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Double Acceptance Sampling Plan for Time Truncated Life Tests Based on Transmuted Generalized Inverse Weibull Distribution

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Abstract: In this paper, a double acceptance sampling plan (DASP) is developed in terms of truncated life tests assuming that the lifetime of a product follows a transmuted generalized inverse Weibull (TGIWD) distribution. With fixing the consumer's confidence level, the minimum required sample sizes of the first and second samples to ensure the specified mean life are obtained. The operating characteristic function values and the minimum ratios of the mean life to the specified life are presented. Some tables are provided and their use is illustrated by a numerical example.

Keywords: Double Acceptance Sampling Plan, Time Truncated Life Tests, Transmuted Generalized Inverse Weibull Distribution, Operating Characteristic Function, Consumer's Risk.

1 Introduction

In this paper, we suggest acceptance sampling plans based on transmuted generalized inverse Weibull distribution with probability density function (pdf) given by

$$f_{TGIW}(x) = \alpha\beta\gamma(\alpha x)^{-\beta-1}e^{-\gamma(\alpha x)^{-\beta}}\left(1+\lambda-2\lambda e^{-\gamma(\alpha x)^{-\beta}}\right),\ \ x\geq 0,\ \beta>0,\ \gamma>0,\ \alpha>0, \tag{1}$$

and $|\lambda|\leq 1$ with cumulative distribution function (cdf) defined as

$$F_{TGIW}(x) = e^{-\gamma(\alpha x)^{-\beta}}\left(1+\lambda-\lambda e^{-\gamma(\alpha x)^{-\beta}}\right). \tag{2}$$

The mean and the moment generating function of the TGIWD are obtained; respectively, as:

$$\mu_{TGIW} = \frac{\gamma^{1/\beta}}{\alpha}\left(1+\lambda-\lambda 2^{1/\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right), \tag{3}$$

and

$$M_X(t) = \sum_{r=0}^{\infty}\frac{t^r\gamma^{\frac{r}{\beta}}}{r!\alpha^r}\Gamma\left(1-\frac{r}{\beta}\right)\left[1+\lambda-\lambda 2^{\frac{r}{\beta}}\right]. \tag{4}$$

The qth quantile, x_q, of the TGIW distribution is

$$x_q = \frac{1}{\alpha}\left\{\frac{1}{\gamma}\log\left(\frac{1+\lambda-\lambda e^{-\gamma(\alpha x)^{-\beta}}}{q}\right)\right\}^{-\frac{1}{\beta}}. \tag{5}$$

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The reliability and hazard rate functions of the transmuted generalized inverse Weibull distribution, respectively, are given by

$$\begin{aligned} R_{TGIW}(x) &= 1 - F_{TGIW}(x) \\ &= 1 - e^{-\gamma(\alpha x)^{-\beta}}\left(1+\lambda-\lambda e^{-\gamma(\alpha x)^{-\beta}}\right), \end{aligned} \tag{6}$$

and

$$\begin{aligned} H_{TGIW}(x) &= \frac{f_{TGIW}(x)}{1-F_{TGIW}(x)} \\ &= \frac{\alpha\beta\gamma(\alpha x)^{-\beta-1}e^{-\gamma(\alpha x)^{-\beta}}\left(1+\lambda-2\lambda e^{-\gamma(\alpha x)^{-\beta}}\right)}{1-e^{-\gamma(\alpha x)^{-\beta}}\left(1+\lambda-\lambda e^{-\gamma(\alpha x)^{-\beta}}\right)}. \end{aligned} \tag{7}$$

The coefficient of kurtosis, coefficient of skewness, and the coefficient of variation of the TGIW distribution, respectively can be obtained as

$$\Psi_1 = \frac{\mu_4 - 4\mu_3\mu_1 + 6\mu_2\mu_1^2}{\left(\mu_2-\mu_1^2\right)^2},\ \Psi_2 = \frac{\mu_3 - 3\mu_2\mu_1 + 2\mu_1^3}{\left(\mu_2-\mu_1\right)^{3/2}},\ \Psi_3 = \sqrt{\frac{\mu_2}{\mu_1}-1}. \tag{8}$$

For more information about the transmuted generalized inverse Weibull distribution, we refer the interested reader to [1].

A double acceptance sampling plan in terms of truncated life tests is proposed when the lifetime of a product follows the TGIW distribution. The acceptance sampling plan is an important subject in statistical quality control. The quality level of products is a very important for both producers and consumers. Since the screening of products is impossible and infeasible due the cost and time, a decision about the product can be taken based on a selected sample from the lot.

The DASP is used to reduce the sample size or producer's risk in the field of quality control. DASP should be used if the decision cannot be taken based on the first sample. Therefore, a second sample should be selected from the lot to make a decision [2].

Several authors considered the DASP under different life time distributions in their research; [3] studied the generalized log-logistic distribution in DASP. [4] proposed DASP based on truncated life tests for the Marshall-Olkin extended exponential distribution. [5] suggested DASP based on truncated life tests using generalized exponential distribution. [6] suggested new ASP for three parameters kappa distribution. [7] proposed ASP for generalized inverted exponential distribution. [8] suggested DASP when the time truncated life tests follows the Maxwell distribution. [9] suggested an ASP for truncated life tests for exponentiated Frechet distribution.

This paper is organized as follows. The suggested double acceptance sampling plan is presented in Section 2. The operating characteristic function is given in Section 3. The minimum variance ratios to the specified life are provided in Section 4. The paper is concluded in Section 5.

2 Double Acceptance Sampling Plan

The DASP based on truncated life time can be described as follows:

1- Draw the first random sample of size n_1 and put them on test during time t_0. If there are c_1 or fewer failures, accept the lot. If c_2+1 failures are observed, stop the test and reject the lot; i.e., $c_1 < c_2$.

2- If the number of failures by t_1 is between c_1+1 and c_2, then draw the second sample of size n_2 and then test the drawn items during another time t_0. If at most c_2 failures are observed from the two samples, i.e., n_1+n_2, accept the lot. Otherwise, reject the lot and terminate the test.

Therefore, the DASP consists of four parameters n_1, n_2, c_1 and c_2. Based on these parameters; the probabilities of acceptance $L(p_1)$ and $L(p_2)$ for the sampling plans $\left(n_1, c_1, \frac{t}{\mu_0}\right)$ and $\left(n_2, c_2, \frac{t}{\mu_0}\right)$ can be calculated under the assumption that the lot is large enough to use the Binomial probability distribution as follows:

$$L(p_1) = \sum_{i=0}^{c_1=0} \binom{n_1}{i} p^i (1-p)^{n_1-i}, \tag{9}$$

$$L(p_2) = \sum_{i=0}^{c_2=2} \binom{n_2}{i} p^i (1-p)^{n_2-i}, \tag{10}$$

respectively, where $p = F(t;\mu) = F\left(\frac{t}{\mu_0}.\frac{\mu_0}{\mu}\right)$ is given in (2). Then, the probability of acceptance in general is given by

$$L(p) = \sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i} \left[\sum_{j=0}^{c_2-j} \binom{n_2}{j} p^j (1-p)^{n_2-j} \right]. \tag{11}$$

Note that if $c_1 = 0$ and $c_2 = 2$, then the probability of acceptance is the total of three different probabilities that can be formulated as:

$P(A) = P$(No failure occurs in the first sample) + P(1 failure occurs in fist sample and zero or one failure occurs in the second sample) + P(Two failures occur in the first sample and zero or one failure occurs in second sample).

In this article, values of the probability of acceptance of a lot for a DASP based on TGIW distribution are obtained at $P^* = 0.75, 0.90, 0.95, 0.99$ for $t/\mu_o = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712$ are presented in Tables (1-2) based on single and double acceptance sampling plans, respectively. These choices of P^* and t/μ_o are computable with [10], [11], [12], [13], [9], and [14].

3 Operating Characteristic Function

The operating characteristic function (OC) is one of the main criteria in the acceptance sampling plan where it represents the probability of accepting a lot. The OC measures the efficiency of a statistical hypotheses test in order to accept or reject a lot. The acceptance sampling plan is suitable if its operating characteristic function values approaches to one. The operating characteristic values of the sampling plan $(n_1, c_1 = 0, t/\mu_o)$ for a given P^* under the TGIWD with $\beta = 3$, $\gamma = 2$ and $\lambda = -0.9$ are presented in Table (1). Furthermore, for the acceptance sampling plan $(n_1, n_2, c_1, c_2, t/\mu_o)$ the OC function values are provided in Table (2).

Based on Tables 1-2, it can be noted that the OC values approach to one, specially for large values of μ/μ_0 based on single and double acceptance sampling plans. However, the values of the probability based on the suggested DASP are larger than their counterpart using the single acceptance sampling plan.

4 Minimum Mean Ratios to the Specified Life and Producer's Risk

The producer's risk (PR) is defined as the probability of rejection of a good lot. i.e., $(\mu \geq \mu_0)$. For the suggested DASP using TGIW distribution and for a given value of the producer's risk θ, we want to find the minimum quality level of μ/μ_0 that asserts the PR to be at most θ. Therefore, μ/μ_0 is the smallest positive number for which $p = F(t;\mu) = F\left(\frac{t}{\mu_0}.\frac{\mu_0}{\mu}\right)$ satisfies the inequality

$$\sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i} \times \left[\sum_{j=0}^{c_2-i} \binom{n_2}{j} p^j (1-p)^{n_2-j} \right] \geq 1-\theta \tag{12}$$

For the proposed acceptance sampling plan $(n_1, n_2, c_1 = 0, c_2 = 2, t/\mu_o)$ at a specified consumer's confidence level P^*, the smallest values of μ/μ_0 satisfying (7) are summarized in Table (3).

Suppose that the experimenter wants to assert that true unknown average life is at least 1000 hours with confidence level 0.95. Also, suppose that the acceptance numbers for this case are $c_1 = 0$ and $c_2 = 2$ with sample sizes $n_1 = 4$ and $n_2 = 6$. Thus, the lot is accepted if within 942 hours no failure is detected with a sample of size 4. For single sampling plan, the probability of accepting the lot is 1 with $\mu/\mu_0 = 4$, 6, 8, 10, 12. The DASP for the same measurements the ratio probability is 1. It is of interest to note here that as the ratio μ/μ_0 increases using DASP the probability of acceptance increases.

The producer's risk with respect to time of experiment for DASP using TGIW distribution for $c_1 = 0$ and $c_2 = 2$ are presented in Table (4) for $P^* = 0.95$. For example, when $\mu/\mu_0 = 2$ (the unknown average life is two times of specified average life) producer's risk in the case of time of experiment being 4712 hours and 628 hours are 0 and 0.910832, respectively. It is found that if the quality level of the product increases the producer's risk decreases.

Table 1: Operating characteristic values of the sampling plan $(n_1, c_1 = 0, t/\mu_o)$ for a given P^* under the TGIWD with $\beta = 3$, $\gamma = 2$ and $\lambda = -0.9$.

P^*	t/μ_o	n_1	$\mu/\mu_o = 2$	4	6	8	10	12
0.75	0.628	7	0.999305	1	1	1	1	1
0.75	0.942	2	0.945325	1	1	1	1	1
0.75	1.257	1	0.797776	0.999899	1	1	1	1
0.75	1.571	1	0.564148	0.996334	0.999999	1	1	1
0.75	2.356	1	0.219915	0.854557	0.996344	0.999977	1	1
0.75	3.141	1	0.099680	0.564489	0.932223	0.996349	0.999900	0.999999
0.75	3.927	1	0.052347	0.348172	0.758400	0.959680	0.996340	0.999775
0.75	4.712	1	0.030652	0.219915	0.564376	0.854557	0.972167	0.996344
0.90	0.628	11	0.998909	1	1	1	1	1
0.90	0.942	3	0.919119	1	1	1	1	1
0.90	1.257	2	0.636446	0.999798	1	1	1	1
0.90	1.571	1	0.564148	0.996334	0.999999	1	1	1
0.90	2.356	1	0.219915	0.854557	0.996344	0.999977	1	1
0.90	3.141	1	0.099680	0.564489	0.932223	0.996349	0.999900	0.999999
0.90	3.927	1	0.052347	0.348172	0.758400	0.959680	0.996340	0.999775
0.90	4.712	1	0.030652	0.219915	0.564376	0.854557	0.972167	0.996344
0.95	0.628	14	0.998611	1	1	1	1	1
0.95	0.942	4	0.893639	1	1	1	1	1
0.95	1.257	2	0.636446	0.999798	1	1	1	1
0.95	1.571	2	0.318263	0.992681	0.999999	1	1	1
0.95	2.356	1	0.219915	0.854557	0.996344	0.999977	1	1
0.95	3.141	1	0.099680	0.564489	0.932223	0.996349	0.999900	0.999999
0.95	3.927	1	0.052347	0.348172	0.758400	0.959680	0.996340	0.999775
0.95	4.712	1	0.030652	0.219915	0.564376	0.854557	0.972167	0.996344
0.99	0.628	21	0.997918	1	1	1	1	1
0.99	0.942	5	0.868866	1	1	1	1	1
0.99	1.257	3	0.507742	0.999697	1	1	1	1
0.99	1.571	2	0.318263	0.992681	0.999999	1	1	1
0.99	2.356	2	0.048363	0.730268	0.992701	0.999955	1	1
0.99	3.141	2	0.009936	0.318648	0.869039	0.992711	0.999800	0.999999
0.99	3.927	1	0.052347	0.348172	0.758400	0.959680	0.996340	0.999775
0.99	4.712	1	0.030652	0.219915	0.564376	0.854557	0.972167	0.996344

5 Conclusions

In this paper, we assumed that the lifetime of the products follows a transmuted generalized inverse Weibull distribution to suggest a double acceptance sampling plan based on TGIWD. For fixed consumer's confidence level, the minimum sample sizes of the required first and second samples to assert the specified mean life are calculated. The operating characteristic values and the minimum ratios of the mean life to the specified life are tabulated. It is shown that the suggested DASP might be more useful than the single acceptance sampling plan.

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Table 2: Operating characteristic values of the sampling plan $(n_1, n_2, c_1 = 0, c_2 = 2, t/\mu_o)$ for a given P^* under the TGIWD with $\beta = 3$, $\gamma = 2$ and $\lambda = -0.9$.

P^*	t/μ_o	n_1	n_2	$\mu/\mu_o = 2$	4	6	8	10	12
0.75	0.628	7	13	1	1	1	1	1	1
0.75	0.942	2	4	0.999679	1	1	1	1	1
0.75	1.257	1	3	0.978535	1	1	1	1	1
0.75	1.571	1	2	0.917203	1	1	1	1	1
0.75	2.356	1	2	0.525293	0.996923	1	1	1	1
0.75	3.141	1	2	0.270223	0.917397	0.999689	1	1	1
0.75	3.927	1	2	0.148965	0.723052	0.985898	0.999935	1	1
0.75	4.712	1	2	0.089168	0.525293	0.917332	0.996923	0.999978	1
0.90	0.628	11	18	1	1	1	1	1	1
0.90	0.942	3	5	0.999113	1	1	1	1	1
0.90	1.257	2	3	0.945621	1	1	1	1	1
0.90	1.571	1	3	0.823783	1	1	1	1	1
0.90	2.356	1	2	0.525293	0.996923	1	1	1	1
0.90	3.141	1	2	0.270223	0.917397	0.999689	1	1	1
0.90	3.927	1	2	0.148965	0.723052	0.985898	0.999935	1	1
0.90	4.712	1	2	0.089168	0.525293	0.917332	0.996923	0.999978	1
0.95	0.628	14	22	1	1	1	1	1	1
0.95	0.942	4	6	0.998149	1	1	1	1	1
0.95	1.257	2	4	0.916228	1	1	1	1	1
0.95	1.571	2	3	0.645316	1	1	1	1	1
0.95	2.356	1	3	0.316503	0.991665	1	1	1	1
0.95	3.141	1	2	0.270223	0.917397	0.999689	1	1	1
0.95	3.927	1	2	0.148965	0.723052	0.985898	0.999935	1	1
0.95	4.712	1	2	0.089168	0.525293	0.917332	0.996923	0.999978	1
0.99	0.628	21	31	1	1	1	1	1	1
0.99	0.942	5	8	0.995988	1	1	1	1	1
0.99	1.257	3	5	0.822283	1	1	1	1	1
0.99	1.571	2	4	0.541254	0.999999	1	1	1	1
0.99	2.356	2	3	0.097317	0.977802	1	1	1	1
0.99	3.141	2	3	0.015734	0.645905	0.997465	1	1	1
0.99	3.927	1	3	0.059866	0.530200	0.964507	0.999809	1	1
0.99	4.712	1	2	0.089168	0.525293	0.917332	0.996923	0.999978	1

Table 3: Minimum ratio of σ/σ_0 for the acceptability of a lot with producer's risk of 0.05 under the TGIWD with $\beta = 3$, $\gamma = 2$ and $\lambda = -0.9$.

P^*	$t/\mu_o = 0.628$	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	1.257	1.562	1.863	2.129	3.193	4.257	5.322	6.385
0.90	1.315	1.656	2.014	2.328	3.193	4.257	5.322	6.385
0.95	1.347	1.721	2.084	2.517	3.491	4.257	5.322	6.385
0.99	1.399	1.787	2.210	2.605	3.774	5.032	5.818	6.385

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Table 4: Producer's risk with respect to time of experiment for double acceptance sampling for $c_1 = 0$ $c_2 = 2$ and $P^* = 0.95$ based on TGIWD with $\beta = 3$, $\gamma = 2$ and $\lambda = -0.9$.

P^*	t/μ_o	n_1	n_2	$\mu/\mu_o = 2$	4	6	8	10	12
0.95	0.628	14	22	0	0	0	0	0	0
0.95	0.942	4	6	0.001851	0	0	0	0	0
0.95	1.257	2	4	0.083772	0	0	0	0	0
0.95	1.571	2	3	0.354684	0	0	0	0	0
0.95	2.356	1	3	0.683497	0.008335	0	0	0	0
0.95	3.141	1	2	0.729777	0.082603	0.000311	0	0	0
0.95	3.927	1	2	0.851035	0.276948	0.014102	0.6.5E-05	0	0
0.95	4.712	1	2	0.910832	0.474707	0.082668	0.003077	2.2E-05	0

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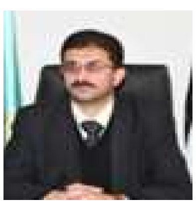

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