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Implementation of Grover quantum search algorithm with two transmon qubits via circuit QED

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Abstract: We propose a scheme for implementing Grover quantum search algorithm based on the qubit-qubit interaction in circuit QED. We show how to implement the proposed quantum search algorithm using two-transmon-qubit in a circuit QED driven by a strong microwave field. The implementation time is a time of nanosecond-scale, which is much smaller than decoherence time and dephasing time, both being the time of microsecond-scale. Numerical simulation under the influence of the Grover algorithm shows that our proposal is realizable with high fidelity. We propose a detailed procedure and experimentally analyze its feasibility. Moreover, the scheme might be experimentally achieved efficiently with presently available techniques.

Keywords: Grover algorithm, transmon qubit, circuit QED, qubit-qubit interaction.

1 Introduction

Quantum computing [1] utilizes quantum coherence [2] and quantum entanglement [3] to solve some problems much faster than on classical turing machines, such as factoring problem [4], search problem [5], phase estimation problem [6] and so on. The quantum algorithms solve problems of significance much more efficiently than their classical counterparts. Among important quantum algorithms, there exist Deutsch-Jozsa algorithm [7], Shor algorithm [8] and Grover search algorithm [9,10]. Grover's quantum search algorithm is an important development in quantum computation. It propose a quantum mechanical algorithm for searching a marked state in an unordered database and it achieved a quadratic speed up over classical search algorithms. In the search algorithm, multiple items are simultaneously examined using a superposition of the corresponding states. The search for the marked item requires only $O(\sqrt{N})$ inquires. Most important, Grover quantum search did not depend for the impact on the unproven difficulty of the factorization problem.

Recently, Dewes [11] operated a superconducting quantum processor consisting of a two transmon qubits coupled by a swapping interaction. With this processor, they implemented the Grover search algorithm among four objects and find that the correct answer is retrieved after a single run with a success probability which is larger than the probability achieved with a classical algorithm. Subsequently, Jiang [12] proposed a simple scheme to implement the two-qubit Grover search algorithm with trapped ions in thermal motion by applying a single standing-wave laser pulse during the two-qubit operation. Similarly, Wang et al. [13] proposed a scheme to implement two-qubit Grover quantum search by using dipole-dipole interaction (DDI) and the atom-cavity interaction (ACI) in cavity Quantum Electrodynamics (QED), driven simultaneously by a strong classical field. In Ref. [14], the authors presented a method to implement the two-qubit Grover search algorithm in trapped ions by applying a single standing-wave laser pulse during the two-qubit operation. In the modified scheme, they need to perform two NOT gates acting on first and second ion for labelling three target states ($|e_1g_2\rangle$, $|g_1e_2\rangle$, and $|g_1g_2\rangle$), and the other target state $|e_1e_2\rangle$ is directly labelled in trapped ions. In the same context, DiCarlo et al. [15] demonstrated a two-qubit superconducting processor and the implementation of the Grover search and Deutsch–Jozsa quantum algorithms. They used a two-qubit interaction,

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tunable in strength by two orders of magnitude on nanosecond timescales, which is mediated by a cavity bus in a circuit quantum electrodynamics architecture. This interaction allows the generation of highly entangled states .Although this processor constitutes an important step in quantum computing with integrated circuits.

In last few years, the circuit quantum electrodynamics has become one of the most promising solid-state candidates for quantum information processing[16]. Moreover , many theoretical proposals and experimental demonstration have been presented for realizing two-qubit gates [11,17] and multiple qubit gates [18] with transmon qubits [19,20] in circuit QED. In this paper, we present a system to implement two-transmon-qubit Grover's algorithm in circuit QED, by using entangling gate J and diffusion transform D with nearest qubit-qubit interaction. The operator J is essentially basis transformations between the classical product state basis and the quantum entangled state basis. We use the system in which the two transmon qubits are capacitively coupled to a superconducting transmission line resonator (TLR) driven by a strong microwave field. On the other hand, the model of the qubits coupling to a single resonator were considered theoretically in Ref. [21] and experimentally in Ref. [22]. The implementation time of this system is much shorter than decoherence time and dephasing time. Our numerical calculation shows that the implementation of this quantum algorithm is feasible in the circuit QED.

The work is organized as follows: In Sec.2, we concretely illustrate the way Grover algorithm via circuit QED with two-transmon-qubit, by using entangling gate J and the diffusion transform D in the case of qubit-qubit interaction. In Sec.3, we study the fidelity and possible experimental of this implementation, we also calculate the implementation time and discuss the result. A concluding summary is given in Sec.4.

2 Implementation of a two-transmon-qubit Grover's algorithm via circuit QED

In this section, we propose a scheme for implementing Grover quantum search algorithm, by using the system of two-transmon qubits based on the qubit-qubit interaction in a circuit QED (see Fig.1). In our scheme, two transmon qubits are capacitively coupled to a superconducting TLR driven by a strong microwave field [17], two capacitively coupled transmon qubits have tunable frequencies controlled by the flux induced in their SQUID loop by a local current line [11]. A microwave field of frequency ω_d is applied to the input wire of the TLR, which can be described below in the Hamiltonian H_D. We consider two transmon qubits each having two-level subspaces driven by a conventional field, this transmon qubits are capacitively coupled to it. The qubit-qubit interaction should be included in circuit QED. The Hamiltonian of the whole system (assuming $\hbar = 1$) [23,24] given by

$$H_s = H_{JC} + H_D + H_{qq} \tag{1}$$

with

$$H_{JC} = \omega_q \sum_{j=1}^{2} \sigma_{z,j} + \omega_r a^+ a + g \sum_{j=1}^{2} (a^+ \sigma_j^- + a \sigma_i^+) \tag{2}$$

$$H_D = \varepsilon(t)(a^+ e^{-i\omega_d t} + a^- e^{i\omega_d t}) \tag{3}$$

$$H_{qq} = \Gamma \sum_{\substack{i,j=1 \\ i \neq j}}^{2} \sigma_i^+ \sigma_j^-, \tag{4}$$

H_{JC} is the resonator plus qubit Hamiltonian that takes the usual Jaynes-Cummings form, H_D is the Hamiltonian of the external driving of the resonator and H_{qq} is the interaction Hamiltonian between qubits. $\sigma_{z,j}$, σ_j^- and σ_j^+ are the collective operators for the two qubits where $\sigma_{z,j} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|), \sigma_j^+ = |e_j\rangle\langle g_j|, \sigma_j^- = |g_j\rangle\langle e_j|$ with $|e_j\rangle(|g_j\rangle)$ is the excited state (ground state) of the transmon qubit. $\omega_r = 1/\sqrt{LC}$ is the resonance frequency of the TLR where the transmission line resonator can be modeled as a simple harmonic oscillator composed of the parallel combination of an inductor L and a capacitor C, ω_q is the transition frequency of the transmon qubit with $\omega_{q_1} = \omega_{q_2}$ (where the qubit definitions are the same which makes the work easier) and ω_d is the frequency of the external drive applied to the TLR. a^+ and a are the creation and annihilation of the resonator mode. g is intensity qubit-TLR coupling, Γ is the force qubit-qubit coupling and $\varepsilon(t)$ is the amplitude of the microwave. In the high E_J/E_C limit the transition frequency between $|g_j\rangle$ and $|e_j\rangle$ is given by $\omega_q = \sqrt{8E_J E_C}/\hbar$ where $E_C = e^2/2C_\Sigma$ and $E_J(\Phi) = E_{J0}|\cos(\pi\Phi/\Phi_0)|$ with $C_\Sigma = C_S + (C_g^{-1} + C_g'^{-1})^{-1}$, here, C_g and C_g' are the gate capacitance, C_S is the additional capacitor and C_Σ is the effective total capacitance. Φ is the external magnetic flux applied to the SQUID loop and $\Phi_0 = h/2e$ is the flux quantum. $E_J(\Phi)$ is the effective Josephson coupling energy, E_C is the charging energy and E_{J0} is the Josephson coupling energy. It is obvious that the frequency of the transmon qubit ω_q can be tuned by external magnetic flux Φ (see Fig. 1).

We work with large amplitude driving fields, in this case, the quantum fluctuations in the drive are very small with respect to the drive amplitude and the drive can be considered for all practical purposes, as a classical field. Here, it is convenient to displace the field operators using the time-dependent displacement operator [25]

$$D(\alpha) = e^{(\alpha a^+ - \alpha^* a)}, \tag{5}$$

Under this transformation, the field a and a^+ goes to $(\alpha + a)$ and $(a^+ + \alpha^*)$, respectively, where α is a complex number representing the classical part of the

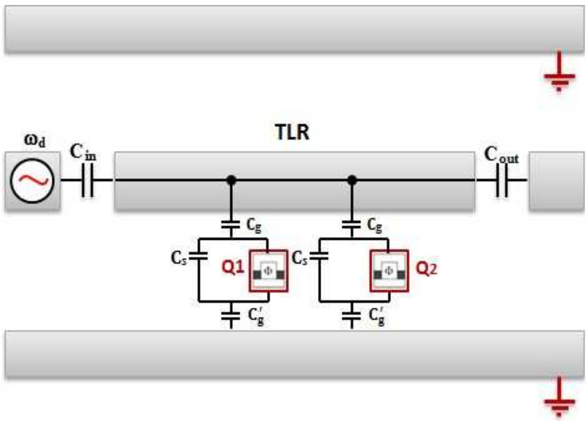

Fig. 1: (Color online) Schematic diagram of a TLR and several two transmon qubits are coupled in a circuit QED (gray) driven by a strong microwave field of frequency ω_d (this field $\sim$ is applied to the input wire of the TLR). The TLR is connected to the input wiring with a capacitor C_{in}, and output wiring with a capacitor C_{out}. The two capacitively coupled transmon qubits have tunable frequencies controlled by the flux induced in their SQUID loop by a local current line.

field. The displaced Hamiltonian reads [26,27]

$$\begin{aligned} H &= D(\alpha)H_s D(\alpha) - iD^{+}(\alpha)\dot{D}(\alpha) \\ &= \omega_q \sum_{j=1}^{2} \sigma_{z,j} + \omega_r a^{+}a + g\sum_{j=1}^{2}(a^{+}\sigma_j^{-} + a\sigma_j^{+}) \\ &\quad - g\sum_{j=1}^{2}(\alpha^{*}\sigma_j^{-} + \alpha\sigma_j^{+}) + \Gamma \sum_{\substack{i,j=1 \\ i\neq j}}^{2} \sigma_i^{+}\sigma_j^{-}, \end{aligned} \tag{6}$$

where we have chosen $\alpha(t)$ to satisfy $\dot{\alpha} = -i\omega_r\alpha - i\varepsilon(t)e^{-i\omega_d t}$. This choice of α is made so as to eliminate the direct drive of microwave field on the TLR, which is described by $Eq.(3)$. In the case where the driving amplitude is independent of time, we get $\alpha = -\frac{\varepsilon(t)}{\omega}e^{-i\omega_d t}$ where $\omega = \omega_r - \omega_d$, then the Hamiltonian H becomes [25,26]

$$\begin{aligned} H &= \omega_q \sum_{j=1}^{2} \sigma_{z,j} + \omega_r a^{+}a + g\sum_{j=1}^{2}(a^{+}\sigma_j^{-} + a\sigma_j^{+}) \\ &\quad + \Omega\sum_{j=1}^{2}(e^{i\omega_d t}\sigma_j^{-} + \alpha e^{-i\omega_d t}\sigma_j^{+}) + \Gamma \sum_{\substack{i,j=1 \\ i\neq j}}^{2} \sigma_i^{+}\sigma_j^{-}, \end{aligned} \tag{7}$$

where $\Omega = g\varepsilon/\omega$. In the rotating wave approximation, when $\Omega \gg \delta$ and $\delta \gg g,\ \Gamma$, the Hamiltonian for the whole system in the interaction picture is (under the assumption that $\omega_d = \omega_q$) [28,29]

$$H_I = H_0 + H_{eff} + H_{Iq} \tag{8}$$

with

$$H_0 = \Omega \sum_{j=1}^{2}(\sigma_j^{+} + \sigma_j^{-}), \tag{9}$$

$$\begin{aligned} H_{eff} &= \frac{1}{2}\lambda \sum_{j=1}^{2}(|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|) + \lambda\left[\sigma_1^{+}\sigma_2^{+} + \sigma_1^{+}\sigma_2^{-}\right. \\ &\quad \left. +H.c.\right], \end{aligned} \tag{10}$$

$$H_{Iq} = \Gamma \sum_{\substack{i,j=1 \\ i\neq j}}^{2}(\sigma_i^{+}\sigma_j^{+} + \sigma_i^{+}\sigma_j^{-} + H.c.). \tag{11}$$

where $\delta = \omega_r - \omega_q$ (δ is the detuning between the transition frequency of each qubit and the frequency of the cavity mode), and $\lambda = \frac{g^2}{2\delta}$. If we define $J_x = \frac{1}{2}\sum_{j=1}^{2} J_{jx}$ with $J_{jx} = \frac{1}{2}\sum_{j=1}^{2}(\sigma_j^{+} + \sigma_j^{-})$, we obtain

$$H_0 = 2\Omega J_x, \tag{12}$$

$$H_{eff} = 2\lambda J_x^2, \tag{13}$$

$$H_{Iq} = 8\Gamma J_{1x}J_{2x}, \tag{14}$$

then we can get the evolution operator $U_I(t)$ as [28]

$$\begin{aligned} U_I(t) &= e^{-iH_0 t}e^{-iH_{eff}t}.e^{-iH_{Iq}} \\ &= e^{-2i\Omega J_x t}e^{-2i\lambda J_x^2 t}.e^{-8i\Gamma J_{1x}J_{2x}t}. \end{aligned} \tag{15}$$

In the subspace spanned by $(|e_1\rangle|e_2\rangle, |e_1\rangle|g_2\rangle, |g_1\rangle|e_2\rangle, |g_1\rangle|g_2\rangle)$, we define the two-qubit Hadamard gate as

$$H^{\otimes 2} = \frac{1}{2}\begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \tag{16}$$

where H_i is the Hadamard gate acting on the i qubit, transforming states as $|g_i\rangle \rightarrow \frac{1}{\sqrt{2}}(|g_i\rangle + |e_i\rangle)$, $|e_i\rangle \rightarrow \frac{1}{\sqrt{2}}(|g_i\rangle - |e_i\rangle)$. Then the evolution operator of the system $U_I(t)$ can be expressed in the same basis as

$$U_I(t) = \frac{1}{2}\begin{bmatrix} Ae^{2i\Gamma t} & -iCe^{-2i\Gamma t} & -iCe^{-2i\Gamma t} & -Be^{2i\Gamma t} \\ -iCe^{-2i\Gamma t} & Ae^{2i\Gamma t} & -Be^{2i\Gamma t} & -iCe^{-2i\Gamma t} \\ -iCe^{-2i\Gamma t} & -Be^{2i\Gamma t} & Ae^{2i\Gamma t} & -iCe^{-2i\Gamma t} \\ -Be^{2i\Gamma t} & -iCe^{-2i\Gamma t} & -iCe^{-2i\Gamma t} & Ae^{2i\Gamma t} \end{bmatrix}, \tag{17}$$

with $A = 1 + cos(2\Omega t)e^{-4i\Gamma t}e^{-i2\lambda t}$, $B = 1 - cos(2\Omega t)e^{-4i\Gamma t}e^{-i2\lambda t}$, and $C = e^{-i2\lambda t}\sin(2\Omega t)$. The entangling gate J is given by [30]

$$J = \begin{bmatrix} cos(\frac{\gamma}{2}) & 0 & 0 & i\sin(\frac{\gamma}{2}) \\ 0 & cos(\frac{\gamma}{2}) & -isin((\frac{\gamma}{2}) & 0 \\ 0 & -isin(\frac{\gamma}{2}) & cos(\frac{\gamma}{2}) & 0 \\ isin\,(\frac{\gamma}{2}) & 0 & 0 & cos(\frac{\gamma}{2}) \end{bmatrix}. \tag{18}$$

If we choose $\Omega t = m\pi$ (m is integer) and $\Gamma t = k\pi$ (k is integer), we can obtain

$$U_I(t) = e^{-i\lambda t}\begin{bmatrix} \cos(\lambda t) & 0 & 0 & -i\sin(\lambda t) \\ 0 & \cos(\lambda t) & -i\sin(\lambda t) & 0 \\ 0 & -i\sin(\lambda t) & \cos(\lambda t) & 0 \\ -i\sin(\lambda t) & 0 & 0 & \cos(\lambda t) \end{bmatrix}, \quad (19)$$

So the entangling gate J can be easily realized in circuit QED. On the other hand, the diffusion transform D is given by [10]

$$D = -I + 2|\Psi_0\rangle\langle\Psi_0| = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}, \quad (20)$$

where the average state $|\Psi_0\rangle = \frac{1}{2}(|e_1\rangle|e_2\rangle + |e_1\rangle|g_2\rangle + |g_1\rangle|e_2\rangle + |g_1\rangle|g_2\rangle)$ and I is the 4×4 identity matrix. If we choose $\lambda t = \frac{\pi}{4} + k\pi$ (k is integer) , $\Gamma t = 2k\pi$ (k is integer), and $\Omega t = \frac{\pi}{4} + k\pi$ (k is integer), we can obtain

$$U_I(t) = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = -D. \quad (21)$$

Then, by choosing an appropriate value of Ω, λ and Γ, we can generate a two-qubit diffusion transform D (different by -1 prefactor). The two-qubit conditional phase gate to label different target states will also be generated in a natrural way.

The Grover search algorithm is an efficient quantum algorithm to look for one item in an unsorted data base of size N. While the most efficient classical algorithm which examines items one by one needs on average $\frac{N}{2}$ queries, the Grover's quantum algorithm uses only $O(\sqrt{N})$. Grover quantum search consists of the following steps:

1. Reset all the qubits in the register to 0 and then perform the Hadamard transform on each of them so that the system is initially in an equal superposition state $|\Psi_0\rangle = (\frac{1}{\sqrt{N}})\sum_{i=1}^{N}|i\rangle$.
2. Repeat operations performed (named Grover iteration) $R = \frac{\pi\sqrt{N}}{4}$ times.

Now, we show how we can use the evolution operator $U_I(t)$ $(Eq.(17))$ and the two-qubit Hadamard gate $(Eq.(16))$ to implement the two-transmon-qubit Grover quantum search in circuit QED. On the other hand, the two-qubit conditional phase gate to label different target states will also be generated in a natrural way. It's easy to find

$$H^{\otimes 2}U_I(t)H^{\otimes 2} = \begin{bmatrix} \Lambda e^{2i\lambda t(h-1)} & 0 & 0 & 0 \\ 0 & \Lambda^{-1} & 0 & 0 \\ 0 & 0 & \Lambda^{-1} & 0 \\ 0 & 0 & 0 & \Lambda e^{-2i\lambda t(h+1)} \end{bmatrix}, \quad (22)$$

with $\Lambda = e^{-2i\Gamma t}$ and $h = \frac{\Omega}{\lambda}$. We choose $\Gamma t_1 = k\pi$, $\lambda t_1 = \frac{\pi}{4}$, $h = 4m+1$, $t_1 = \frac{\pi}{4\lambda}$, $\frac{\Omega}{\lambda} = 4m+1$, we have [12]

$$H^{\otimes 2}U_I(t_1)H^{\otimes 2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = I_{g_1 g_2}. \quad (23)$$

Thus, the $I_{g_1 g_2}$ phase operation is obtained and the target state $|g_1\rangle|g_2\rangle$ is labeled. Similarly, target state $|e_{1\rangle}|e_2\rangle$ can be labeled by setting $\Gamma t_2 = k\pi$, $\lambda t_2 = \frac{\pi}{4}$,

$h = 4m+3$, $t_2 = \frac{\pi}{4\lambda}$, $\frac{\Omega}{\lambda} = 4m+3$, we have

$$H^{\otimes 2}U_I(t_2)H^{\otimes 2} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_{e_1 e_2}. \quad (24)$$

The slight modification of the operations $I_{e_1 e_2}$ or $I_{g_1 g_2}$ by the NOT gate $\sigma_{x,2}$ acting on qubit 2, allows us to find the states $|e_1\rangle|e_2\rangle$ or $|g_1\rangle|g_2\rangle$ [14] as

$$\sigma_{x,2}I_{g_1 g_2}\sigma_{x,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_{g_1 e_2}, \quad (25)$$

$$\sigma_{x,2}I_{e_1 e_2}\sigma_{x,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_{e_1 g_2}. \quad (26)$$

In this way, we can implement the operations I_α easily up to a global phase $(|\alpha\rangle = |e_1\rangle|e_2\rangle, |e_1\rangle|g_2\rangle, |g_1\rangle|e_2\rangle, |g_1\rangle|g_2\rangle$, respectively). Therefore, given a specific λ, by choosing an appropriate Ω, intensity qubit-TLR coupling g, and the qubit-qubit coupling strength Γ, we can generate all the two-transmon-qubit operations necessary in the two-transmon-qubit Grover search algorithm.

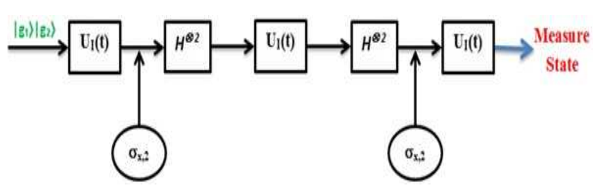

Fig. 2: Quantum circuit for the implemented two-qubit Grover quantum search algorithm. Here, $H^{\otimes 2}$ represents a two-qubit Hadamard gate on an input query transmon qubit, $\sigma_{x,2}$ is the NOT gate and $U_I(t)$ described by Eq.(17).

To carry out our scheme, in a circuit QED, two transmon qubits are prepared in the state $|g_1\rangle|g_2\rangle$, then the entangling gate J acts on it. The first operation $U_I(t)$ is for the entangling gate J, after $H^{\otimes 2}$ operation, are in the initial average state. Then, they undergo the operations in Fig.2 from the left to the right. For searching $|g_1\rangle|g_2\rangle$ or $|e_1\rangle|e_2\rangle$, our implementation is straightforward because the qubits interact with the cavity and the classical field simultaneously. While to search $|e_1\rangle|g_2\rangle$ or $|g_1\rangle|e_2\rangle$, since NOT gates are only performed on qubit 2. The next operation $U_I(t)$ is for the diffusion transform D(see Fig.2).

3 Fidelity and discussion

Let us now study the fidelity of the system for finding the target state. In order to check the validity of our scheme. We assume that the circuit QED is initially in a Fock state $|n\rangle$, the fidelity of implementing the conditional phase operation is given by [31,32]

$$F = |\langle \Psi(t)|U(t)|g_1g_2\rangle|^2. \qquad (27)$$

Where $|\Psi(t)\rangle = (\alpha_1|g_1g_2\rangle+\alpha_2|g_1e_2\rangle+\alpha_3|e_1g_2\rangle+\alpha_4|e_1e_2\rangle) \otimes \beta_n|n\rangle$ represents the final state the whole system after the gate operations that the initial state $|g_1g_2\rangle$ followed by an ideal phase operation, and $U(t)$ describers the overall evolution operator of the system are performed in a real situation. Our numerical calculation shows the influence of the photon number operations on the fidelity, where this fidelity decreases with the increase of the photon number in the case of $\Gamma = 1/\delta$ (see Fig. 3).

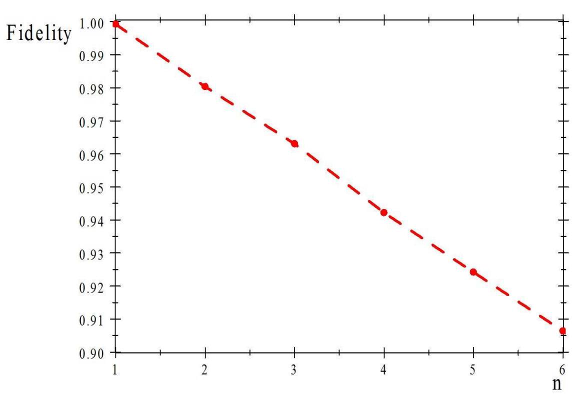

Fig. 3: Numerical results represent the fidelity for different initial circuit Fock state $|n\rangle$ of the two-qubit Grover quantum search, in which $g = 2\pi \times 200MHz$, $\delta = 20g$, $\Omega = 81\times\lambda$ and $\Gamma = 1/\delta$.

We also numerically simulate the relationship between the fidelity of the system for finding the searched state and the force qubit-qubit coupling Γ. Even for $n = 5$, the fidelity $F > 90\%$. However, with the increase of the force qubit-qubit coupling Γ, we obtain a high fidelity (see Fig. 4).

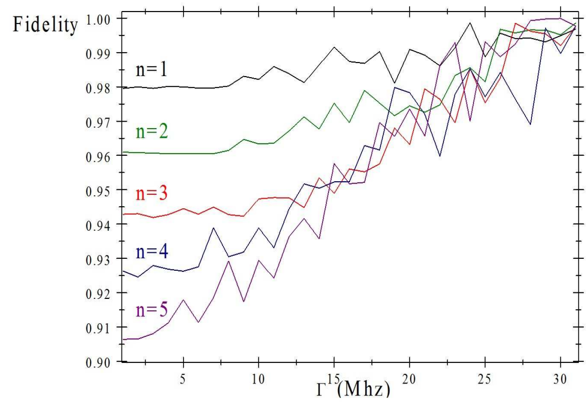

Fig. 4: Numerical results for the fidelity of the The fidelity of the two-qubit Grover quantum search versus the force qubit-qubit coupling Γ with the increase of the photon number n. The parameters used in the numerical calculation are $g = 2\pi \times 200MHz$, $\delta = 20g$ and $\Omega = 81\times\lambda$.

Now, we briefly discuss experimental feasibility of the current scheme. Compared with the usual charge qubit, the transmon qubit is immune to the $1/f$ charge noise and it has much longer dephasing time as a result of the transmon qubits chosen in the system. In recent experiments, it was showen that the decoherence times T_1 and dephasing time T_2 can be made to be on the order of $20-100\ \mu s$ for transmon qubits (when $E_J/E_C = 50$) [33, 34]. In addition, the coupling strength is $g = 2\pi \times 200MHz$ [18], $\delta = 10g$ ($\delta \gg g$, Γ), where $\Gamma = (2k+1)/\delta$ ($k = 0,1,...,n$) [13] and $\Omega \approx 10\delta$ ($\Omega \gg \delta$). The implementation time is $T_{imp} = 5\pi/g$. So, the direct calculation shows that the operation time required to implement the Grover's algorithm with two-transmon-qubit will be $T_{imp} = 12.5ns$, which is much shorter than decoherence times T_1 and dephasing time T_2, this being a strong indication of the feasibility of our proposed protocol. So our proposal is realizable with presently available circuit QED techniques. Furthermore, it should be pointed out that the Rabi frequency Ω during the two-qubit gate is about $2\pi \times 20GHz$ and should be slightly adjusted to satisfy the condition $\frac{\Omega}{\lambda} = 4m+1$ or $\frac{\Omega}{\lambda} = 4m+3$ mentioned above. Our scheme may have potential applications in multipartite entanglement. The charge fluctuations are principal only in low-frequency region and can be reduced by the echo technique [35] and by controlling the gate voltage to the degeneracy point, but an effective technique for suppressing charge fluctuations and keep the state coherent for a longer time is highly desired.

Next, we generalize our scheme to the case of multi-qubits. It is well known that a one-qubit unitary gate and a two-qubit conditional phase gate are universal for building quantum computers. Thus, the multi-qubit conditional phase operation required for implementing Grover quantum search in our scheme can be realized by combining the two-qubit conditional phase operation described by $Eq.((23), (24), (25)$ and $(26))$ and the one-bit unitary operation that is easily realized by using classical microwave field and qubit-resonator interaction. For example, the three-qubit conditional phase operation is:

$$I_{g_1g_2g_3} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (28)$$

Furthermore, the two-qubit controlled-NOT operation can be obtained by combining the two-qubit conditional phase operation ($Eq.((23), (24), (25)$ and $(26)))$ with two single-qubit H transforms, which are performed on the target qubit before and after the two-qubit conditional phase operation ($Eq.((23), (24), (25)$ and $(26)))$, respectively (see Fig. 5 *b*). Similarly, the multi-qubit conditional phase operation can be realized by using the same method. Then, the multi-qubit Grover quantum search can be implemented successfully.

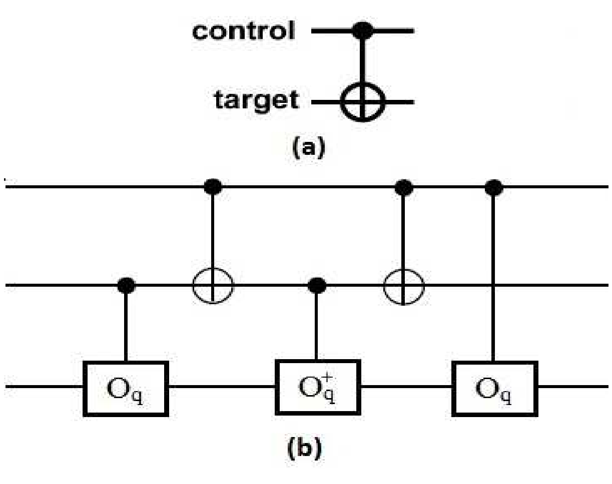

Fig. 5: (a) The symbol of a controlled-NOT gate. **(b)** Proposed quantum circuit for implementing three-qubit conditional phase operation. Here, O_q is a one-qubit unitary operation.

4 Conclusion

In conclusion, we have proposed a simple scheme for implementing two-qubit Grover search algorithm in circuit QED by introducing the qubit-qubit interaction with using the entangling gate J and the diffusion transform D. We have presented a method to implement the proposed quantum search algorithm with two-transmon-qubit capacitively coupled to a superconducting in circuit QED driven by a strong microwave field with Rabi frequency $\Omega = 2\pi \times 400MHz$.The two capacitively coupled transmon qubits have tunable frequencies controlled by the flux induced in their SQUID loop by a local current line. First, the tow-transmon-qubit prepared in the initial average state $|g_1g_2\rangle$. Subsequently, the two-qubit undergo the operations in Fig.2 from the left to the right, the implementation is straightforward. Then, after the three operations $H^{\otimes 2}$, $U_I(t)$, $H^{\otimes 2}$ we obtain $|g_1\rangle|g_2\rangle$ or $|e_1\rangle|e_2\rangle$, while to search $|e_1\rangle|g_2\rangle$ or $|g_1\rangle|e_2\rangle$ since two NOT gates $\sigma_{x,2}$ are only performed on qubit 2. Finally, the final state of the two-qubit will be measured in output of the system. Thus, the implementation time calculated is much shorter than the decoherence time and the dephasing time. In addition, the numerical simulation shows that our implementation is good enough to demonstrate a two-qubit Grover's search with high fidelity. Therefore, the present scheme might be realizable using the presently available techniques, and the experimental implementation of the present scheme would be a important step toward more complex quantum algorithm, serving to show the power of the transmon-qubit system for quantum information processing.

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