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# Likelihood Estimation of Exponentiated Exponential Distribution under Step Stress Partially Accelerated Life Testing Plan Using Progressive Type-I Censoring

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**Abstract:** Recently, partially accelerated life testing has become quite important in reliability and life testing studies. This paper discusses maximum likelihood estimation method in step-partially accelerated life tests when the lifetimes of items under use condition follow the exponentiated exponential distribution. Based on progressively type-I censored samples; the point and interval maximum likelihood estimations for the considered parameters and the tampering coefficient are obtained in closed forms. The observed Fisher information matrix is derived to calculate confidence intervals for the considered parameters. The performances of the resulting estimators of the developed model parameters are evaluated and investigated in terms of mean squared errors by using a Monte Carlo simulation method.

**Keywords:** Partially Accelerated Life Testing, Progressive Type-I Censoring, Exponentiated Exponential Distribution, Maximum Likelihood Estimation, Simulation Study.

## 1 Introduction

It is very hard to obtain information concerning the lifetime of an item or system with high reliability under usual operating conditions. In such problems, an experimental process called accelerated life testing (ALT) is conducted, where systems are tested under higher stress than normal, to find and induce their failure information. The stress loadings are allowed to increase at some pre-assigned time points such that the required information on the lifetime parameters can be obtained more quickly than under normal operating conditions. Commonly used stress patterns are constant stress and step stress [1]. Thus, accelerated life tests (ALTs) or partially accelerated life tests (PALTs) are used to shorten the lives of test items and to reduce the experimental time and the cost incurred in the experiment. Under step-stress PALT (SSPALT), a test item is first subjected to normal (use) condition and, if it does not fail for a specified time, then it is run at accelerated condition until the test terminates.

Although PALT procedure can be conducted to shorten the test time in an experiment, it still costs much time for an experimenter to wait for all the units to be failed. Therefore, censoring schemes have been an important tool to consider. The most commonly used censoring schemes are Type-I and Type-II censoring schemes [2]. Suppose there are n items under consideration in a particular experiment. Under the conventional Type-I censoring scheme, the experiment continues up to a pre-specified time T. On the other hand, the conventional Type-II censoring scheme requires the experiment to continue until a pre-specified number of failures $m \leq n$ occurs. Both these censoring schemes do not allow the experimenter to remove the units from the experiment at points other than the terminal point. This allowance will be important when a compromise between reduced time of experimentation and the observations of at least some extreme lifetimes are sought. Also when some of the surviving units in the experiment those are removed early one can be used for some other test. These reasons lead us into the area of progressive censoring. Here, in this study, we propose to use type-I progressive censoring on exponentiated exponential distribution using PALT procedures.

A lot of literature is available on SS-PALT analysis, for example, see Goel [3], DeGroot and Goel [4], Bhattacharyya and Soejoeti [5], Bai and Chung [6], Abdel-Ghani [7] and Abdel-Ghaly et al. [8], Abdel-Ghani [9], Recently, Ismail [10] studied the estimation and optimal design problems for the Gompertz distribution in SS-PALT with type I censored data. Also, SSPALT has been studied under hybrid censoring, see Ismail [11]. In addition, Ismail [12] has considered SSPALT using the progressive Type-II censoring scheme.

The newness in this study is to apply the step PALTs to the exponentiated exponential distribution using progressively

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type-I censored data and then estimate the parameters under consideration using maximum likelihood method of estimation.

Based on the progressive censoring scheme, few interesting studies have been made under ALT, for example Viveros and Balakrishnan [13], Balakrishnan and Sindhu ([14], [15]), Balasooriya and Balakrishnan [16], Ng, et al ([17], [18]), Gouno et al [19], Balasooriya and Low [20] and Soliman [21]. Abdel-Hamid [22] considered the constant-partially accelerated life tests for Burr type-XII distribution with progressive type-II censoring. Wu, et al [23] discussed the same problem considering progressive type-I censoring with grouped data.

The exponentiated exponential distribution has been quite extensively used in reliability analysis to analyze many lifetime data and has been effectively used in place of two-parameter Weibull or Gamma distribution. The distribution has received considerable attention in the field of reliability and lifetime data. Gupta and Kundu [24] compared the performance various estimation procedures of the distribution parameters. Abdel-Hamid and Al-Hussaini [25] studied the estimation of the EE parameters in step-stress ALT under type-I censoring. Chen and Lio [26] considered the parameter estimation of EE distribution using progressive type-I interval censoring. Recently David Han [27], under the time constraint, presented the estimation in step stress life tests with complementary risks from the exponentiated exponential distribution.

The rest of the paper is organised as follows: In Section 2, a description of the model and a discussion of progressive type-I censoring scheme is presented. Closed forms of the maximum likelihood estimates (MLEs) of the parameters under consideration are derived in Section 3. Simulation studies are provided in the section 4. Lastly, conclusions and future possible research is discussed in section 5.

## 2 Model Description and Test Method

In this section, a design is framed to estimate the parameters and the tampering coefficient in SSPALT under type-I progressive censoring scheme assuming that the failure times follow exponentiated exponential distribution.

### *2.1 Basic assumptions*

1. Under SSPALT, the product is first tested at a normal stress level $S_0$ and at time τ the same is increased to $S_1$, $S_0 < S_1$.
2. The total lifetime $T$ of a unit under normal and accelerated conditions is given by

$$T = \begin{cases} Y, & 0 < Y \le \tau, \\ \tau + (Y - \tau)/\lambda, & Y > \tau, \end{cases} \quad (1)$$

where $Y$ is the lifetime of an experimental unit at normal conditions, $\tau$ is the stress change time and $\lambda(>1)$ is the tampering coefficient.

3. Suppose the random variable $Y$ has exponentiated exponential distribution with scale parameter $\beta(>0)$ and a shape parameter $\alpha(>0)$. Thus the cumulative density function (CDF) of $Y$ is given by

$$F(y) = \left(1 - e^{-\beta y}\right)^{\alpha}, \quad y > 0. \quad (2)$$

### *2.2 The Testing under Progressive type-I Censoring Scheme*

Under PALT scheme, the procedure for applying progressive type-I censoring is given below. Suppose n items are placed under test and each is initially run under normal stress condition until time $\tau_1(>0)$. At this time point, the number of failure units $n_1$ are counted and $R_1$ units are randomly withdrawn from the experimental process. When time $\tau_2(>0)$ is reached, at this point $n_2$ failed units are counted and $R_2$ of the surviving $n - n_1 - R_1$ units are withdrawn from the test. The process will continue and similarly at time point $\tau_k(>0)$, the $n_k$ failed units are counted and $R_k$ units are removed from the test. Here, at this time point all the remaining $n - n_k - R_k$ surviving units are placed under accelerated condition and run until time $\tau_{k+1}$ at which point the number of failures, $n_{k+1}$ are counted and $R_{k+1}$ surviving units are removed from the test. The test procedure is continued at accelerated condition in the same way until $\tau_K$ is reached and at this point $R_K = n - \sum_{i=1}^{K} n_i - \sum_{i=1}^{K-1} R_i$ surviving units are removed, thereby terminate the test. The above censoring times $\tau_1, \ldots, \tau_k, \ldots, \tau_K$ are fixed in advance. The observed data in the SSPALT for exponentiated exponential distribution under progressive type-I censoring is given as

$$\begin{aligned} & t_{11} \le \ldots \le t_{1n_1} \le \tau_1 \le t_{21} \le \ldots \le t_{2n_2} \le \ldots \le t_{k1} \le \ldots \le t_{kn_k} \le \tau_k \le \\ & t_{(k+1)1} \le \ldots \le t_{(k+1)n_{k+1}} \le \tau_{k+1} \le \ldots \le t_{K1} \le \ldots \le t_{Kn_K} \le \tau_K . \end{aligned} \quad (3)$$

Using equation (1) and variable transformation, we form the probability function of a unit under step stress PALT as,

$$g(t) = \begin{cases} f_1(t) = \alpha\beta\left(1 - \exp(-\beta t)\right)^{\alpha-1} \exp(-\beta t) \\ f_2(t) = \lambda\alpha\beta\left\{1 - \exp\left(-\beta\left(\tau_k + \lambda(t - \tau_k)\right)\right)\right\}^{\alpha-1} \exp\left\{-\beta\left(\tau_k + \lambda(t - \tau_k)\right)\right\}, t > \tau_k \end{cases} \quad (4)$$

The survival function is given by

$$S(t)=\begin{cases} S_1(t)=1-\left\{1-\exp(-\beta t)\right\}^{\alpha}, \\ S_2(t)=1-\left\{1-\exp\left(-\beta\left(\tau_k+\lambda(t-\tau_k)\right)\right)\right\}^{\alpha}, \end{cases} \quad (5)$$

## 3 Estimation Method

The MLE is used here because it is very sound and gives the estimates of the parameters with good statistical properties. Here, in this section we describe the point and interval estimation of the tampering coefficient and parameters of exponentiated exponential model based on progressive type-I censoring. The likelihood function using the censoring times $(\tau_1,\dots,\tau_k,\dots,\tau_K)$ and the progressive type-I censored sample in (3), is formed as

$$L(\alpha,\beta,\lambda;t)=\prod_{i=1}^{k}\prod_{j=1}^{n_i} f_1(t_{ij})\left[S_1(\tau_i)\right]^{R_i}\prod_{i=k+1}^{K}\prod_{j=1}^{n_i} f_2(t_{ij})\left[S_2(\tau_i)\right]^{R_i}. \quad (6)$$

Using the value of equations (4) in the above likelihood function and taking logarithm on both sides, we get

$$\ell(\alpha,\beta,\lambda;t)=NIn\alpha\beta+N_2In\lambda-\beta\left[N_2\tau_k+\sum_{i=1}^{k}\sum_{j=1}^{n_i}t_{ij}\right]+(\alpha-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}In\left(1-\exp(-\beta t_{ij})\right)+$$
$$\sum_{i=1}^{k}R_in_iIn\left[1-\left(1-\exp(-\beta\tau_i)\right)^{\alpha}\right]-\sum_{i=k+1}^{K}\sum_{j=1}^{n_i}\left[\beta\lambda(t_{ij}-\tau_k)-(\alpha-1)In\left(1-\exp(-\beta\varphi_t)\right)\right]+$$
$$\sum_{i=k+1}^{K}R_in_iIn\left[1-\left\{1-\exp(-\beta\varphi_\tau)\right\}^{\alpha}\right] \quad (7)$$

Where $\varphi_t=\tau_k+\lambda(t_{ij}-\tau_k)$, $\varphi_\tau=\tau_k+\lambda(\tau_i-\tau_k)$, $t_{ij}$ is the $j^{th}$ unit in the $i^{th}$ semi closed time interval $(\tau_{i-1},\tau_i]$, $\tau_0=0$, $N_1=\sum_{i=1}^{k}n_i$ and $N_2=\sum_{i=k+1}^{K}n_i$, is the number of units which get failed before and after the time point $\tau_k$, respectively. Also $N=N_1+N_2$. In the rest of the paper we will denote $t_{ij}$ by $t$.

### 3.1 Point estimation

In this subsection, we discuss the process of obtaining the point ML estimates of parameters and tampering coefficient of the model formed in section 3. We equate the partial derivatives of equation (7) to zero with respect to the each parameter in the parameter set Θ = (α, β, λ).

$$\frac{\partial\ell}{\partial\alpha}=\frac{N}{\alpha}+\sum_{i=1}^{k}\sum_{j=1}^{n_i}In\,A-\sum_{i=1}^{k}R_in_i\frac{B^{\alpha}}{1-B^{\alpha}}In\,B+\sum_{i=k+1}^{K}\sum_{j=1}^{n_i}In\,C-\sum_{i=k+1}^{K}R_in_i\frac{D^{\alpha}}{1-D^{\alpha}}In\,D=0, \quad (8)$$

$$\frac{\partial\ell}{\partial\beta}=\frac{N}{\beta}-N_2\tau_k-\sum_{i=1}^{k}\sum_{j=1}^{n_i}t\left\{\frac{1-\alpha e^{-\beta t}}{A}\right\}-\sum_{i=1}^{k}R_in_i\tau_i\frac{\alpha B^{\alpha-1}}{1-B^{\alpha}}e^{-\beta\tau_i}-$$
$$\sum_{i=k+1}^{K}\sum_{j=1}^{n_i}\left[\lambda(t-\tau_k)-\frac{(\alpha-1)}{C}\varphi_te^{-\beta\varphi_t}\right]-\sum_{i=k+1}^{K}R_in_i\varphi_\tau\frac{\alpha D^{\alpha-1}}{1-D^{\alpha}}e^{-\beta\varphi_\tau}=0, \quad (9)$$

$$\frac{\partial\ell}{\partial\lambda}=\frac{N_2}{\lambda}-\beta\sum_{i=k+1}^{K}\sum_{j=1}^{n_i}(t-\tau_k)\frac{1-\alpha e^{-\beta\varphi_t}}{C}-\alpha\beta\sum_{i=k+1}^{K}R_in_i(\tau_i-\tau_k)\frac{D^{\alpha-1}}{1-D^{\alpha}}e^{-\beta\varphi_\tau}=0, \quad (10)$$

Where,

$A=1-e^{-\beta t}$, $B=1-e^{-\beta\tau_i}$, $C=1-e^{-\beta\phi_t}$, $D=1-e^{-\beta\phi_\tau}$.

Equations (8), (9) and (10) are non-linear equations as these are functions of population parameters, which are themselves functions of the solutions of these equations. Due to this difficulty, it is not possible to find exact solution and in order to obtain the MLEs of α, β, and λ, their solutions will be obtained numerically by using Newton Raphson method.

### 3.2 Interval estimates

We know, the asymptotic variance-covariance matrix of α, β, and λ is obtained by inverting the fisher information matrix, $I=\left[-\frac{\partial^2 In\,L}{\partial\omega_i\partial\omega_j}\right], i,j=1,2,3;$ where $\omega_1=\alpha,\omega_2=\beta,\omega_3=\lambda.$ The elements of fisher information are given by;

$$\frac{\partial^2\ell}{\partial\alpha^2}=-\frac{N}{\alpha^2}-\sum_{i=1}^{k}R_in_iB^{\alpha}\left(\frac{In\,B}{1-B^{\alpha}}\right)^2-\sum_{i=k+1}^{K}R_in_iD^{\alpha}\left(\frac{In\,D}{1-D^{\alpha}}\right)^2, \quad (11)$$

$$\frac{\partial^2\ell}{\partial\beta^2}=-\frac{N}{\beta^2}-\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(\frac{t}{A}\right)^2(\alpha-1)e^{-\beta t}-\sum_{i=1}^{k}R_in_i\tau_i^2\alpha B^{\alpha-1}e^{-\beta\tau_i}\left\{\frac{B^{\alpha}-1+B^{-1}e^{-\beta\tau_i}(B^{\alpha}-1+\alpha)}{(1-B^{\alpha})^2}\right\}-$$
$$\sum_{i=k+1}^{K}\sum_{j=1}^{n_i}\left(\frac{\phi_t}{C}\right)^2(\alpha-1)e^{-\beta\phi_t}-\sum_{i=k+1}^{K}R_in_i\phi_\tau^2\alpha D^{\alpha-1}e^{-\beta\phi_\tau}\left\{\frac{D^{\alpha}-1+D^{-1}e^{-\beta\phi_\tau}(D^{\alpha}-1+\alpha)}{(1-D^{\alpha})^2}\right\}, \quad (12)$$

$$\frac{\partial^2\ell}{\partial\lambda^2}=-\frac{N_2}{\lambda^2}-\beta\sum_{i=k+1}^{K}\sum_{j=1}^{n_i}\beta(t-\tau_k)^2e^{-\beta\phi_t}\frac{(\alpha-1)}{C^2}-$$
$$\alpha\beta^2\sum_{i=k+1}^{K}R_in_i(\tau_i-\tau_k)^2D^{\alpha-1}e^{-\beta\phi_\tau}\left\{\frac{e^{-\beta\phi_\tau}(\alpha+D^{\alpha}-1)-(D+D^{\alpha+1})}{(1-D^{\alpha})^2}\right\}, \quad (13)$$

$$\frac{\partial^2\ell}{\partial\alpha\partial\beta}=\frac{\partial^2 l}{\partial\beta\partial\alpha}=\sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{te^{-\beta t}}{A}-\sum_{i=1}^{k}\tau_iR_in_iB^{\alpha-1}e^{-\beta\tau_i}\left\{\frac{1+\alpha InB-B^{\alpha}}{(1-B^{\alpha})^2}\right\}+$$
$$\sum_{i=k+1}^{K}\sum_{j=1}^{n_i}\frac{\phi_te^{-\beta\phi_t}}{C}-\sum_{i=k+1}^{K}\phi_\tau R_in_iD^{\alpha-1}e^{-\beta\phi_\tau}\left\{\frac{1+\alpha InD-D^{\alpha}}{(1-D^{\alpha})^2}\right\}, \quad (14)$$

$$\frac{\partial^2\ell}{\partial\alpha\partial\lambda}=\frac{\partial^2\ell}{\partial\lambda\partial\alpha}=\sum_{i=k+1}^{K}\sum_{j=1}^{n_i}\beta(t-\tau_k)\frac{e^{-\beta\phi_t}}{C}-\sum_{i=k+1}^{K}\beta R_in_iD^{\alpha-1}(\tau_i-\tau_k)e^{-\beta\phi_\tau}\left\{\frac{1+\alpha InD-D^{\alpha}}{(1-D^{\alpha})^2}\right\}, \quad (15)$$

$$\frac{\partial^2\ell}{\partial\beta\partial\lambda}=\frac{\partial^2\ell}{\partial\lambda\partial\beta}=\sum_{i=k+1}^{K}\sum_{j=1}^{n_i}(t-\tau_k)\left\{1-(\alpha-1)\frac{e^{-\beta\phi_t}}{C^2}\right\}-$$
$$\sum_{i=k+1}^{K}\alpha R_in_iD^{\alpha-1}(\tau_i-\tau_k)e^{-\beta\phi_\tau}\left[\frac{(1-D^{\alpha})\left\{1+\beta\phi_\tau(\alpha-1)e^{-\beta\phi_\tau}-\beta\phi_\tau\right\}+\alpha\beta\phi_\tau D^{\alpha-1}e^{-\beta\phi_\tau}}{(1-D^{\alpha})^2}\right], \quad (16)$$

therefore, we have the approximate 100(1-υ)% confidence intervals for α, β and λ as

$$\hat{\alpha}\pm Z_{\upsilon/2}\sqrt{\mathrm{var}(\hat{\alpha})},\ \hat{\beta}\pm Z_{\upsilon/2}\sqrt{\mathrm{var}(\hat{\beta})},$$
$$\hat{\lambda}\pm Z_{\upsilon/2}\sqrt{\mathrm{var}(\hat{\lambda})}, \quad (18)$$

where $Z_{\upsilon/2}$ is the $100(1-\upsilon)\%$ percentile of a standard normal variate.

## 4 Simulation Studies

Since the analytical comparison of the estimators in complicated expressions is almost impossible to compute. Therefore, Monte Carlo method of simulation is carried out to compute them.The study is carried out to compute the relative absolute biases (RABs), mean squared errors (MSEs) and 90% approximate confidence intervals (CIs) of the model parameters. Based on 10000 simulations, the results are estimated and reported in tabular form. The simulation study is carried according to the following steps.

1. Generate a random sample of size $n$ from uniform distribution U(0,1) and obtain the order statistics $\left(U_{1:n}, U_{2:n}, \ldots, U_{n:n}\right)$

2. Given the values of parameters $(\alpha, \beta)$, stress change time $\tau_k$, acceleration factor $\lambda$ and censoring time $\tau_K$; we define $n_1^*$ and $n_2^*$ such that

$$U_{n_1^*:n} \le \left(1-e^{-\beta\tau_k}\right)^{\alpha} < U_{n_1^*:n}$$

and $$U_{n_2^*:n-n_1^*} \le \left(1-e^{-\beta\left(\tau_k-\lambda(\tau_K-\tau_k)\right)}\right)^{\alpha} < U_{n_2^*+1:n-n_1^*}$$

3. From step 2, the ordered observations $t_{1:n}^* < \ldots < t_{n_1^*:n}^* \le \tau_k < t_{n_1^*+1:n}^* < \ldots < t_{n_1^*+n_2^*:n}^* \le \tau_K$ are calculated as follows

$$t_{i:n}^* = \begin{cases} -\beta^{-1} In\left(1-U_{i:n}^{1/\alpha}\right), & 1 \le i \le n_1^*, \\ \tau_k + \left(\tau_k + \beta^{-1} In\left(1-U_{i::n}^{1/\alpha}\right)\right)/\lambda, & n_1^*+1 \le i \le n_1^*+n_2^*. \end{cases}$$

   Where the ordered observations $t_{i:n}^*, i=1,\ldots,n_1^*+n_2^*$ represent the type-I censored sample generated from the exponentiated exponential distribution under PALT.

4. For given values of $k \,\&\, K$, apply the progressive type-I censoring scheme to the observations generated in step 3 to obtain the observations given in expression (3), where

$$n_1^* = \sum_{i=1}^{k} n_i + R_i \quad \text{and} \quad n_2^* = \sum_{i=1}^{K} n_i + R_i \,.$$

5. Finally, we consider the following four progressive censoring schemes and for each setting, the bias and MSEs based on 10000 simulations are estimated and reported in tabular form.

Scheme1:

$$R_1 = R_2 = \ldots = R_{K-1} = 0 \;\&\; R_K = n - \sum_{I=1}^{K} n_i;$$

Scheme 2:

$$R_1 = R_2 = \ldots = R_k = 0 \;, R_{k+1} = \ldots = R_{K-1} = 1$$
$$\&\; R_K = n - \sum_{I=1}^{K} n_i - (K-k-1);$$

Scheme 3:

$$R_1 = R_2 = \ldots = R_k = 1 \;, R_{k+1} = \ldots = R_{K-1} = 0$$
$$\&\; R_K = n - \sum_{I=1}^{K} n_i - k;$$

Scheme 4:

$$R_1 = R_2 = \ldots = R_{K-1} = 1 \;\&\; R_K = n - \sum_{I=1}^{K} n_i - K + 1;$$

**Table 1:** Mean values of MLEs with Bias and MSEs when $\alpha, \beta, \lambda, \tau_k \,\&\, \tau_K$ are set at 0.60, 0.70, 1.1, 4 & 6 respectively.

| n | schemes | Estimates of α | | | Estimates of β | | | Estimates of λ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | MLE | Bias | MSE | MLE | Bias | MSE | MLE | Bias | MSE |
| 20 | 1 | 0.732 | 0.247 | 0.213 | 0.556 | 0.238 | 0.258 | 1.217 | 0.230 | 0.222 |
| | 2 | 0.710 | 0.350 | 0.310 | 0.769 | 0.332 | 0.366 | 1.049 | 0.297 | 0.312 |
| | 3 | 0.478 | 0.298 | 0.276 | 0.766 | 0.256 | 0.282 | 1.003 | 0.255 | 0.240 |
| | 4 | 0.510 | 0.376 | 0.389 | 0.567 | 0.385 | 0.393 | 1.302 | 0.317 | 0.359 |
| 30 | 1 | 0.512 | 0.235 | 0.191 | 0.645 | 0.196 | 0.206 | 1.300 | 0.204 | 0.183 |
| | 2 | 0.671 | 0.339 | 0.254 | 0.655 | 0.251 | 0.298 | 1.002 | 0.241 | 0.269 |
| | 3 | 0.690 | 0.287 | 0.212 | 0.743 | 0.205 | 0.246 | 1.045 | 0.210 | 0.209 |
| | 4 | 0.571 | 0.367 | 0.286 | 0.768 | 0.295 | 0.309 | 1.198 | 0.276 | 0.281 |
| 50 | 1 | 0.676 | 0.210 | 0.143 | 0.765 | 0.131 | 0.154 | 1.202 | 0.141 | 0.138 |
| | 2 | 0.734 | 0.329 | 0.219 | 0.729 | 0.196 | 0.240 | 1.038 | 0.208 | 0.191 |
| | 3 | 0.751 | 0.256 | 0.163 | 0.654 | 0.145 | 0.179 | 1.067 | 0.154 | 0.146 |
| | 4 | 0.665 | 0.356 | 0.219 | 0.681 | 0.220 | 0.266 | 1.042 | 0.217 | 0.212 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 75 | 1 | 0.576 | 0.150 | 0.121 | 0.739 | 0.108 | 0.130 | 1.008 | 0.107 | 0.101 |
| | 2 | 0.678 | 0.259 | 0.184 | 0.710 | 0.168 | 0.169 | 1.054 | 0.159 | 0.147 |
| | 3 | 0.545 | 0.162 | 0.139 | 0.675 | 0.124 | 0.136 | 1.189 | 0.140 | 0.130 |
| | 4 | 0.551 | 0.280 | 0.211 | 0.755 | 0.188 | 0.191 | 1.187 | 0.192 | 0.186 |
| 100 | 1 | 0.663 | 0.105 | 0.095 | 0.755 | 0.089 | 0.090 | 1.098 | 0.067 | 0.067 |
| | 2 | 0.677 | 0.159 | 0.121 | 0.776 | 0.138 | 0.124 | 1.056 | 0.114 | 0.111 |
| | 3 | 0.566 | 0.116 | 0.108 | 0.664 | 0.110 | 0.107 | 1.088 | 0.101 | 0.096 |
| | 4 | 0.619 | 0.161 | 0.136 | 0.881 | 0.153 | 0.150 | 1.041 | 0.155 | 0.126 |
| 150 | 1 | 0.620 | 0.049 | 0.043 | 0.641 | 0.051 | 0.037 | 1.123 | 0.031 | 0.032 |
| | 2 | 0.611 | 0.093 | 0.089 | 0.702 | 0.090 | 0.081 | 1.109 | 0.098 | 0.060 |
| | 3 | 0.678 | 0.061 | 0.059 | 0.711 | 0.067 | 0.052 | 1.130 | 0.062 | 0.043 |
| | 4 | 0.623 | 0.091 | 0.101 | 0.692 | 0.104 | 0.096 | 1.089 | 0.095 | 0.078 |

**Table 2:** Mean values of MLEs with Bias and MSEs when $\alpha, \beta, \lambda, \tau_k$ & $\tau_K$ are set at 0.65, 0.75, 1.2, 5 & 7 respectively.

| n | Schemes | Estimates of α | | | Estimates of β | | | Estimates of λ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | MLE | Bias | MSE | MLE | Bias | MSE | MLE | Bias | MSE |
| 20 | 1 | 0.682 | 0.192 | 0.266 | 0.812 | 0.227 | 0.267 | 1.341 | 0.341 | 0.311 |
| | 2 | 0.704 | 0.228 | 0.302 | 0.802 | 0.285 | 0.293 | 1.447 | 0.387 | 0.404 |
| | 3 | 0.635 | 0.206 | 0.280 | 0.789 | 0.248 | 0.273 | 1.130 | 0.344 | 0.332 |
| | 4 | 0.592 | 0.249 | 0.321 | 0.723 | 0.311 | 0.311 | 1.436 | 0.441 | 0.456 |
| 30 | 1 | 0.719 | 0.138 | 0.201 | 0.799 | 0.202 | 0.220 | 1.470 | 0.271 | 0.259 |
| | 2 | 0.549 | 0.205 | 0.254 | 0.763 | 0.248 | 0.249 | 1.092 | 0.361 | 0.346 |
| | 3 | 0.593 | 0.144 | 0.223 | 0.804 | 0.220 | 0.227 | 1.278 | 0.261 | 0.267 |
| | 4 | 0.599 | 0.235 | 0.290 | 0.693 | 0.283 | 0.280 | 1.042 | 0.341 | 0.376 |
| 50 | 1 | 0.704 | 0.112 | 0.145 | 0.732 | 0.160 | 0.139 | 1.138 | 0.141 | 0.178 |
| | 2 | 0.642 | 0.154 | 0.198 | 0.697 | 0.197 | 0.178 | 1.421 | 0.233 | 0.267 |
| | 3 | 0.678 | 0.128 | 0.167 | 0.770 | 0.173 | 0.144 | 1.155 | 0.245 | 0.188 |
| | 4 | 0.632 | 0.181 | 0.211 | 0.741 | 0.217 | 0.211 | 1.119 | 0.285 | 0.284 |
| 75 | 1 | 0.687 | 0.102 | 0.151 | 0.780 | 0.117 | 0.119 | 1.289 | 0.145 | 0.123 |
| | 2 | 0.645 | 0.128 | 0.182 | 0.712 | 0.134 | 0.151 | 1.067 | 0.210 | 0.171 |
| | 3 | 0.617 | 0.116 | 0.167 | 0.779 | 0.125 | 0.137 | 1.448 | 0.197 | 0.131 |
| | 4 | 0.621 | 0.161 | 0.197 | 0.741 | 0.178 | 0.188 | 1.022 | 0.231 | 0.200 |
| 100 | 1 | 0.635 | 0.081 | 0.108 | 0.756 | 0.089 | 0.061 | 1.289 | 0.087 | 0.076 |
| | 2 | 0.648 | 0.105 | 0.135 | 0.782 | 0.118 | 0.119 | 1.147 | 0.166 | 0.105 |
| | 3 | 0.607 | 0.090 | 0.116 | 0.781 | 0.107 | 0.078 | 1.110 | 0.094 | 0.088 |
| | 4 | 0.672 | 0.126 | 0.133 | 0.736 | 0.134 | 0.144 | 1.301 | 0.187 | 0.122 |
| 150 | 1 | 0.633 | 0.033 | 0.061 | 0.746 | 0.019 | 0.040 | 1.211 | 0.044 | 0.032 |
| | 2 | 0.662 | 0.073 | 0.087 | 0.743 | 0.069 | 0.067 | 1.289 | 0.065 | 0.051 |
| | 3 | 0.659 | 0.047 | 0.063 | 0.758 | 0.040 | 0.052 | 1.167 | 0.051 | 0.044 |
| | 4 | 0.636 | 0.086 | 0.101 | 0.746 | 0.078 | 0.088 | 1.198 | 0.099 | 0.065 |

## 5 Concluding Remarks and Further Studies

In this study, we considered the likelihood estimation of exponentiated exponential distribution parameters and acceleration factor under step stress partially accelerated life testing plan using progressive type-I censoring. Using Newton-Raphson method, we obtain the numerical values of MLEs of model parameters. Their performances are analyzed and discussed in terms of bias and MSE. It has seen that, as the sample size increases the biases and MSEs of the estimated parameters decreases. This indicates that the maximum likelihood estimators are consistent and asymptotically normally distributed. As a future work, Bayesian inferences under the SSPALT assuming the same censoring proposed in this article will be considered.

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