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Alternative Approach for Quantum Computation in a Cavity QED

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Abstract: We propose an approach to achieve quantum computation with atomics qubits in a cavity QED. We encode a single qubit on a pair of atoms. The qubit is typically encoded by two two-levels atoms with one is in the ground state while the other in the excited state. We propose an universal set of gates including two rotations X and Z on the Bloch sphere of each single qubit and a Controlled NOT gate of each two-qubit.

Keywords: Quantum gate, cavity *QED*

1 Introduction:

Usually the quantum information processing in a cavity QED uses either the atoms or photons as qubits. The problem of storing and manipulating entangled atomic and photon states has recently received much attention in the context of recent proposals for implementing quantum logic gates, such as photons in the polarization degree of freedom (DOF) [1,2,3] and those in both the polarization and the spatial-mode DOFs (the hyper-parallel photonic quantum computing) [4,5,6], nuclear magnetic resonance [7,8,9,10], quantum dots [11,12,13,14,15], diamond nitrogenvacancy center [16,17,18], superconduting qubits [19,20], superconducting resonators (microwave photons) [21,22], and hybrid quantum systems [23,24]. Atomic systems are excellent quantum memories and are more suitable for large scale quantum computation. Whereas photons are robust against decoherence and can be easily transmitted over long distances. We will discuss the setup where atoms are the qubits and photons are used to manipulate the atoms. A single two-level atom coupled to a single cavity mode is one of the simplest quantum systems. Each qubit can be represented as a linear combination of the two atom states (i.e the ground state $|g\rangle$ and the excited state $|e\rangle$). In this work we adapt a different approach by encoding a single qubit on a pair of atoms. If we consider two atoms 1 and 2 with levels ($|g_1\rangle$, $|e_1\rangle$) and ($|g_2\rangle$, $|e_2\rangle$) respectively, the qubit is typically encoded by two two-levels atoms with one is in the level $|g\rangle$ while the other in the level $|e\rangle$. The idea of encoding a single qubit on a pair of atoms is similar to dual-rail qubit representation [34,35] which is encoded by the presence of a single photon in one or the other of two optical cavity modes. The concept of using the représentation $|g_1e_2\rangle$ and $|e_1g_2\rangle$ is not new [36,37,38], but existing methods for quantum logic do not, to our knowledge, use the all advantageous of this representation.

2 Universal Set of Logic Gates:

If we let two atoms 1 and 2 simultaneously interacting with a single mode in a cavity QED and if we assume the atoms are intially in the states $|g_1e_2\rangle$ or $|e_1g_2\rangle$ (see figure1). Then the state $|g_1e_2\rangle$ encode the qubit state $|-\rangle$ and $|e_1g_2\rangle$ encode the qubit state $|+\rangle$ (see table.1):

For a universal set of gates, we need a multi-qubit-entangling gate and two rotations on the Bloch sphere of each single qubit [25]. We choose the rotations about x and z axes for a single qubit and a CNOT gate for an entangling multi-qubit gate to achieve the universal set of gates.

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Table 1: The qubit is encoded by two atoms with one is in the level $|g\rangle$ and the other in the level $|e\rangle$.

atoms states		qubit
$\|g_1e_2\rangle$	$\longmapsto$	$\|-\rangle$
$\|e_1g_2\rangle$	$\longmapsto$	$\|+\rangle$

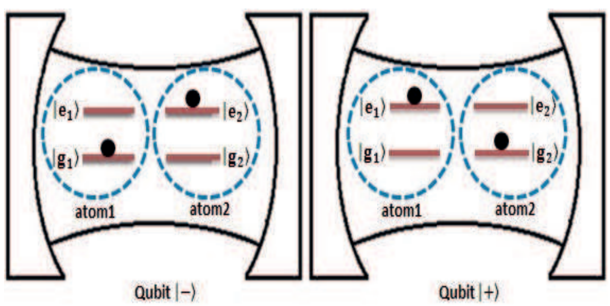

Fig. 1: The alternative representation of qubit which is encoded as a two atoms with one is in the level $|g\rangle$ and the other in the level $|e\rangle$ ($|g_1e_2\rangle \equiv |-\rangle$ and $|e_1g_2\rangle \equiv |+\rangle$).

2.1 Rotation about the x-axis

To construct a rotation about the x-axis, we consider two identical two-level atoms numbered 1 and 2 simultaneously interacting with a single-mode cavity field with frequency ω_a and driven by a classical field with frequency ω. The two atoms are initially in the states $|g_1e_2\rangle$ or $|e_1g_2\rangle$. We will see that the photon-number dependent parts in the effective Hamiltonian are canceled with the assistant of a strong classical driving field.

The general Hamiltonian H for the system, with the dipôle and rotating wave approximations, can be written as ($\hbar = 1$) [26,27,28]:

$$H = \omega_0 \sum_{j=1}^{N} S_{z,j} + \omega_a \hat{a}^+ \hat{a} + \sum_{j=1}^{N} \left[g \left(\hat{a}^+ S_j^- + \hat{a} S_j^+ \right) + \Omega \left(S_j^+ e^{-i\omega t} + S_j^- e^{i\omega t} \right) \right] \quad (1)$$

where N is the number of atoms (here $N = 2$). $\hat{a}$ and $\hat{a}^+$ is the boson operators for the cavity mode, g is the atom-field coupling constant, Ω is the Rabi frequency of the classical field, ω_0 is the frequency for atomic transition and $S_{z,j} = \frac{1}{2}\left(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|\right)$, $S_j^+ = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$.

Assuming $\omega_0 = \omega$ and $\delta = \omega_0 - \omega_a$, we have following Hamiltonian in the interaction picture [26,27,28]

$$H_i = \sum_{j=1}^{N} \left[g \left(e^{-i\delta t} \hat{a}^+ S_j^- + e^{i\delta t} \hat{a} S_j^+ \right) + \Omega \left(S_j^+ + S_j^- \right) \right] \quad (2)$$

When $\Omega \gg \delta, g$ and $\delta \gg g$, we can obtain the evolution operator of the system in the interaction picture [26,27]

$$U_I(t) = e^{-iH_0 t} e^{-iH_e t} \quad (3)$$

with

$$H_0 = \Omega \sum_{j=1}^{N} \left(S_j^+ + S_j^- \right) \quad (4)$$

and

$$H_e = \frac{\lambda}{2} \left[\sum_{j=1}^{N} \left(|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j| \right) + \sum_{j,k=1, i\neq j}^{N} \left(S_j^+ S_k^+ + S_j^+ S_k^- + H.C \right) \right] \quad (5)$$

where $\lambda = \frac{g^2}{2\delta}$. Then, the time evolution of the initial states $|g_1e_2\rangle$ and $|e_1g_2\rangle$ ($N = 2$), can be given as [26]:

$$\begin{aligned}
|g_1e_2\rangle \longmapsto e^{-i\lambda t} \{ &\eta_1 [(\delta_1 |g_1\rangle - i\delta_2 |e_1\rangle)(\delta_1 |e_2\rangle - i\delta_2 |g_2\rangle)] \\
&- i\eta_2 [(\delta_1 |e_1\rangle - i\delta_2 |g_1\rangle)(\delta_1 |g_2\rangle - i\delta_2 |e_2\rangle)] \} \\
|e_1g_2\rangle \longmapsto e^{-i\lambda t} \{ &\eta_1 [(\delta_1 |e_1\rangle - i\delta_2 |g_1\rangle)(\delta_1 |g_2\rangle - i\delta_2 |e_2\rangle)] \\
&- i\eta_2 [(\delta_1 |g_1\rangle - i\delta_2 |e_1\rangle)(\delta_1 |e_2\rangle - i\delta_2 |g_2\rangle)] \} \quad (6)
\end{aligned}$$

with $\delta_1 = \cos(\Omega t)$, $\delta_2 = \sin(\Omega t)$, $\eta_1 = \cos(\lambda t)$ and $\eta_2 = \sin(\lambda t)$.

The global phase factor $e^{-i\lambda t}$ is omitted in the following equations. By using the definition of qubits ($|g_1e_2\rangle \equiv |-\rangle$ and $|e_1g_2\rangle \equiv |+\rangle$) and by setting $\Omega t = \pi$, one finds that:

$$\begin{aligned}
|-\rangle &\longmapsto \cos(\lambda t)|-\rangle - i\sin(\lambda t)|+\rangle \\
|+\rangle &\longmapsto \cos(\lambda t)|+\rangle - i\sin(\lambda t)|-\rangle \quad (7)
\end{aligned}$$

so that, finally the x-rotation operation becomes:

$$R_x(t) = \cos(\lambda t) I - i\sin(\lambda t)\sigma_x$$

σ_x is the pauli-X gate and I is the identity gate.

2.2 Rotations about the z-axis

we adopt the following scheme to achieve the rotation about z-axis [see figure2]. If we take two different two-levels atoms passing through a cavity QED and simultaneously interacting with a single-mode cavity. We assume that the system {atom1+atom2+cavity mode} is initially in one of the states $|g_1, e_2, 1\rangle$ or $|e_1, g_2, 1\rangle$. We consider a very high detuning between the first atom and the cavity mode, and a large detuning between the second atom and the cavity mode.

We know that for a large detuning, a system with a two level atom interacting with a single mode cavity, remains in its initial state and a phase shift can be produced as [29]:

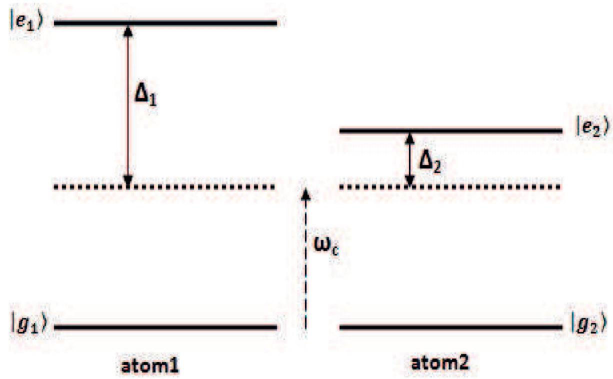

Fig. 2: Representation of two two-levels atoms with a very high detuning Δ_1 for the first atom and a large detuning Δ_2 for the second atom.

$$|g,n\rangle \longmapsto e^{i\Phi(n)}|g,n\rangle$$
$$|e,n\rangle \longmapsto e^{-i\Phi(n+1)}|e,n\rangle \quad (8)$$

with $\Phi(n)$ can be expressed as [29]:

$$\Phi(n) = \frac{\Delta}{2v}\int_0^L dz\left[\sqrt{1+n\left(\frac{g(z)}{\Delta/2}\right)^2}-1\right] \quad (9)$$

where v is the velocity of the atom passing through the cavity, L is the cavity length, n is the number of photons in cavity and $g(z)$ is the coupling constant which in our case is independent of z.

Whereas for a very high detuning the system remains in its initial state. From these considerations, we can say that the first atom state remains unchnaged while for the evolution of the second atom, we will introduce a phase shift as:

$$|g_1,e_2,1\rangle \longmapsto e^{-i\frac{2g_2^2}{\Delta_2}t}|g_1,e_2,1\rangle$$
$$|e_1,g_2,1\rangle \longmapsto e^{i\frac{g_2^2}{\Delta_2}t}|e_1,g_2,1\rangle \quad (10)$$

that we can write as ($|g_1e_2\rangle \equiv |-\rangle$ and $|e_1g_2\rangle \equiv |+\rangle$):

$$|-\rangle \longmapsto e^{-i\frac{g_2^2}{2\Delta_2}t}e^{-i\frac{3g_2^2}{2\Delta_2}t}|-\rangle$$
$$|+\rangle \longmapsto e^{-i\frac{g_2^2}{2\Delta_2}t}e^{i\frac{3g_2^2}{2\Delta_2}t}|+\rangle \quad (11)$$

So, the z-rotation operation can be constructed as (the global phase facor $e^{-i\frac{g_2^2}{2\Delta_2}t}$ is omitted):

$$R_z(t) = \cos(\frac{3g_2^2}{2\Delta_2}t)I - i\sin(\frac{3g_2^2}{2\Delta_2}t)\sigma_z \quad (12)$$

σ_z is the pauli-Z gate.

2.3 Entangling Multi-Qubits Gate

In addition to the one qubit x-rotation and z-rotation gates, we also need to an entangling multi-qubit gate to complete a universal set of gates. The CNOT gate, which is the most obvious condidate for a multi-qubit entangling gate, can be implemented by using the definition of such logical qubits.

We consider four identical two-level atoms numbered 1, 2, 3 and 4 simultaneously interacting with a single mode cavity field with frequency ω_a and driven by a classical field with frequency ω. The four atoms are initially in the states $|g_1e_2g_3e_4\rangle$, $|g_1e_2e_3g_4\rangle$, $|e_1g_2g_3e_4\rangle$, or $|e_1g_2e_3g_4\rangle$. We have the same scheme as that of section 2.1, but with four atoms instead of two. We use the equations (1) to (5) with $N=4$ to develop the evolution operator of eq.(3). We define $S_x = \frac{1}{2}\sum_{j=1}^4\left(S_j^+ + S_j^-\right)$. Then the eqs. (4) and (5) reduce to $H_0 = 2\Omega S_x$ and $H_e = 2\lambda S_x^2$ respectively and the eq. (3) becomes:

$$U_I(t) = e^{-i2\Omega t S_x}e^{-i2\lambda t S_x^2} \quad (13)$$

where $\lambda = \frac{g^2}{2\delta}$.

By considering the matrix representation of S_x, we develop the evolution operator $U_I(t)$ in the space spanned by the basis states $\{|g_1g_2g_3g_4\rangle, |g_1g_2g_3e_4\rangle, |g_1g_2e_3g_4\rangle, |g_1g_2e_3e_4\rangle, |g_1e_2g_3g_4\rangle, |g_1e_2g_3e_4\rangle, |g_1e_2e_3g_4\rangle, |g_1e_2e_3e_4\rangle, |e_1g_2g_3g_4\rangle, |e_1g_2g_3e_4\rangle, |e_1g_2e_3g_4\rangle, |e_1g_2e_3e_4\rangle, |e_1e_2g_3g_4\rangle, |e_1e_2g_3e_4\rangle, |e_1e_2e_3g_4\rangle, |e_1e_2e_3e_4\rangle\}$ as:

$$U_I(t) = \alpha S_x^2 + \beta I + \gamma J + \mu S_x + \nu S_x^{'} \quad (14)$$

with

$$\alpha = \frac{\left(e^{-i8\lambda t}\cos(4\Omega t)-1\right)}{4},$$
$$\beta = \frac{\left(4e^{-i2\lambda t}\cos(2\Omega t) - e^{-i8\lambda t}\cos(4\Omega t)+5\right)}{8},$$
$$\gamma = \frac{\left(-4e^{-i2\lambda t}\cos(2\Omega t) + e^{-i8\lambda t}\cos(4\Omega t)+3\right)}{8},$$
$$\mu = -\frac{i\left(2e^{-i2\lambda t}\sin(2\Omega t) + e^{-i8\lambda t}\sin(4\Omega t)\right)}{4},$$
$$\nu = -\frac{i\left(-2e^{-i2\lambda t}\sin(2\Omega t) + e^{-i8\lambda t}\sin(4\Omega t)\right)}{4}$$

I is the matrix Identité $I = I_1 \otimes I_2 \otimes I_3 \otimes I_4$ et $J = \sigma_{x,1} \otimes \sigma_{x,2} \otimes \sigma_{x,3} \otimes \sigma_{x,4}$.

The matrix representation of S_x and $S_x^{'}$ are defined as:

$$S_x = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix},$$

$$S'_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

We use the logical qubits (i.e $|g_1e_2\rangle \equiv |-_1\rangle$, $|e_1g_2\rangle \equiv |+_1\rangle$, $|g_3e_4\rangle \equiv |-_2\rangle$ and $|e_3g_4\rangle \equiv |+_2\rangle$), then we would be interested in the time evolution of the initial states $|g_1e_2g_3e_4\rangle$, $|g_1e_2e_3g_4\rangle$, $|e_1g_2g_3e_4\rangle$ and $|e_1g_2e_3g_4\rangle$:

$$\begin{aligned} |g_1e_2g_3e_4\rangle \to \frac{1}{2}[&\alpha\,|g_1g_2g_3g_4\rangle + \mu\,|g_1g_2g_3e_4\rangle + \nu\,|g_1g_2e_3g_4\rangle \\ &+\alpha\,|g_1g_2e_3e_4\rangle + \mu\,|g_1e_2g_3g_4\rangle \\ &+2(\alpha+\beta)\,|g_1e_2g_3e_4\rangle + \alpha\,|g_1e_2e_3g_4\rangle \\ &+\mu\,|g_1e_2e_3e_4\rangle + \nu\,|e_1g_2g_3g_4\rangle + \alpha\,|e_1g_2g_3e_4\rangle \\ &+2\gamma|e_1g_2e_3g_4\rangle + \nu\,|e_1g_2e_3e_4\rangle + \alpha\,|e_1e_2g_3g_4\rangle \\ &+\mu\,|e_1e_2g_3e_4\rangle + \nu\,|e_1e_2e_3g_4\rangle + \alpha\,|e_1e_2e_3e_4\rangle] \end{aligned} \tag{15}$$

$$\begin{aligned} |g_1e_2e_3g_4\rangle \to \frac{1}{2}[&\alpha\,|g_1g_2g_3g_4\rangle + \nu\,|g_1g_2g_3e_4\rangle + \mu\,|g_1g_2e_3g_4\rangle \\ &+\alpha\,|g_1g_2e_3e_4\rangle + \mu\,|g_1e_2g_3g_4\rangle + \alpha\,|g_1e_2g_3e_4\rangle \\ &+2(\alpha+\beta)\,|g_1e_2e_3g_4\rangle + \mu\,|g_1e_2e_3e_4\rangle \\ &+\nu\,|e_1g_2g_3g_4\rangle + 2\gamma|e_1g_2g_3e_4\rangle + \alpha\,|e_1g_2e_3g_4\rangle \\ &+\nu\,|e_1g_2e_3e_4\rangle + \alpha\,|e_1e_2g_3g_4\rangle + \nu\,|e_1e_2g_3e_4\rangle \\ &+\mu\,|e_1e_2e_3g_4\rangle + \alpha\,|e_1e_2e_3e_4\rangle] \end{aligned} \tag{16}$$

$$\begin{aligned} |e_1g_2g_3e_4\rangle \to \frac{1}{2}[&\alpha\,|g_1g_2g_3g_4\rangle + \mu\,|g_1g_2g_3e_4\rangle + \nu\,|g_1g_2e_3g_4\rangle \\ &+\alpha\,|g_1g_2e_3e_4\rangle + \nu\,|g_1e_2g_3g_4\rangle + \alpha\,|g_1e_2g_3e_4\rangle \\ &+2\gamma|g_1e_2e_3g_4\rangle + \nu\,|g_1e_2e_3e_4\rangle + \mu\,|e_1g_2g_3g_4\rangle \\ &+2(\alpha+\beta)\,|e_1g_2g_3e_4\rangle + \alpha\,|e_1g_2e_3g_4\rangle \\ &+\mu\,|e_1g_2e_3e_4\rangle + \alpha\,|e_1e_2g_3g_4\rangle + \mu\,|e_1e_2g_3e_4\rangle \\ &+\nu\,|e_1e_2e_3g_4\rangle + \alpha\,|e_1e_2e_3e_4\rangle] \end{aligned} \tag{17}$$

$$\begin{aligned} |e_1g_2e_3g_4\rangle \to \frac{1}{2}[&\alpha\,|g_1g_2g_3g_4\rangle + \nu\,|g_1g_2g_3e_4\rangle + \mu\,|g_1g_2e_3g_4\rangle \\ &+\alpha\,|g_1g_2e_3e_4\rangle + \nu\,|g_1e_2g_3g_4\rangle + 2\gamma|g_1e_2g_3e_4\rangle \\ &+\alpha\,|g_1e_2e_3g_4\rangle + \nu\,|g_1e_2e_3e_4\rangle + \mu\,|e_1g_2g_3g_4\rangle \\ &+\alpha\,|e_1g_2g_3e_4\rangle + 2(\alpha+\beta)\,|e_1g_2e_3g_4\rangle \\ &+\mu\,|e_1g_2e_3e_4\rangle + \alpha\,|e_1e_2g_3g_4\rangle + \nu\,|e_1e_2g_3e_4\rangle \\ &+\mu\,|e_1e_2e_3g_4\rangle + \alpha\,|e_1e_2e_3e_4\rangle] \end{aligned} \tag{18}$$

If we choose:

$$\lambda t = \frac{\pi}{4} \tag{19}$$

and

$$\Omega t = (2k+1)\frac{\pi}{2} \tag{20}$$

we can get $\alpha = 0$, $\beta = \frac{1+i}{2}$, $\gamma = \frac{1-i}{2}$, $\mu = 0$, $\nu = 0$ and the time evolution of the initial states of the system becomes:

$$\begin{cases} |g_1e_2g_3e_4\rangle \to \beta\,|g_1e_2g_3e_4\rangle + \gamma|e_1g_2e_3g_4\rangle \\ |g_1e_2e_3g_4\rangle \to \beta\,|g_1e_2e_3g_4\rangle + \gamma|e_1g_2g_3e_4\rangle \\ |e_1g_2g_3e_4\rangle \to \gamma|g_1e_2e_3g_4\rangle + \beta\,|e_1g_2g_3e_4\rangle \\ |e_1g_2e_3g_4\rangle \to \gamma|g_1e_2g_3e_4\rangle + \beta\,|e_1g_2e_3g_4\rangle \end{cases} \tag{21}$$

We represent $U_I(t)$ in the subspace spanned by the basis states of the previous wavevector $\{|-_1-_2\rangle, |-_1+_2\rangle, |+_1-_2\rangle, |+_1+_2\rangle\}$, one finds:

$$U_I^{'}(t) = \begin{array}{|c|c|c|c|} \hline \frac{1+i}{2} & 0 & 0 & \frac{1-i}{2} \\ \hline 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ \hline 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ \hline \frac{1-i}{2} & 0 & 0 & \frac{1+i}{2} \\ \hline \end{array} \tag{22}$$

This matrix is equivalent to CNOT up to one-bit operations (see figure.3).

Hence, we can achieve the CNOT gate. We take the interaction time t such that $\lambda t = \frac{\pi}{4}$ and next we can choose the Rabi frequency Ω appropriately to satisfy eq (20).

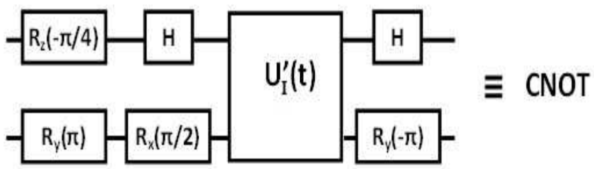

Fig. 3: Representation of CNOT gate where H represents the Hadamard transformation and $(R_x(\theta), R_y(\theta)$ and $R_z(\theta))$ represent respectively a rotation of angle θ around axis (x, y and z).

2.3.1 Discussion:

To investigate the experimental feasibility of this proposal, let us consider Rydberg atoms which interact with a high Q cavity. The photon decay time is about $T_c = 10^{-3}s$ [32] and the coupling constant is $g = 50 \times 2\pi KHz$ [32]. Thus, the interaction time is about $\tau = \frac{\pi}{4\lambda} = \frac{\pi\delta}{2g^2}$. Setting $\delta = 10g$, we have $\tau \simeq 5.10^{-5}s$, which is much smaler than the photon decay time.

-Fidelity: In the derivation of effective Hamiltonian in eq. (5) and by assuming $\Omega \gg \delta, g$, we have neglected the terms oscillating fast [27]:

$$\Delta H(t) = \sum_{j=1}^{4} g \left[e^{-i\delta t} a^+ \left(\frac{1}{2}\sigma_j^+ e^{i\Omega t} - \frac{1}{2}\sigma_j^- e^{-i\Omega t} \right) + H.C \right] \tag{23}$$

In this equation, we use the atomic basis: $|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle)$ and $|-_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle)$ to define $\sigma_j^+$and σ_j^-: $\sigma_j^+ = |+_j\rangle\langle -_j|$ and $|-_j\rangle\langle +_j|$ (Do not confuse with the notation of the logical qubit that we have discussed in this proposal).

These terms induce Stark shifts on the states $|g_j\rangle$ and $|e_j\rangle$ and could reduce the fidelity of the gate. Here we calculate the dependence of fidelity considering these errors. We can also write $\Delta H(t)$ as:

$$\Delta H(t) = \sum_{j=1}^{4} ig \left[\sin(\Omega t) S_z^j - \cos(\Omega t) S_y^j \right] a e^{i\delta t} + H.C \tag{24}$$

With $S_z^j = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$ and $S_y^j = \frac{1}{2i}(|e_j\rangle\langle g_j| - |g_j\rangle\langle e_j|)$. For the S_z^j terms, we can use the spin-echo technique to eliminate the errors. Then we will only study the effect of the S_y^j terms on the gate operation. For these terms, we use the method described in Ref.[33] to study their effect.

Changing to the interaction picture, we may find the propagator $U_I(t)$ from the Dyson series:

$$U_I(t) = 1 - i\int_0^{t'} dt' \Delta H_I(t') - \int_0^{t'}\int_0^{t''} dt' dt'' \Delta H_I(t') \Delta H_I(t'') + \ldots \tag{25}$$

where the interaction Hamiltonian $\Delta H_I(t)$ is given by: $\Delta H_I(t) = U^+(t)\Delta H(t) U(t)$. We can treat $U(t)$ as a constant during the integration because $\Delta H(t)$ is oscillating much faster than the propagator. Then we get:

$$U_I(t) = 1 - i\frac{2g}{\Omega}\sin(\Omega t)\sum_{j=1}^{4} U^+(t) S_y^j U(t) - \frac{g^2}{\Omega^2}\sum_{j,k=1}^{4}(1-\cos(2\Omega t))U^+(t) S_y^j S_y^k U(t) + \ldots \tag{26}$$

Near the time $\tau = \frac{\pi\delta}{2g^2}$ and for the initial state $|g_1e_2g_3e_4\rangle$, $|g_1e_2e_3g_4\rangle$, $|e_1g_2g_3e_4\rangle$ or $|e_1g_2e_3g_4\rangle$, we obtain the fidelity:

$$F \simeq 1 - \frac{2g^2}{\Omega^2}(1-\cos(2\Omega\tau)) \tag{27}$$

Next , we show the plot of fidelity as a function of $\frac{\Omega}{g}$ in figure.4. We find that high fidelity is obtained for $\frac{\Omega}{g} \geq 10$, which is in good agreement with the approximation made by neglecting the terms oscillating fast $(\Omega \gg \delta, g)$

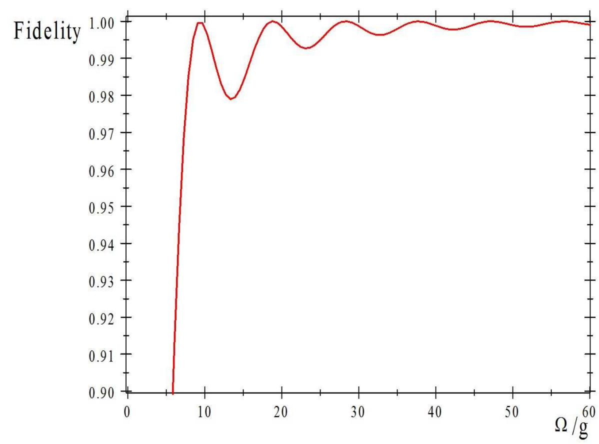

Fig. 4: Fidelity as a function of $\frac{\Omega}{g}$ for the implementation of CNOT gate$(\delta = 10g)$. Here we consider the errors introduced by the Stark Shifts.

It should be noted that the estimation of Fidelity is obtained in the interaction picture.

3 Conclusion

In summary, we have shown that the quantum gates in cavity can be realized when the qubits are encoded by two circular Rydberg atoms with one is in the ground state and the other in the excited state. We have also seen that this logical representation of qubits can realize a universal set of logic gates. In this regard we have implemented two rotations X and Z on the Bloch sphere of each single qubit and a Controlled NOT gate of each two qubit. The system seem promissing for scalability and through all these methods, we believe that more complex gates could be built up. However, the atom-cavity interaction appear to be in the range of practicality and other applications such as the creation of entangled states in cavity resonators may be achieved with these logical qubits.

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