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Lie Group and Numerical Communication of Heat and Mass Transfer Characteristics on MHD Dissipative Free Convection Steady Fluid Flow over an Inclined Permeable Plate under Second Order Chemical Reaction

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Abstract: The current analysis is made to tap the untapped characteristics of steady two-dimensional magneto hydr dynamic, viscous incompressible fluid flow over an inclined permeable surface along heat and mass transfer phenomena. The effect logs regarding free convection is identified by incorporating viscous dissipation term in energy equation. Then, the physical problem is mathematically modelled in terms of coupled non-linear partial differential equations. Lie's scaling group of transformations are attained and utilized to gain non-linear ordinary differential equations. For numerical computations of these equations, we have employed shooting method conjectured with fifth order Runge-Kutta algorithm. Furthermore, the effect logs of involved physical parameters over velocity, temperature and concentration profiles are discussed with the aid of graphs. The present work is certified through numeric values of skin friction, Nusselt and Sherwood numbers by providing comparison with existing literature.

Keywords: Lie group analysis, An inclined permeable plate, MHD flow, Viscous dissipation, Free convection, Second order chemical reaction, Shooting method.

1 Literature Assessment

It is commonly known that most of the physical phenomena that arises in industrial and engineering area can be described by ordinary and partial differential equations. For example, wave propagation and heat flows are identified by way of partial differential equations (PDE's). Further, majority of the physical phenomena regarding fluid dynamics, plasma physics, electricity, quantum mechanics, shallow water waves structures and such type of many other models are controlled around the domain of validity with the aid of partial differential equations.

Therefore, PDE's have a central role and become a mathematical tool to provide complete description of these natural phenomena of industrial and engineering communications. Consequently, it becomes very essential to be familiar with all traditional and recently developed methods to trace out the solution of partial differential equations, and the way of implementation of such type of methods. However, in current analysis, we restricted our investigation to Lie-group analysis. This procedure was derived from the "invariance of a differential equation by a continuous group of symmetries". This important finding not only unified the existing solution techniques but also provided a significant extension to these techniques, which ultimately led to the development of the "theory of continuous groups" known as "Lie groups of transformations" [1].

Such a theory, yields substantial impact on numerous branches of applied and pure mathematics. The most busy integration algorithms for ordinary as well as partial differential equations are unifies by this method and has played a key role in the context of mathematical traits of solution system directed by continuous equations. The equations under boundary layer approximations received especial attention and interest for physical point of view because they can admit a outsized invariant solutions i.e. mainly analytic solutions. Prandtls boundary layer equations admit different many symmetry groups. Symmetry groups or merely symmetries are nothing but invariant transformations which retains the structural form of the concerning physical equations under inspection. The irregular motion of fluid particles generates non-linearity in fluid flow governing partial differential equations and this

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makes hard to give complete solution of undergoing system of equations.

To overcome this, most of the researchers preferred similarity solutions especially in the area of fluid mechanics. Further, in frame of scaling group of transformations, the group-invariant solutions are the well-known similarity solutions [2]. In this communication, we have used scaling group transformations, to trace out the complete set of symmetries for current physical model and then to entertained best one to provide similarity solution. In this regard, numerous attempts has been made to identified physical phenomena like Yurusoy and Pakdemirli [3] acknowledged symmetry reductions for unsteady three dimensional non-Newtonian fluid models. Yurusoy et al. [4] studied the creeping flow for second grade model by way of Lie group analysis.

The symmetry groups and similarity solution are given by Kalpakides and Balassas [5] to insights the characteristics of free convective boundary layer model. Ibrahim et al. [6] used Lie-group analysis to explored the influence of magnetic field and radiative assumption on mass transfer along free convection flow over a semi-infinite flat surface. The effect logs of natural convection in the presence of heat and mass transfer past an inclined surface was studied (through Lie-group analysis) by Sivasankaran et al. [7]. Later on, Lie group analysis for heat and mass characteristics with natural convection along an inclined porous surface by way of heat generation was identified by Sivasankaran et al. [8]. Jalil and Asghar [9] examined the natural convection with heat and mass transfer over an inclined surface. Further, a Lie group analysis of mass transfer properties on mixed convection flow past a stretching surface in the presence of suction/injection was probed by Jalil et al. [10].

Hamad and Ferdows [11] presented the Lie group insights for stagnation point flow adjacent to porous heated stretching plate. They considered nanofluid model along suction/injection and absorption/generation. A Lie group analysis for power law fluid model was given by Jalil and Asghar [12]. The viscous dissipation effects on magnetohydrodynamic boundary layer flow over an inclined surface via Lie group analysis was studied by Reddy [13]. The impact of variable viscosity adjacent to the heated surface by way of Lie group analysis was examined by Hassan et al. [14].

The flow characteristics was identified by Asghar et al. [15] on a stretching rotating disk. A Lie group analysis was utilized in this communication. Afify and Elgazery [16] studied impact of chemical reaction on MHD stagnation point flow towards a heated porous stretching sheet by way of Lie group analysis. Recently, Lie group analysis for effects of second order chemical reaction on MHD free convection dissipative fluid flow adjacent to the an inclined porous sheet was presented by Malik and Rehman [17]. In fact researchers presented different studies to identify the physical influences of involved flow controlling parameters namely, magneto-hydrodynamic flows and axisymmetric flow by utilizing numerical and analytical techniques to mention just a few [18-27].

The above literature assessment reflects that the most of the studies are made by considering lower order chemical reactions. The heat mass characteristics on steady two dimensional magneto-hydrodynamic incompressible free convection dissipative fluid flow adjacent to an inclined permeable sheet with undergoing second order chemical reaction were still not addressed. Therefore, the main purpose of our study is to extend the effort of Reddy et al. [28] by considering additional effect i-e high order chemical reaction. The current computational investigation is the source to understand physical significance of the steady, two dimensional, incompressible, viscous fluid flow (chemically reacting) adjacent to the permeable plate under viscous dissipation effect.

We have established Lie group transformations for our physical model, under which invariant property is admitted by governing partial differential equations and prescribed endpoints conditions. Ultimately similarity variables are found with the aid of relative symmetries for our flow model. So the independent variable is dropped by one and system of ordinary differential equations is created. The solution for this system is obtained by shooting technique along fifth order R-K (Runge-Kutta) algorithm. The conformity and stability of present communication is established by providing comparison with previously published work Reddy et al. [28].

2 Flow Formulation

Consider two dimensional steady boundary layer MHD flow of incompressible electrically conducting viscous dissipating fluid over a semi-infinite acutely inclined permeable plate with the manifestation of chemically reactive species (submit to a second order chemical reaction). The domain for the fluid flow is $\hat{y} > 0$ and it is assumed to be along $\hat{x} - \text{axis}$ (in the direction of permeable plate). Whereas $\hat{y} - \text{axis}$ is considered normal to the $\hat{x} - \text{axis}$. Since in the present analysis, the Reynolds number is less than unity so that we neglected the interruption of an induced magnetic field. In the similar fashion, the Soret and Dufour influences are ignored because foreign mass concentration level is assumed to be lesser. The temperature at the surface is supposed to be greater as compared to ambient fluid.

The flow governing equations (under the usual boundary layer and Boussinesq's approximations) of viscous fluid model can be written as:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0, \tag{1}$$

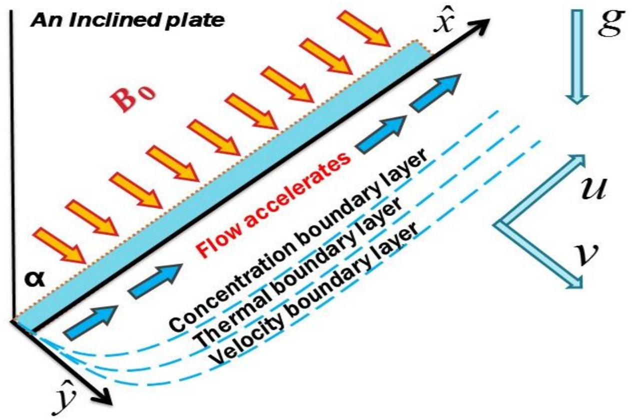

Fig. 1. Schematic diagram for physical flow model.

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{u}}{\partial \hat{y}}=\nu\frac{\partial^2 \hat{u}}{\partial \hat{y}^2}-\frac{\sigma B_0^2}{\rho}\hat{u}-\frac{\nu\hat{u}}{K_p}+(g\beta_T(\hat{T}-\hat{T}_\infty)+g\beta_C(\hat{C}-\hat{C}_\infty))\cos\alpha, \quad (2)$$

$$\hat{u}\frac{\partial \hat{T}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{T}}{\partial \hat{y}}=\frac{\kappa}{\rho c_p}\frac{\partial^2 \hat{T}}{\partial \hat{y}^2}+\frac{\mu}{\rho c_p}(\frac{\partial \hat{u}}{\partial \hat{y}})^2, \quad (3)$$

$$\hat{u}\frac{\partial \hat{C}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{C}}{\partial \hat{y}}=D\frac{\partial^2 \hat{C}}{\partial \hat{y}^2}-K_r(\hat{C}-\hat{C}_\infty)^m, \quad (4)$$

The respective endpoint conditions are prescribed as:

$$\hat{u}=0,\ \ \hat{v}=0,\ \ \hat{T}=\hat{T}_w,\ \ \hat{C}=\hat{C}_w \ \ at \ \ \hat{y}=0,$$

$$\hat{u}\to 0,\ \ \hat{T}\to\hat{T}_\infty,\ \ \hat{C}\to\hat{C}_\infty,\ \ as \ \ \hat{y}\to\infty. \quad (5)$$

Proceeding with analysis, the non-dimensional quantities are given by:

$$u=\frac{\hat{u}}{U_\infty},\ \ v=\frac{\hat{v}}{U_\infty}, x=\frac{\hat{x}U_\infty}{\nu},\ \ y=\frac{\hat{y}U_\infty}{\nu}, K=\frac{K_pU_\infty^3}{\nu^3}, M=\frac{\sigma B_0^2\nu}{U_\infty^3},$$

$$\Pr=\frac{\nu}{\alpha}, Sc=\frac{\nu}{D}, Ec=\frac{U_\infty^2}{c_p(\hat{T}_w-\hat{T}_\infty)},\ \theta=\frac{\hat{T}-\hat{T}_\infty}{\hat{T}_w-\hat{T}_\infty},\ \phi=\frac{\hat{C}-\hat{C}_\infty}{\hat{C}_w-\hat{C}_\infty},$$

$$Gr=\frac{\nu g\beta_T\left(\hat{T}_w-\hat{T}_\infty\right)}{U_\infty^3},\ Gm=\frac{\nu g\beta_C\left(\hat{C}_w-\hat{C}_\infty\right)}{U_\infty^3},\ Cr=\frac{K_r\nu(\hat{C}_w-\hat{C}_\infty)^m}{U_\infty^2(\hat{C}_w-\hat{C}_\infty)}. \quad (6)$$

By utilizing Eq. (6) into Eqs. (1)-(4), we acquired the transformed equations given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (7)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-(M+K^{-1})u+(Gr\theta+Gm\phi)\cos\alpha, \quad (8)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+Ec(\frac{\partial u}{\partial y})^2, \quad (9)$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-Cr\phi^m, \quad (10)$$

the corresponding endpoint conditions are:

$$u=0,\ v=0,\ \theta=1,\ \phi=1\ at\ \ y=0,$$

$$u\to 0,\ \theta\to 0,\ \phi\to 0\ \ as\ \ \ y\to\infty. \quad (11)$$

The stream function $\tilde{\Psi}$ is prescribed as:

$$u=\frac{\partial\tilde{\Psi}}{\partial y}, v=-\frac{\partial\tilde{\Psi}}{\partial x},$$

through Eqs. (8)-(10), we have obtained :

$$(\frac{\partial^2\tilde{\Psi}}{\partial x\partial y}\frac{\partial\tilde{\Psi}}{\partial y}-\frac{\partial^2\tilde{\Psi}}{\partial y^2}\frac{\partial\tilde{\Psi}}{\partial x})=\frac{\partial^3\tilde{\Psi}}{\partial y^3}-(M+K^{-1})\frac{\partial\tilde{\Psi}}{\partial y}+(Gr\theta+Gm\phi)\cos\alpha, \quad (12)$$

$$(\frac{\partial\theta}{\partial x}\frac{\partial\tilde{\Psi}}{\partial y}-\frac{\partial\theta}{\partial y}\frac{\partial\tilde{\Psi}}{\partial x})=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+Ec(\frac{\partial^2\tilde{\Psi}}{\partial y^2})^2, \quad (13)$$

$$(\frac{\partial\phi}{\partial x}\frac{\partial\tilde{\Psi}}{\partial y}-\frac{\partial\phi}{\partial y}\frac{\partial\tilde{\Psi}}{\partial x})=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-K_r\phi^m, \quad (14)$$

the endpoint conditions takes the form:

$$\frac{\partial\tilde{\Psi}}{\partial y}=\frac{\partial\tilde{\Psi}}{\partial x}=0,\ \theta=1,\ \phi=1\ \ at\ \ \ y=0,$$

$$\frac{\partial\tilde{\Psi}}{\partial y}\to 0,\ \theta\to 0,\ \phi\to 0\ \ as\ \ \ y\to\infty. \quad (15)$$

We have to find transformations group by utilizing elementary set of one parameter scaling transformations so that it could be helpful for us to trace out the solution of Eqs. $(12)-(14)$. Therefore, considered transformations (Reddy et al. [28]) as follows:

$$\Delta:\ x^\bullet=xe^{\varepsilon\lambda_1},\ y^\bullet=ye^{\varepsilon\lambda_2},\ \tilde{\Psi}^\bullet=\tilde{\Psi}e^{\varepsilon\lambda_3},\ u^\bullet=ue^{\varepsilon\lambda_4},\ v^\bullet=ve^{\varepsilon\lambda_5},\ \theta^\bullet=\theta e^{\varepsilon\lambda_6},\ \phi^\bullet=\phi e^{\varepsilon\lambda_7}, \quad (16)$$

ε is a small parameter and $\lambda_1, \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6$ and λ_7 denotes transformation

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parameters. The point-transformations given in Eq. (16) reduces the coordinates $(x, y, \tilde{\Psi}, u, v, \theta, \phi)$ into $(x^\bullet, y^\bullet, \tilde{\Psi}^\bullet, u^\bullet, v^\bullet, \theta^\bullet, \phi^\bullet)$ as a new coordinates. Now substituting Eq. (16) in Eqs. (12)-(14), we have

$$e^{\varepsilon(\lambda_1+2\lambda_2-2\lambda_3)}\left(\frac{\partial^2\tilde{\Psi}^\bullet}{\partial x^\bullet \partial y^\bullet}\frac{\partial\tilde{\Psi}^\bullet}{\partial y^\bullet}-\frac{\partial\tilde{\Psi}^\bullet}{\partial x^\bullet}\frac{\partial^2\tilde{\Psi}^\bullet}{\partial y^{\bullet 2}}\right)=\begin{bmatrix} e^{\varepsilon(3\lambda_2-\lambda_3)}\dfrac{\partial^3\tilde{\Psi}^\bullet}{\partial y^{\bullet 3}}-e^{\varepsilon(\lambda_2-\lambda_3)}(M+K^{-1})\dfrac{\partial\tilde{\Psi}^\bullet}{\partial y^\bullet} \\ +(e^{-\varepsilon\lambda_6}Gr\theta+e^{-\varepsilon\lambda_7}Gm\phi)\cos\alpha \end{bmatrix}, \tag{17}$$

$$e^{\varepsilon(\lambda_1+\lambda_2-\lambda_3-\lambda_6)}\left(\frac{\partial\theta^\bullet}{\partial x^\bullet}\frac{\partial\tilde{\Psi}^\bullet}{\partial y^\bullet}-\frac{\partial\theta^\bullet}{\partial y^\bullet}\frac{\partial\tilde{\Psi}^\bullet}{\partial x^\bullet}\right)=e^{\varepsilon(2\lambda_2-\lambda_6)}\frac{1}{\Pr}\frac{\partial^2\theta^\bullet}{\partial y^{\bullet 2}}+e^{\varepsilon(4\lambda_2-2\lambda_3)}Ec\left(\frac{\partial^2\tilde{\Psi}^\bullet}{\partial y^{\bullet 2}}\right)^2, \tag{18}$$

$$e^{\varepsilon(\lambda_1+\lambda_2-\lambda_3-\lambda_7)}\left(\frac{\partial\phi^\bullet}{\partial x^\bullet}\frac{\partial\tilde{\Psi}^\bullet}{\partial y^\bullet}-\frac{\partial\phi^\bullet}{\partial y^\bullet}\frac{\partial\tilde{\Psi}^\bullet}{\partial x^\bullet}\right)=e^{\varepsilon(2\lambda_2-\lambda_7)}\frac{1}{Sc}\frac{\partial^2\phi^\bullet}{\partial y^{\bullet 2}}-e^{-\varepsilon(m\lambda_7)}K_r\phi^{\bullet m}, \tag{19}$$

the transformed endpoint conditions are given as:

$$\frac{\partial\tilde{\Psi}^\bullet}{\partial y^\bullet}=\frac{\partial\tilde{\Psi}^\bullet}{\partial x^\bullet}=0,\ \theta^\bullet=1,\ \phi^\bullet=1\ \ at\ \ y^\bullet=0,$$

$$\frac{\partial\tilde{\Psi}^\bullet}{\partial y^\bullet}\to 0,\ \theta^\bullet\to 0,\ \phi^\bullet\to 0\quad as\quad y^\bullet\to\infty. \tag{20}$$

The transformations $\Delta,$ makes system invariant under the relations given by:

$$\lambda_1+2\lambda_2-2\lambda_3=3\lambda_2-\lambda_3=-\lambda_6=-\lambda_7=\lambda_2-\lambda_3,$$
$$\lambda_1+\lambda_2-\lambda_3-\lambda_7=2\lambda_2-\lambda_7=-m\lambda_7,$$
$$\lambda_1+\lambda_2-\lambda_3-\lambda_6=2\lambda_2-\lambda_6=4\lambda_2-2\lambda_3,$$

by means of elementary algebra, we have

$$\lambda_2=\frac{1}{4}\lambda_1,\lambda_3=\frac{3}{4}\lambda_1,\ \lambda_4=\frac{1}{2}\lambda_1,\ \lambda_5=-\frac{1}{4}\lambda_1,\ \lambda_6=0,\ \lambda_7=0,$$

So, the one-parameter group of transformations is given by:

$$\Delta:\ x^\bullet=xe^{\varepsilon\lambda_1},\ y^\bullet=ye^{\varepsilon\frac{\lambda_1}{4}},\ \tilde{\Psi}^\bullet=\tilde{\Psi}e^{\varepsilon\frac{3\lambda_1}{4}},\ u^\bullet=ue^{\varepsilon\frac{\lambda_1}{2}},\ v^\bullet=ve^{-\varepsilon\frac{\lambda_1}{4}},\ \theta^\bullet=\theta,\ \phi^\bullet=\phi,$$

Taylor's method up to $o\left(\varepsilon^2\right)$, gives

$$x^\bullet-x\approx\varepsilon(x\lambda_1)\ ,\ y^\bullet-y\approx\varepsilon(y\frac{\lambda_1}{4}),\ \tilde{\Psi}^\bullet-\tilde{\Psi}=\varepsilon(\tilde{\Psi}\frac{3\lambda_1}{4}),$$

$$u^\bullet-u\approx\varepsilon(u\frac{\lambda_1}{2}),\ v^\bullet-v\approx\varepsilon(-v\frac{\lambda_1}{4}),\ \theta^\bullet-\theta\approx 0,\ \phi^\bullet-\phi\approx 0,$$

The supplementary characteristic equations will be written as :

$$\frac{dx}{x\lambda_1}=\frac{dy}{y\frac{\lambda_1}{4}}=\frac{d\tilde{\Psi}}{\tilde{\Psi}\frac{3\lambda_{11}}{4}}=\frac{du}{u\frac{\lambda_1}{2}}=\frac{dv}{-v\frac{\lambda_1}{4}}=\frac{d\theta}{0}=\frac{d\phi}{0}, \tag{21}$$

we get concerning similarity transformations by solving Eq. (21), as given as:

$$\eta=x^{-\frac{1}{4}}y,\ \tilde{\Psi}^\bullet=x^{\frac{3}{4}}f(\eta),\ \theta^\bullet=\theta,\ \phi^\bullet=\phi, \tag{22}$$

through Eqs. $(17)-(19)$ with endpoint conditions Eq. (20), we have:

$$f'''+\frac{3}{4}ff''-\frac{1}{2}f'^2+(M+K^{-1})f+(Gr\,\theta+Gm\,\phi)Cos\alpha, \tag{23}$$

$$\theta''+\frac{3}{4}\Pr f\,\theta'+\Pr Ecf''^2=0, \tag{24}$$

$$\phi''+\frac{3}{4}Scf\phi'-ScK_r\phi^m=0, \tag{25}$$

with transformed endpoint conditions:

$$f=0,\ f'=0,\ \theta=1,\ \phi=1\ \ at\ \ \eta=0,$$
$$f'\to 0,\ \theta\to 0,\ \phi\to 0\quad as\quad \eta\to\infty. \tag{26}$$

3 Solution Methodology

The physical problem given by system of Eqs. (23)-(25) cannot be solved by shooting method because the imposed endpoint conditions i-e Eq. (26) are not at coincident locations regarding independent variable η. So that the above concerned system of equations along endpoints Eq. (26) are primarily converted into a system of first order equations. For this purpose we have taken

$$k_2=f',$$
$$k_3=k_2'=f'',$$
$$k_5=\theta',$$
$$k_7=\phi',$$

by hosting fresh variables in Eqs. (23)-(25), we obtained

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$$\begin{bmatrix} k'_1 \\ k'_2 \\ k'_3 \\ k'_4 \\ k'_5 \\ k'_6 \\ k'_7 \end{bmatrix} = \begin{bmatrix} k_2 \\ k_3 \\ \frac{1}{2}k_2^2 - \frac{3}{4}k_1k_3 + \left(M + K^{-1}\right)k_2 - \left(Grk_4 + Gmk_6\right)Cos\alpha \\ l_5 \\ \Pr[-\frac{3}{4}k_1k_5 - Eck_3^2] \\ k_7 \\ Sc[-\frac{3}{4}k_1k_7 + Cr(k_6)^m] \end{bmatrix}, \tag{27}$$

the endpoint conditions are now in coincident locations under new variables, $k_1, k_2, k_3, k_4, k_5, k_6, k_7$ and given as :

$$k_1(0) = 0,\ k_2(0) = 0, k_3(0) = \text{imposed guess},\ k_4(0) = 1,$$

$$k_5(0) = \text{imposed guess},\ k_6(0) = 1,\ k_7(0) = \text{imposed guess}. \tag{28}$$

To trace out the integration of Eq. (27) being a initial value problem we have used numeric values (guessed) of $k_3(0)$ indicates $f''(0)$, $k_5(0)$ implies $\theta'(0)$ and $k_7(0)$ i-e $\phi'(0)$. The imposed guess values are choose in such way the additional endpoint conditions holds absolutely:

$$k_2(\infty) = 0,\ k_4(\infty) = 0,\ k_6(\infty) = 0. \tag{29}$$

This scheme is reviewed until and unless the convergence criteria i-e error tolerance of 10^{-6} is achieved.

4 Results and their Interpretation

The computations were carried out with the aid of shooting method along Runge–Kutta algorithm for different physical parameters, namely permeability parameter *K*, magnetic parameter *M*, Schmidt number *Sc*, Prandtl number *Pr*, Eckert number *Ec*, order of reaction *m*, chemical reaction parameter *Cr*, thermal Grashof number *Gr* and species Grashof number *Gm*. We have compared our numerical results of skin friction coefficient, heat and mass transfer rate with those existing in open literature in **Tables 1-3,** which yields an excellent agreement. The effects logs of certain involved parameters on velocity, temperature and concentration profiles are sort out and presented graphically in **Figs. 2-13**. In this endorsement, K = 1.0, Pr = 0.71 and Sc = 0.6, M = 1.0, Cr = 0.5, α = 30º, Ec = 0.01, Gr = 2.0 and Gm = 2.0, are used as a default parametric values for all graphic results unless directed on the appropriate graphs where needed.

It is noticed from **Table 1** that the skin friction coefficient, wall temperature and concentration gradient increases for inciting values of thermal and solutal Grashof number. Whereas we have found opposite attitude towards permeability parameter *K* and magnetic parameter *M*. **Table 2** is used to demonstrate the influence of Prandtl number *Pr*, Eckert number *Ec* and an inclination *α*. It is observed that the wall temperature gradient (heat transfer rate) decreases for greater values of an inclination and Eckert number while it shows opposite attitude towards Prandtl number *Pr*. **Table 3** is constructed to examine the influence of Sc (Schmidt number) and *Cr* (chemical reaction parameter). It is apparent that the mass transfer rate increases for larger values Sc and *Cr*.

4.1 Velocity Profiles

Figs. 2-8 are used to demonstrate the effect of involved parameters which includes an inclination *α*, magnetic parameter *M*, permeability parameter *K*, Eckert number *Ec*, chemical reaction parameter *Cr*, thermal Grashof number *Gr* and species Grashof number *Gm*.

The interpretation with respect to physical interest for thermal Grashof number *Gr* is given by **Fig. 2.** The relative impact of the thermal buoyancy force corresponds *Gr*. Which implies increment in *Gr*, and it yields increment in thermal buoyancy force in a flow field and hence velocity profile increases. The sharp evolution is observed **(from Fig. 2)** regarding velocity adjacent to the permeable plate. The flow velocities shoots for extreme and then properly decays to zero far from the surface.

Table 1. Comparison of $-\theta'(0)$, $f''(0)$ and $-\phi'(0)$ over *Gr*, *Gm*, *M* and *K*

				Present outcomes			Reddy et al. [28]		
Gr	**Gm**	**M**	**K**	$-\theta'(0)$	$f''(0)$	$-\phi'(0)$	$-\theta'(0)$	$f''(0)$	$-\phi'(0)$
2.0	2.0	1.0	1.0	0.3766	1.6761	0.6304	0.37536	1.67872	0.632846
3.0	-	-	-	0.4000	2.0278	0.6418	0.404755	2.06767	0.648809
4.0	0.2	-	-	0.4232	2.4066	0.6602	0.429506	2.44141	0.66312
-	3.0	-	-	0.3986	2.0585	0.6408	0.399334	2.03459	0.646135
-	4.0	1.0	-	0.4207	2.3959	0.6501	0.420826	2.38237	0.658602
-	-	2.0	-	0.3454	1.4838	0.6137	0.348147	1.49761	0.619813
-	-	3.0	1.0	0.3279	1.3087	0.6184	0.327933	1.36399	0.610413
-	-	-	2.0	0.3454	1.4833	0.6137	0.348174	1.49761	0.619813
-	-	-	3.0	0.3279	1.3083	0.6114	0.327933	1.36399	0.610413

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Table 2. Comparison of $-\theta'(0)$, $f''(0)$ and $-\phi'(0)$ over *Pr*, *Ec* and *α*.

			Present outcomes			**Reddy et al. [28]**		
Pr	***Ec***	***α***	$-\theta'(0)$	$f''(0)$	$-\phi'(0)$	$-\theta'(0)$	$f''(0)$	$-\phi'(0)$
0.71	0.01	30^0	0.3766	1.6761	0.6304	0.37536	1.67872	0.632846
1.0	-	-	0.3297	1.6405	0.6294	0.429375	1.64902	0.629509
2.0	-	-	0.5748	1.5740	0.6264	0.570278	1.57721	0.622038
-	0.1	-	0.3327	1.6809	0.6306	0.337882	1.68425	0.633256
-	0.2	30^0	0.2906	1.6930	0.6367	0.29549	1.69051	0.633719
-	0.01	45^0	0.3540	1.3992	0.6214	0.352877	1.39653	0.621139
-	-	60^0	0.3161	1.0984	0.6091	0.318826	1.01526	0.60434

Table 3. Comparison of $-\theta'(0)$, $f''(0)$ and $-\phi'(0)$ over *Sc and Cr*.

		Present results			Reddy et al. [28]		
Sc	*Cr*	$-\theta'(0)$	$f''(0)$	$-\phi'(0)$	$-\theta'(0)$	$f''(0)$	$-\phi'(0)$
0.5	0.5	0.3801	1.6972	0.5968	_	_	_
0.6	-	0.3777	1.6893	0.6089	0.37536	1.76872	0.632846
0.78	-	0.3763	1.6761	0.6304	0.369773	1.65106	0.716312
0.1	0.5	0.3647	1.6545	0.7406	0.364526	1.62341	0.805778
0.6	1.0	0.3628	1.6211	0.8096	0.36603	1.6253	0.828406
-	2.0	0.3620	1.6209	0.8252	0.355191	1.55591	1.12767

It is noticed that the flow velocities declines monotonically from peak level to free stream zero, so admitting the far away boundary conditions. The impact of *Gm* (solutal Grashof number) on velocity profile is painted in **Fig. 3.** It is found that higher values of *Gm* takes increment in velocity profile. The effect logs of acute angle α over flow velocity is depicted in **Fig. 4.** It is seen that the flow velocity deceases when we elevate the surface, this fact is due to buoyancy effect. The elevated plate give birth to cosα as a gravity component. So when we increase inclination of surface, the buoyancy force contribution drops by a factor of cosα. Hence, the driven force reduces which brings decline in flow velocity. The effect logs of permeability parameter *K* over velocity of the fluid is examined in **Fig. 5**, it is found that the flow velocity is decreasing by increasing *K (*permeability parameter).

Fig. 6 is used to studied the influence of magnetic field over velocity profile. It is observed that the application of magnetic field generates resistive-type force i.e Lorentz force and this force is responsible for reduction in fluid motion and hence velocity profile decreases. The effect logs via *Cr* (chemical reaction parameter) over the velocity profile are explored in **Fig. 7**. It reflects that the velocity profile shows decline attitude towards increasing values of *Cr*. The effect of Eckert number Ec over velocity profile is sketched in **Fig. 8**, it is observed that for higher values of Eckert number velocity profiles increases.

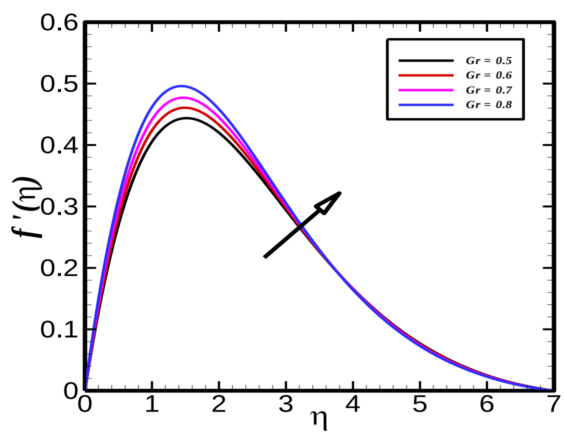

Fig. 2. Impact of thermal Grashof number over velocity profile.

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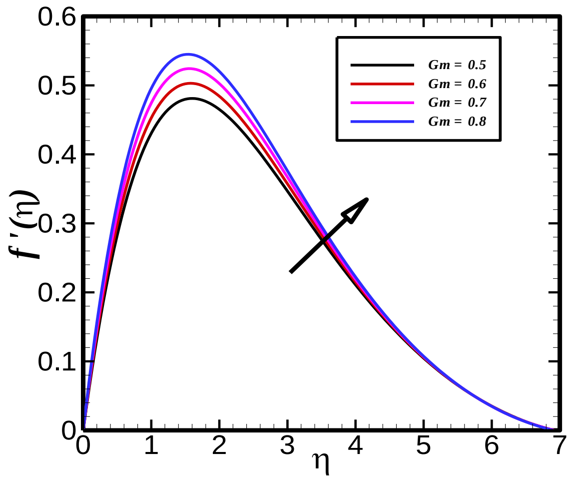

Fig. 3. Impact of solutal Grashof number over velocity profile.

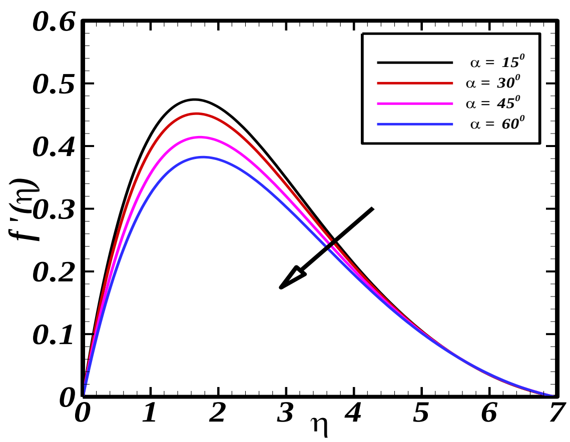

Fig. 4. Impact of an inclination over velocity profile.

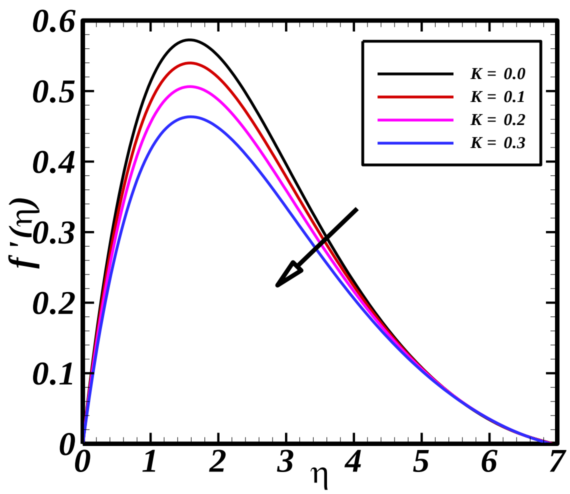

Fig. 5. Impact of permeability parameter over velocity profile.

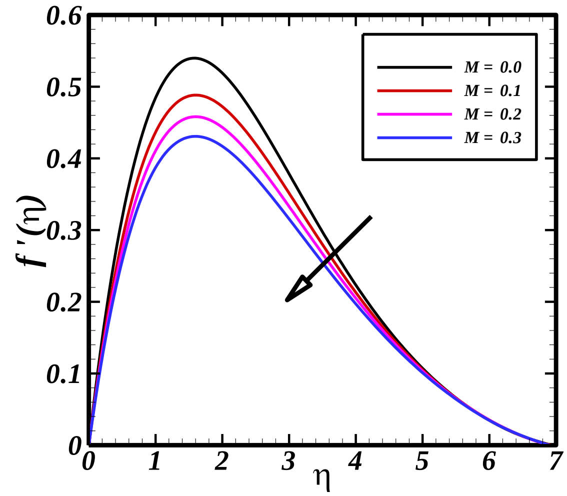

Fig. 6. Impact of magnetic parameter over velocity profile.

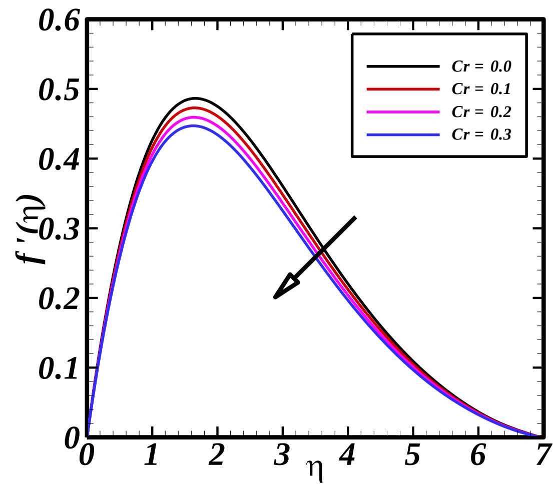

Fig. 7. Impact of chemical reaction parameter over velocity profile.

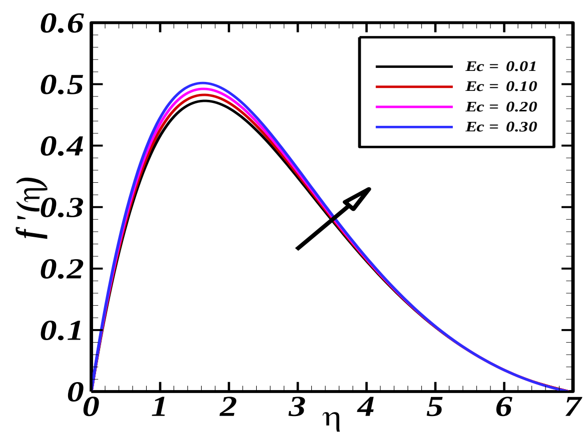

Fig. 8. Impact of Eckert number over velocity profile.

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4.2 Temperature Profiles

The impact of M as a magnetic parameter on flow field is described in **Fig. 9**. The increasing values of M generates Lorentzian retardation. Hence, supplementary work is required to overcome this resistive force. Such a work is then dissipated in terms of thermal energy which brings warmness in fluid throughout the boundary layer so the temperature rises. The effect of permeability parameter K over temperature profile is probed in **Fig. 10**.

It is noticed that the huge resistance is produced by altering the permeability parameter K. Due to this, trifling changes arises in the flow field and hence inciting attitude for fluid temperature is witnessed. **Fig. 11** is used to examined the influence of Cr (chemical reaction parameter). The chemical energy is fairly transformed into thermal energy by means of increasing numeric values of Cr, which brings increment of fluid temperature. The effect logs of Eckert number on fluid temperature is identified by **Fig. 12.** It was expected that the temperature will shoot up due to repeatly viscous dissipation. The cooling of the surface reflects the progressive numeric values of Ec i-e drop of heat from the surface to the surroundings (fluid). Furthermore, thermal boundary layer (in the sense of magnitude) expressively large under viscous dissipation effect. The combined impact of Pr (Prandtl number) and m (high order chemical reaction) is explained by **Fig. 13**.

The inverse relations between Pr with thermal conductivity narrates that the higher numeric values of Prandtl number brings weakness in thermal diffusivity, so the thermal boundary layer becomes thinner. Further, it is also witnessed that the temperature profile shows inciting attitude towards altering values of m.

Fig. 14 paints the combined effect logs of Sc (Schmidt number) and m (high order chemical reaction). It is noticed that by increasing Sc, the decreasing behavior of concentration profile is achieved. Furthermore, the comparative impact of momentum to species diffusion is signifies by way of Sc (Schmidt number) because it is defined by ratio of the momentum to the mass diffusivity. More specifically Sc =1 relates the equal rate of diffusion in the flow regime for both momentum and species. In this context, the order of magnitude of both momentum and concentration boundary layer will be same.

Whereas, the momentum diffusion is faster that species for the case of $Sc > 1$ and opposite for the case of $Sc < 1$ i-e species diffusion is slower than that of momentum diffusion. The impact of order of chemical reaction on fluid concentration is aslo illustrated in **Fig. 14**. For the increasing values order of chemical reaction ($m = 0$, $m = 1$, $m = 2$.), the concentration profile increases.

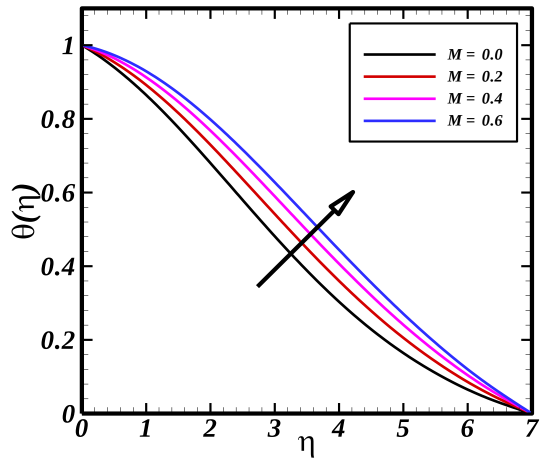

Fig. 9. Impact of magnetic parameter over temperature profile.

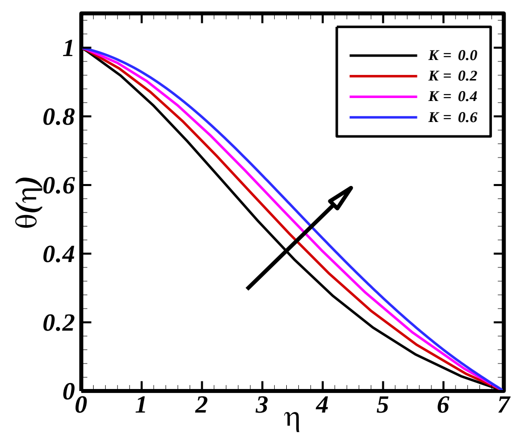

Fig. 10. Impact of permeability parameter over temperature profile.

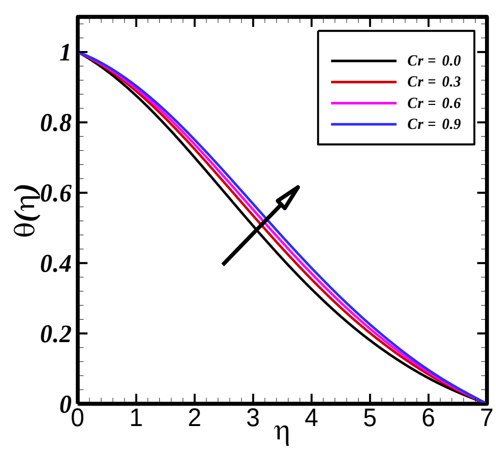

Fig. 11. Impact of chemical reaction parameter over temperature profile.

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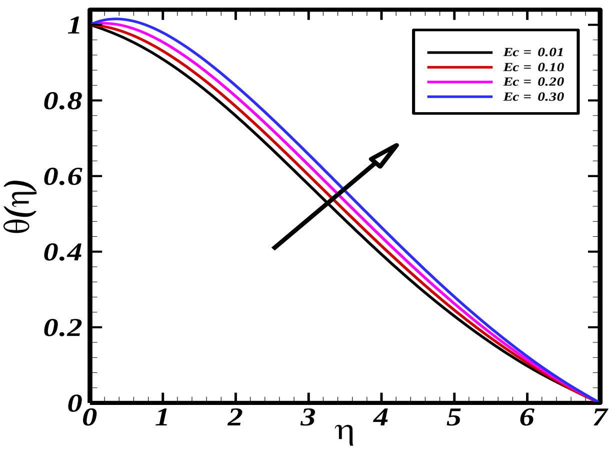

Fig. 12. Impact of Eckert number over temperature profile.

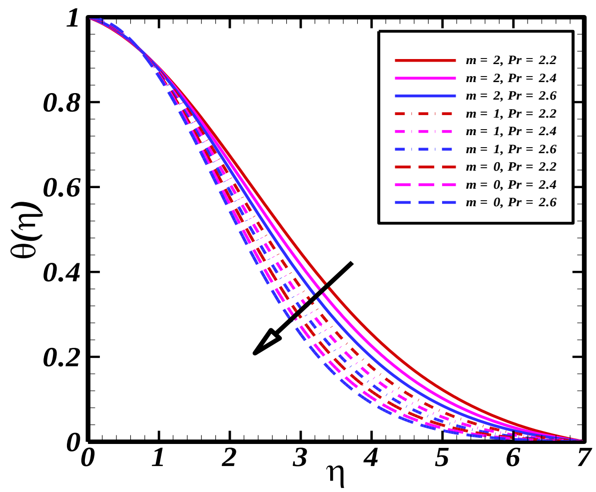

Fig. 13. Impact of order of chemical reaction and Prandtl number over temperature.

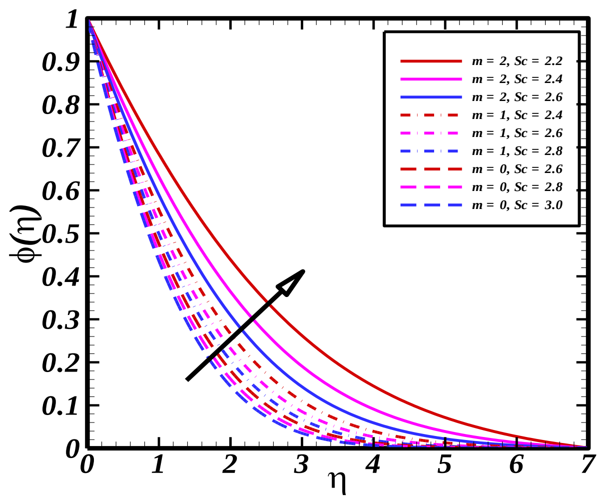

Fig. 14. Impact of order of chemical reaction and Schmidt number over concentration.

5 Itemized Key Points

In this communication, we have scrutinized the heat and mass transfer characteristics via Lie group analysis on free convection dissipative fluid flow adjacent to an inclined permeable plate with second order chemical reaction. In the light of physical interest, the worth mentioning observations on flow field profiles are enumerated as follows:

- The permeability parameter enhances temperature profile and has opposite attitude towards velocity profile i-e to suppress the velocity of fluid.
- The magnetic field parameter reduces the flow field velocity which yields inciting nature of temperature profile.
- The viscous parameter has substantial effect over the flow field which yields increase in temperature and velocity profiles.
- The flow field velocity decreases across the boundary layer for chemical reaction parameter while reverse attitude is witnessed for temperature distribution.
- Thermal and solutal Grashof number has same effect on velocity i-e an increase in fluid velocity. Whereas, it show decline for inclination angle of the plate regarding fluid velocity.
- The temperature and concentration profiles increases for high order chemical reaction and decreases for Prandtl and Schmidt numbers respectively (as likely).
- The conformity and stability of present communication is established by providing comparison with previously published literature.

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