutions of velocity field and temperature are obtained. . The theory of the growth of a single vapour bubble in a superheated liquid has been considered by several authors. The inertia controlled growth was presented by Rayleigh [7], who determined he first equation of motion for a spherical bubble growth (or collapse).

The asymptotic solution, presented by Plesset and Zwick [17], considered thermal diffusion controlled growth, neglecting liquid inertia, and provided a zero-order approximate solution for the bubble wall temperature with the assumption of a thin thermal boundary layer with error

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of less than 10% [17]. Their solution [15] was in good agreement with the experimental data of Degarabedian [18] in moderately superheated water up to 6 °C. Forster. In this paper, the peristaltic bubbly flow of viscous incompressible Newtonian flow in a vertical cylindrical tube is analyzed in details. The growth of vapour bubbles between two-phase densities inside a tube is studied under the effect of some physical parameters. The influence of some various physical parameters on the flow is observed. The temperature and the heat transfer are discussed through graphs.

2 Analysis

Consider the peristaltic bubbly flow of a viscous incompressible Newtonian fluid through a vertical tube. The flow is generated by sinusoidal wave trains propagating with constant speed along the wall of the outer rube. The axisymmetric cylindrical polar coordinate system is chosen such that the coordinate is along the center line of the tube and coordinate along the radial coordinate.

The wall of the tube is maintained at a temperature T_0 and at the center we have used axisymmetric condition on temperature as in Fig. 1, which depicts the physical model of the problem. where$a$ is the radius of the tube, b is the amplitude of the wave, λ is the wavelength and t is the time. The flow is unsteady in the fixed frame (z, r). However, in a coordinate system moving with the propagation velocity c.

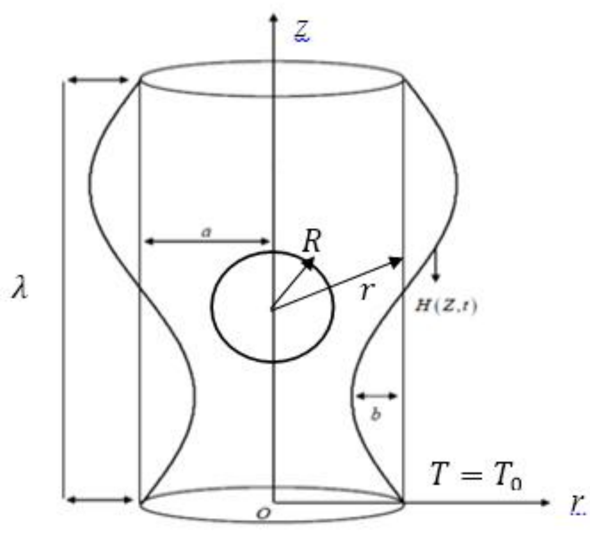

Fig. 1 Physical Model.

The mathematical model of the physical problem is described by the conservation equations of mass, momentum, and heat transfer as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u)+\frac{\partial w}{\partial z}=0 \qquad (1)$$

$$\rho\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)-\frac{1}{r^2}+\frac{\partial^2}{\partial z^2}\right\}u \qquad (2)$$

$$\rho\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{\partial^2 w}{\partial z^2}\right\}+\rho g\alpha(T-T_0) \qquad (3)$$

$$\rho\, c_p\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)+Q_0, \qquad (4)$$

where p is the pressure, T is the temperature, Q_0 is the constant heat addition/absorption, c_p is the specific heat at constant pressure, k is the thermal conductivity and ρ is the density of the fluid. Introducing the dimensionless variables as follows:

$$\bar{r}=\frac{r}{a},\quad \bar{z}=\frac{z}{\lambda},\quad \bar{w}=\frac{w}{c},\quad \bar{u}=\frac{u}{c\delta},\quad \bar{p}=\frac{pa^2}{\mu c\lambda},$$

$$\bar{\theta}=\frac{T-T_0}{T_0},\quad \bar{\delta}=\frac{a}{\lambda},\quad h=1+e\sin(2\pi z),\quad e=\frac{b}{a},$$

$$R_e=\frac{\rho ca}{\mu},\quad G=\frac{a^3 g\alpha T_0}{v^2},\quad Pr=\frac{\mu c_p}{k}\quad \beta=\frac{a^2 Q_0}{kT_0}, \qquad (5)$$

where, R_e is the Reynolds numbers, δ is the wave number, e is the amplitude ratio, G is the Grashof number, Pr is the Prandtl number, and β is the non-dimensional heat source parameter. Substituting by the equation (5) into the equations (1-4), we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u)+\frac{\partial w}{\partial z}=0 \qquad (6)$$

$$R_e\delta^3\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\delta^2\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)-\frac{u}{r^2}+\delta^2\frac{\partial^2 u}{\partial z^2}\right\} \qquad (7)$$

$$R_e\delta\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\delta^2\frac{\partial^2 w}{\partial z^2}+G\theta \qquad (8)$$

$$R_e\,Pr\,\delta\left(u\frac{\partial\theta}{\partial r}+w\frac{\partial\theta}{\partial z}\right)=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\delta^2\frac{\partial^2\theta}{\partial z^2}+\beta. \qquad (9)$$

When the wavelength is large ($\delta \ll 1$), the Reynolds number is quite small ($R_e \to 0$) and the equations (7-9) becomes

$$\frac{\partial p}{\partial r}=0 \qquad (10)$$

$$\frac{\partial p}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+G\theta \qquad (11)$$

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\beta=0. \qquad (12)$$

The dimensionless volume flow rate in the fixed frame of reference is given by

$$q=2\int_0^h wrdr, \qquad (13)$$

the corresponding dimensionless boundary conditions are

$$\frac{\partial w}{\partial r}=0 \qquad at \qquad r=0 \qquad (14)$$

$$w=-1 \qquad at \qquad r=h \qquad (15)$$

$$\frac{\partial\theta}{\partial r}=0 \qquad at \qquad r=0 \qquad (16)$$

$$\theta=0 \qquad at \qquad r=h. \qquad (17)$$

Solving Eq. (12) using the Eqs. (16) and (17), we get

$$\theta = \frac{\beta}{4}(h^2 - r^2), \tag{18}$$

Substituting by Eq. (18) into the Eq. (11) and solving Eq. (11) with the boundary conditions of Eqs. (14) and (15), we get

$$w = \frac{1}{4}\frac{dp}{dz}(r^2 - h^2) - \frac{G\,\beta}{4}\left(\frac{h^2 r^2}{4} - \frac{r^4}{16}\right) + \frac{3\,G\,\beta\,h^4}{64} - 1, \tag{19}$$

The volume flow rate is given by

$$q = -\frac{dp}{dz}\left(\frac{h^4}{8}\right) + \frac{G\,\beta\,h^6}{96} - h^2. \tag{20}$$

From the Eq. (20), we have

$$w = \left(\frac{G\,\beta\,h^2}{24} - \frac{2q}{h^4} - \frac{2}{h^2}\right)(r^2 - h^2) - \frac{G\,\beta}{4}\left(\frac{h^2 r^2}{4} - \frac{r^4}{16}\right) + \frac{3\,G\,\beta\,h^4}{64} - 1. \tag{21}$$

On the basis of continuity Eq. (1), we find that, the velocity of cylindrical coordinates of the vapour bubble, can be written as

$$w(r,t) = \frac{\varepsilon\,R\dot{R}}{r}. \tag{22}$$

From the Eq. (22), we can obtain the velocity of vapour bubble radius in a vertical cylindrical tube as the form

$$\dot{R}(r,t) = \left(\frac{1}{\varepsilon}\right)\left(\left(\frac{G\,\beta\,h^2}{24} - \frac{2q}{h^4} - \frac{2}{h^2}\right)(r^2 - h^2) - \frac{G\,\beta}{4}\left(\frac{h^2 r^2}{4} - \frac{r^4}{16}\right)\right) + \frac{3\,G\,\beta\,h^4}{64\,\varepsilon} - \frac{1}{\varepsilon}, \tag{23}$$

$$R\,(t) = \sqrt{\frac{M}{A}}\tan\left[\tan^{-1}\frac{R_0}{\sqrt{\frac{M}{A}}} - \frac{\sqrt{M\,A}}{\boldsymbol{\varepsilon}}(t - t_0)\right], \tag{24}$$

where,

$$M = -1 - \left(\frac{-2q}{h^2} - 2\right),$$

$$A = \frac{-2q}{h^4} - \frac{2}{h^2},$$

$$\varepsilon = \left(1 - \frac{\rho_v}{\rho_l}\right).$$

3 Discussion and Results

The problem of bubble growth in a vertical cylindrical tube is studied for Newtonian fluid with peristaltic bubbly flow. The physical problem is described by Fig. 1, the mathematical model is formulated by the system of equations (1-4) in a two dimensional cylindrical coordinates (r,z), with boundary conditions (14-17), and solved analytically for the long wavelength $\delta \ll 1$. The growth of vapour bubble radius is obtained by Eq. (24). Temperature distribution and velocity distribution are derived by Eqs. (18), and (21) respectively. Moreover, the unsteady bubble radius is formulated in cylindrical coordinates (r,z).The temperature distribution inside a superheated water under atmospheric pressure (P_∞ =247kPa and T_s=400 K). The physical values are calculated by Haar et al. [6] as given by Table 1. Moreover, by using Mathematica program (version 7.0), the following graphs that demonstrate the effect of the physical parameters on velocity distribution, temperature distribution, and the radius of vapour bubble. The velocity distribution in terms of parameter r for different values of Grashof number G, and heat source parameter β are shown in Fig. 2 (a, b). It is observed that the velocity distribution is proportional with Grashof number$G$ and heat source parameter β. In Fig. 3, the velocity distribution is plotted in terms of (r,z) with two different values of volume rate q. It is clearly, the velocity distribution shifted for the upper values with the increasing of volume rate q. On the other hand, temperature distribution in terms of (r,z), is shown in Fig. 4. It observed increases in the middle part of the channel and decreases near the channel walls. The streamlines are calculated from Eqs. (21), (18), and plotted in Figs. 5, 6 respectively. Figs. 7(a, b) shows the temperature distribution in terms of parameter r for different values of amplitude ratio e and heat source parameter β, is respectively. It is observed that the temperature distribution is proportional with amplitude ratio e and heat source parameter β.

Temperature distribution in terms of parameter z for different values of amplitude ratio e and heat source parameter β are shown in Figs. 8 (a, b). It is observed that the temperature distribution is proportional with amplitude ratio e and heat source parameter β. The behavior of vapour bubble for two different values of density fraction ε, amplitude ratio e are shown in Fig. 9 (a, b). When increasing value of density fraction ε, amplitude ratio e, then the behavior of vapour bubble shifted for lower values and this is agreement with Prosperetti [17].It is observed that the bubble velocity is decreasing with increasing of time t in Figs. 10 (a, b). In Figs. 11 (a, b).

The velocity of vapour bubble is proportionally inversely with the radius of vapour bubble under the effect of Grashof number G and heat source parameter β and shifted for the upper values with increasing of Grashof number G and heat source parameter β and this is agreement with Ref. [4]. The influence the amplitude ratio e on the velocity of vapour bubbles in terms of R is shown in Fig. 11 (c). It is reveals the same behavior in Figs. 11 (a, b), but here the irregular increment, the reason for this is due to peristaltic bubbly flow in a vertical cylindrical tube. The growth of vapour bubble compared with Mohammadein et. al. model is shows in Fig. 12. It is observed that the present model performs at lower values than Mohammadein model with fraction density $\varepsilon = 0.4$ and initial velocity $\dot{R}_0 = 5 \times 10^{-5}\mathrm{m/s}$.

Table: 1 Parameters' values used in the present problem.

Parameter	Value	Unite	Parameter	Value	Unite
ρ_v	0.597	kg/m^3	k	0.6857	W
ρ_l	958.3	kg/m^3	σ	0.0535	$/mk^0$
ρ_g	1.37	kg/m^3	$\Delta\theta_0$	2.5	kg/s^2
R_0	1×10^{-5}	m	T_0	273.15	k^0
c_p	4240	$J(kgk^0)$			
L	533000	Jkg			
$\dot{R}_0$	$R_0\varphi_0^{\frac{-1}{3}}$	m/s			
R_m	10^{-4}	m			

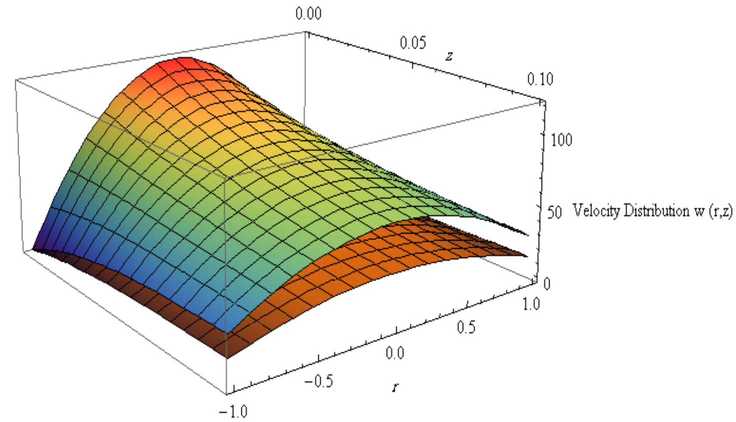

Fig. 3 The Velocity distribution is plotted in 3Das afunctionof r, z with the different values of volume flow rate$q$.

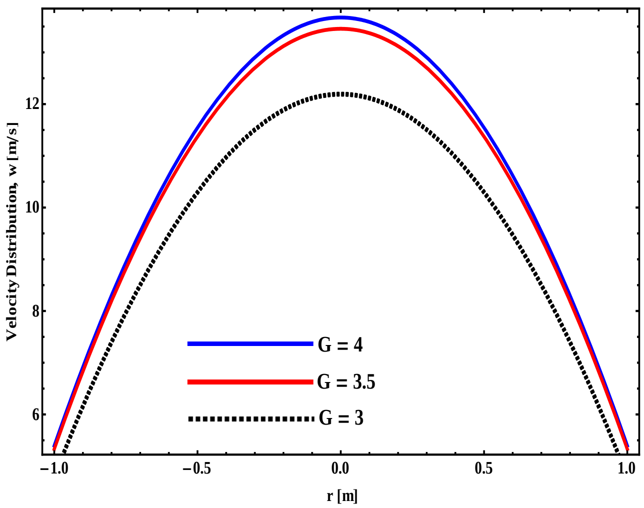

Fig. 2 (a) The Velocity distribution is plotted as a function of$r$ with the different Values of Grashof number G (G=3, 3.5, 4).

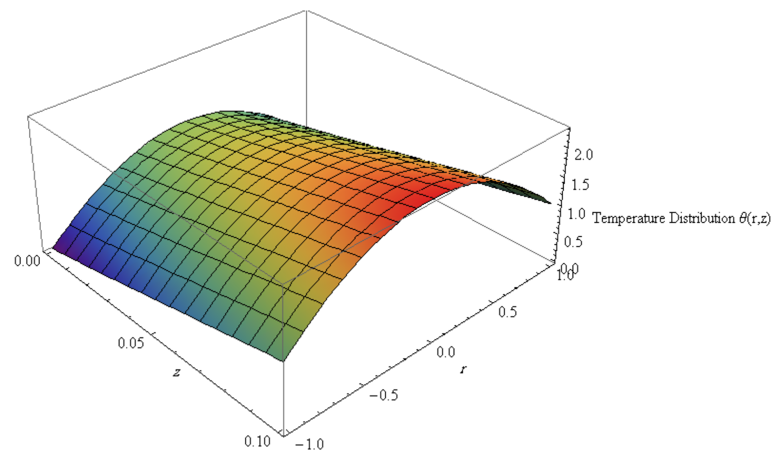

Fig.4 Temperature distribution $\theta(r,z)$ is plotted in 3D as a function of r, z.

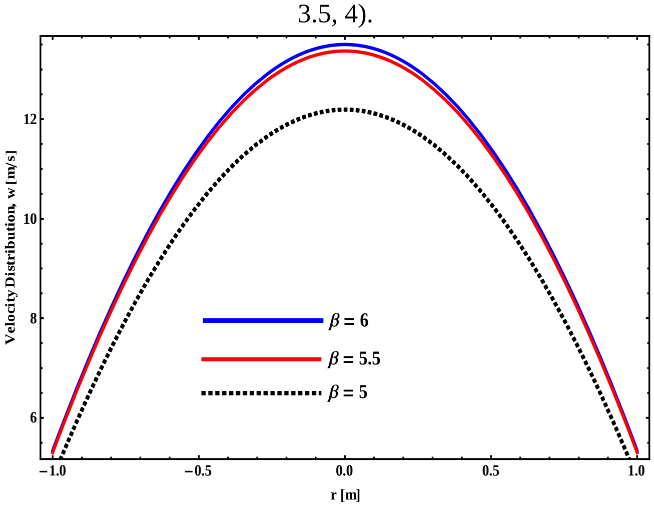

Fig. 2 (b) The Velocity distribution is plotted as a function of r with the different Values of Grashof number $(\beta = 5, 5.5, 6\ )$.

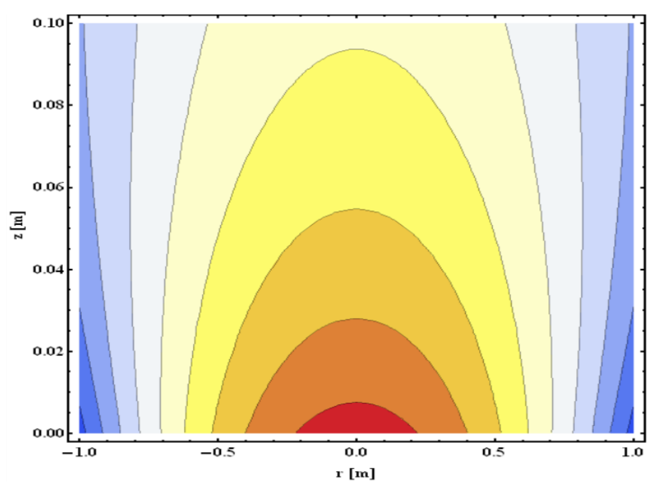

Fig. 5 Streamlines of velocity distribution $w(r,z)$is plotted as a function of r, z.

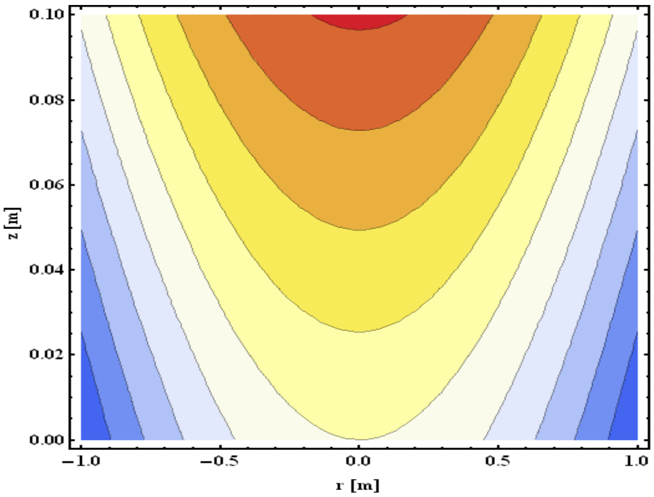

Fig. 6 Streamlines of temperature distribution $\theta(r,z)$ is plotted as a function of r, z.

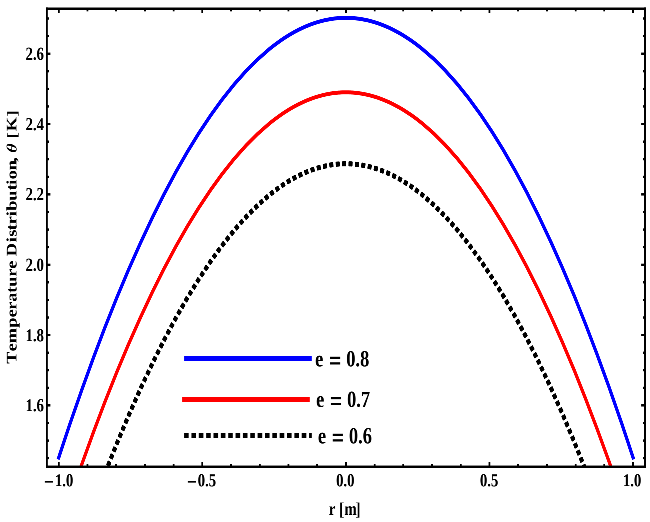

Fig. 7 (a) Temperature distribution $\theta(r,z)$ is plotted as a function of r with the different amplitude factor e (e =0.6, 0.7, 0.8).

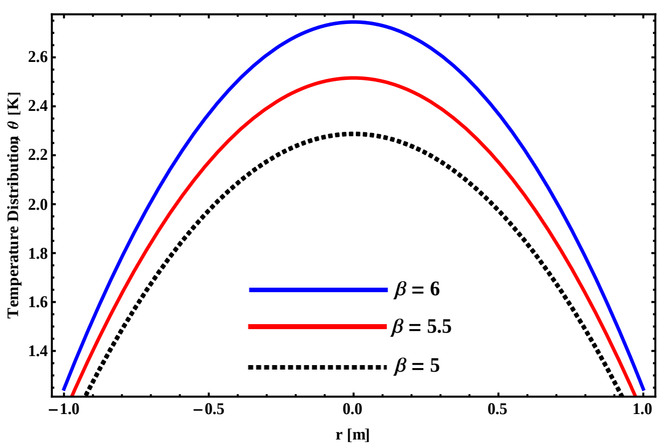

Fig. 7 (b) Temperature distribution $\theta(r,z)$ is plotted as a function of r with the different Values of heat source parameter β (β =5, 5.5, 6).

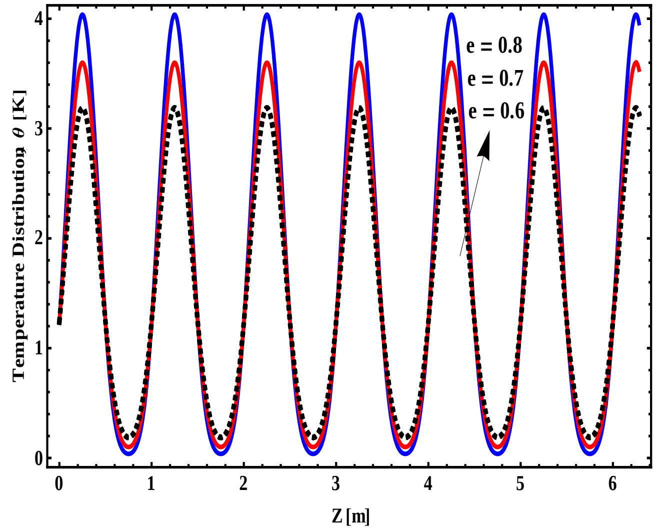

Fig. 8(a) Temperature distribution $\theta(r,z)$ is plotted as a function of z with the different amplitude factor e (e =0.6, 0.7, 0.8).

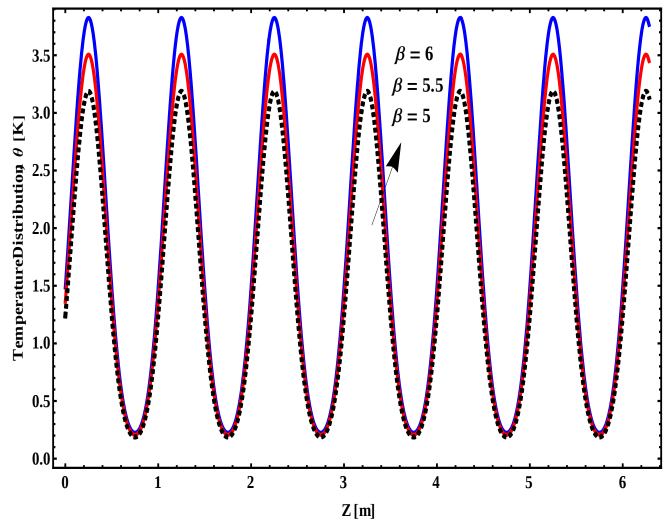

Fig. 8 (b) Temperature distribution $\theta(r,z)$ is plotted as a function of z with the different Values of heat source parameter β (β =5, 5.5, 6).

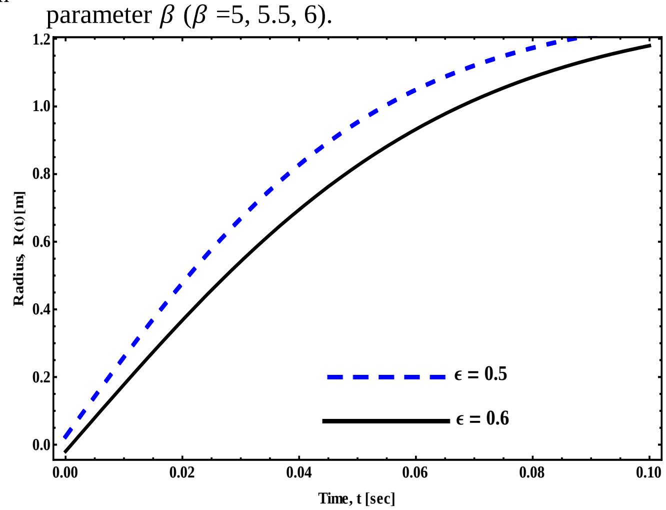

Fig. 9 (a) The radius of vapour bubble is plotted as a function of time t for two different values of parameter ε ($\varepsilon = 0.5, 0.6$).

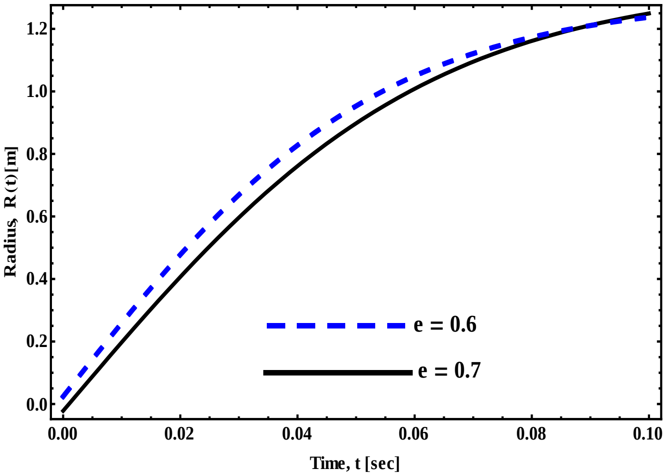

Fig. 9 (b) The radius of vapour bubble is plotted as afunction of time t for two different values of amplituderatio e ($e = 0.5, 0.6$).

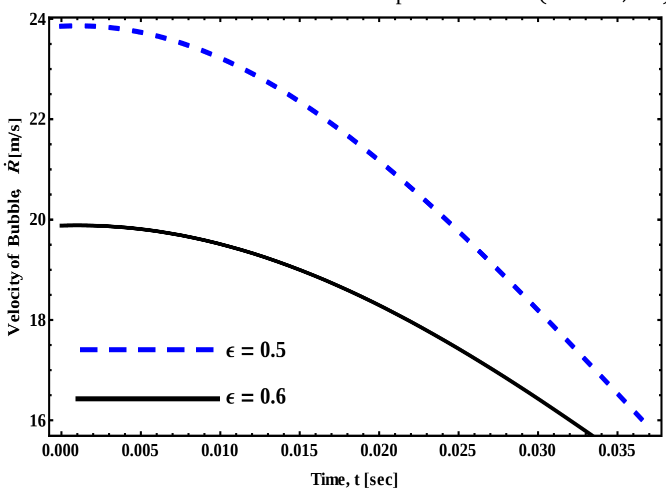

Fig. 10 (a) The velocity of bubble radius is plotted as a function of time t with the different values of ε ($\varepsilon = 0.5, 0.6$).

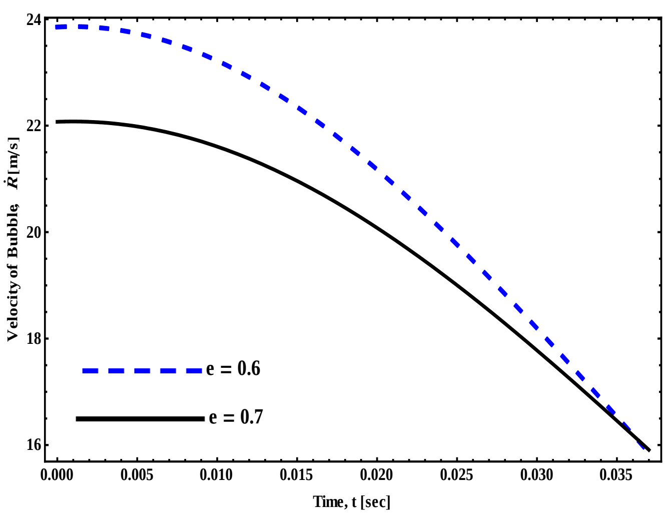

Fig. 10 (b) The Velocity of bubble radius is plotted as a function of time t with the different values of amplitude ratio e ($e = 0.6, 0.7$).

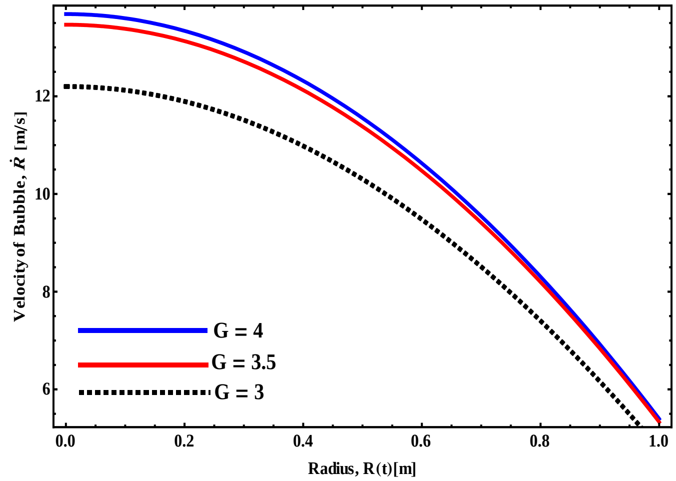

Fig. 11 (a) The Velocity of bubble radius is plotted as a function of bubble radius R with the different values of Grashofnumber G (G=3, 3.5, 4).

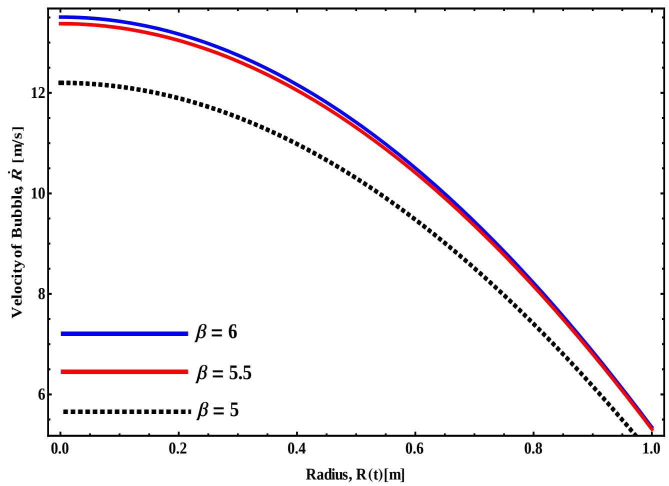

Fig. 11(b) The velocity of bubble radius is plotted as a function of bubble radius R with the different values of heat source parameter β (β =5, 5.5, 6).

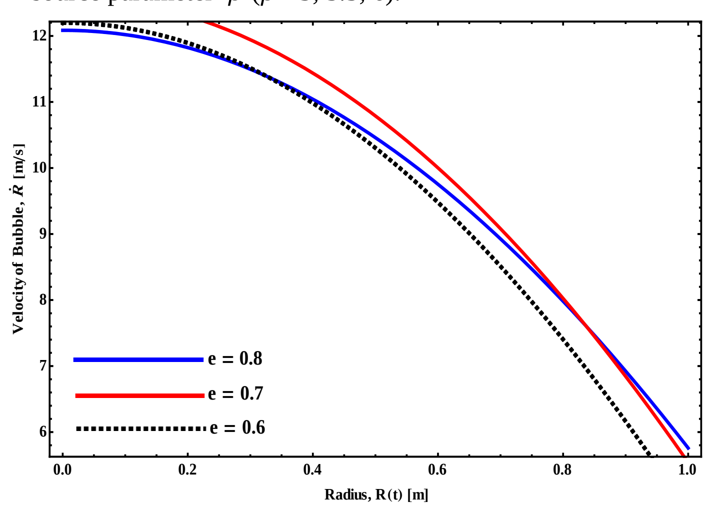

Fig. 11 (c) The velocity of bubble radius is plotted as a function of bubble radius R with the different amplitude factor e (e =0.6, 0.7, 0.8).

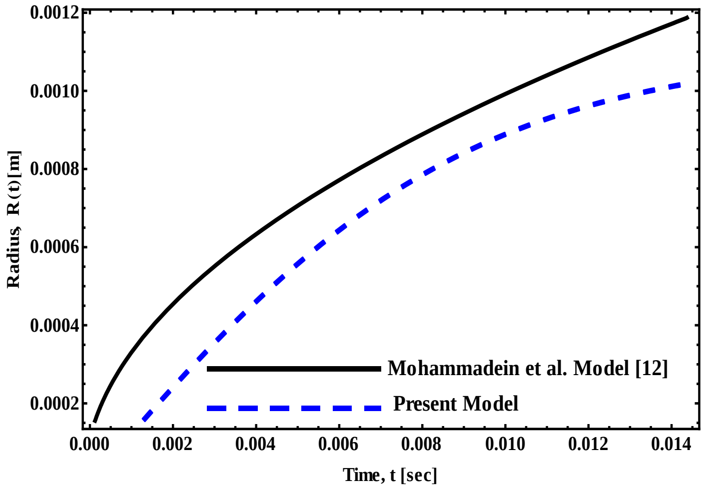

Fig. 12 The comparison of present model with Mohammadein et al. model [12].

4 Conclusion

The peristaltic bubbly flow of a non-viscous incompressible Newtonian fluid in a vertical cylindrical tube with observing of vapour bubbles is formulated. The flow is generated by sinusoidal wave trains propagating with constant speed c along the wall of the outer tube. The effect of various physical parameters on velocity distribution, temperature distribution, and growth of vapour bubble radius are illustrated through the graphs. The velocity distribution in the mixture is proportional with different values of Grashof number G and heat source parameter β. The velocity distribution increasing with increasing of volume flow rate q. Temperature distribution in the mixture is proportional with different values of amplitude ratio e, and heat source parameterβ. The behavior of bubble radius in the mixture decreases with the increasing values of amplitude ratio e. The growth of vapour bubble in the present model performed lower values than that obtained by Mohammadein model [12]. The temperature, velocity distributions, and behavior of bubble radius in vertical cylindrical tube are obtained as special case when the amplitude ratio tends to zero.

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Nomenclature

a Radius of the tube

b Amplitude of the wave

λ Wavelength

c_p Specific heat of liquid at constant pressure $(\mathrm{Kg}^{-1}\mathrm{J/k})$

ρ_v, ρ_l Density of vapour and liquid $(\mathrm{Kg\ m}^{-3})$

Q_0 Constant heat addition/absorption

k Thermal conductivity

r Radial coordinate

z Coordinate z

R Instantaneous bubble radius (m)

$\dot{R}$ Instantaneous radial velocity of bubble boundary

t Time of bubble growth (s)

T Temperature of liquid(K^0)

T_0 Initial temperature of liquid (K^0)

w Liquid velocity

θ Temperature distribution

e Amplitude Factor

ε Constant defined by Eq. 25

G Grashof number

Pr Prandtl number

β Non-dimensional heat source parameter

q Dimensionless volume flow rat

R_e Reynolds number

δ Wave number

Subscript

l Liquid

v Vapour

P Pressure

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