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Common Random Fixed Point Theorems of Akram-Contraction Mappings in Cone Random Metric Spaces

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Abstract: In this paper, we prove a unique common random fixed point theorems in the framework of cone random metric spaces for generalized M_J-contraction and generalized $M_{J\psi}$-contraction condition. An example to justify our theorems is given. Our results extends some previous work related to cone random metric spaces from the current existing literature.

Keywords: common random fixed point, cone random metric spaces, generalized $M_{J\psi}$-contraction condition.

1 Introduction

Fixed point theory has the diverse applications in different branches of mathematics, statistics, engineering and economics in dealing with the problems arising in approximation theory, potential theory, game theory, theory of differential equations, theory of integral equations and others. Developments in the investigation on fixed points of non-expansive mappings, contractive mappings in different spaces like metric spaces, Banach spaces, Fuzzy metric spaces and cone metric spaces have almost been saturated. The study of random fixed point theorems was initiated by the Prague school of probabilistic in 1950 [10,11,29].

Common random fixed point theorems are stochastic generalization of classical common fixed point theorems. Random methods have revolutionized the financial markets. The survey article by Bharucha-Reid [8] in 1976 attracted the attention of several mathematicians and gave wings to the theory. The results of Špaček and Hanš in multi-valued contractive mappings was extended by Itoh [15]. Now this theory has become the full fledged research area and various ideas associated with random fixed point theory are used to give the solution of nonlinear system see[5,6,7,12,22,26]. Common random fixed points, random coincidence points of a pair of compatible random operators and fixed point theorems for contractive random operators in Polish spaces are obtained by Papageorgiou [19,20] and Beg [3,4].

In [13] Huang and Zhang generalized the concept of metric spaces by replacing the set of real numbers with an ordered Banach space, hence they have defined the cone metric spaces. They also described the convergence of sequences and introduced the notion of completeness in cone metric spaces. They have proved some fixed point theorems of contractive mappings on complete cone metric space with the assumption of normality of a cone. According to this concept, several other authors [1,14,25,28] studied the existence of fixed points and common fixed points of mappings satisfying contractive type condition on a normal cone metric space.

In 2008, the assumption of normality in cone normal spaces is deleted by Rezapour and Hamlbarani [25], which is an important event in developing fixed point theory in cone metric spaces.

Akram et al. [2] introduced a new class of contraction mappings called A-contraction, which is proper super class of Kannan's [16], Bianchini's [9] and Reich's [23] type contractions as follows:

Definition 1.1. A self mapping $T : X \to X$ of a metric space (X,d) is said to be A-contraction if it satisfies the condition

$$d(Tx,Ty) \le \alpha(d(x,y),d(x,Tx),d(y,Ty)), \text{ for all } x,y \in X, \quad (1)$$

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and some $\alpha \in A$, where A is the set of all functions $\alpha : \mathbb{R}_+^3 \to \mathbb{R}_+$ satisfying

(A_1) α is continuous on the set $\mathbb{R}_+^3$ (with respect to the Euclidean metric on $\mathbb{R}^3$).

(A_2) $a \leq kb$ for some $k \in [0,1)$ whenever $a \leq \alpha(a,b,b)$ or $a \leq \alpha(b,a,b)$ or $a \leq \alpha(b,b,a)$ for all $a,b \in \mathbb{R}$.

Definition 1.2. [17] A function ψ: $\mathbb{R}_+ \to \mathbb{R}_+$ is called a comparison function if it satisfies the following conditions

(i) ψ is monotone increasing, $\psi(t) < t$ for some $t > 0$,

(ii) $\psi(0) = 0$,

(iii) $\lim_{n\to\infty} \psi^n(t) = 0$, $\forall t \geq 0$.

A comparison function satisfying $\sum_{n=0}^{\infty} \psi^n(t)$ converges for all $t > 0$ is called a (c)-comparison function.

In 2016, Olatinwo and Omidire [18] proved some fixed point theorems for some new classes of contraction mappings which are more general than the A-contractions of Akram et al. They introduced the following definitions:

Definition 1.3. A self mappings $S,T : X \to X$ of a metric space (X,d) is said to be generalized M_J-contraction if S and T satisfy the contractive condition

$$d(Tx,Ty) \leq \alpha \begin{pmatrix} d(Sx,Sy), d(Sx,Tx), d(Sy,Ty), \\ [d(Sx,Tx)]^r [d(Sy,Tx)]^s d(Sx,Ty), \\ d(Sy,Tx)[d(Sx,Tx)]^m \end{pmatrix}, \tag{2}$$

for all $x,y \in X$, $r,s,m \in \mathbb{R}_+$ and some $\alpha \in M_J$, where M_J is the set of all functions $\alpha : \mathbb{R}_+^5 \to \mathbb{R}_+$ satisfying

(M_{J1}) α is continuous on the set $\mathbb{R}_+^5$ (with respect to the Euclidean metric on $\mathbb{R}^5$),

(M_{J2}) If any of the conditions $a \leq \alpha(b,b,a,c,c)$ or $a \leq \alpha(b,b,a,b,b)$ or $a \leq \alpha(a,b,b,b,b,)$ for all $a,b,c \in \mathbb{R}_+$, then there exists $k \in [0,1)$ such that $a \leq kb$.

Definition 1.4. A self mappings $S,T : X \to X$ of a metric space (X,d) is said to be generalized $M_{J\psi}$-contraction if S and T satisfy the contractive condition

$$d(Tx,Ty) \leq \alpha \begin{pmatrix} d(Sx,Sy), d(Sx,Tx), d(Sy,Ty), \\ [d(Sx,Tx)]^r [d(Sy,Tx)]^s d(Sx,Ty), \\ d(Sy,Tx)[d(Sx,Tx)]^m \end{pmatrix}, \tag{3}$$

for all $x,y \in X$, $r,s,m \in \mathbb{R}_+$ and some $\alpha \in M_{J\psi}$, where $M_{J\psi}$ is the set of all functions $\alpha : \mathbb{R}_+^5 \to \mathbb{R}_+$ satisfying

$(M_{J\psi 1})$ α is continuous on the set $\mathbb{R}_+^5$ (with respect to the Euclidean metric on $\mathbb{R}^5$),

$(M_{J\psi 2})$ If any of the conditions $a \leq \alpha(b,b,a,c,c)$ or $a \leq \alpha(b,b,a,b,b)$ or $a \leq \alpha(a,b,b,b,b,)$ for all $a,b,c \in \mathbb{R}_+$, then there exists a continuous (c)-comparison function $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ such that $a \leq \psi(b)$.

According to the above definitions we extend the contractive conditions (2) or (3) to a stochastic form and we obtain common random fixed point theorems under these conditions in random cone metric spaces. Our main results generalize and extend many deterministic results as Olatinwo [17,18], Akram et al. [2], Rashwan and Hammad [21] and many others in complete metric spaces

2 Preliminaries

Definition 2.1. [25] Let (E,τ) be a topological vector space. A subset P of E is called a cone if the following conditions satisfied:

(c_1) P is closed, nonempty and $P \neq \{0\}$;

(c_2) $a,b \in \mathbb{R}$, $a,b \geq 0$ and $x,y \in P \Rightarrow ax+by \in P$;

(c_3) If $x \in P$ and $-x \in P \Rightarrow x = 0$.

For a given cone $P \subset E$, we define a partial ordering $\leq$ with respect to P by $x \leq y$ iff $y - x \in P$. We shall write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in P^\circ$, where P° indicate to the interior of P.

Definition 2.2. [13,30] Let X be a nonempty set. Assume that the mapping $d : X \times X \to E$ satisfies

(d_1) $0 \leq d(x,y)$ for all $x,y \in X$ and $d(x,y) = 0 \Leftrightarrow x = y$;

(d_2) $d(x,y) = d(y,x)$ for all $x,y \in X$;

(d_3) $d(x,y) \leq d(x,z) + d(z,y)$ $x,y,z \in X$.

Then d is called a cone metric [13] or K-metric [30] on X and (X,d) is called a cone metric space [13].

The concept of a cone metric space is more general than that of a metric space, because each metric space is a cone metric space where $E = \mathbb{R}$ and $P = [0,+\infty)$.

Example 2.1. [13] Let $E = \mathbb{R}^2$, $P = \{(x,y) \in \mathbb{R}^2 : x \geq 0, y \geq 0\}$, $X = \mathbb{R}$ and $d : X \times X \to E$ defined by $d(x,y) = (|x-y|, \mu(|x-y|))$ where $\mu \geq 0$ is a constant. Then (X,d) is a cone metric space with normal cone P where $K = 1$.

Example 2.2. [24]Let $E = l^2$, $P = \{\{x_n\}_{n\geq 1} \in E : x_n \geq 0$, for all n$\}$, (X,ρ) a metric space and $d : X \times X \to E$ defined by $d(x,y) = \{\frac{\rho(x,y)}{2^n}\}_{n\geq 1}$. Then (X,d) is a cone metric space.

Clearly, the above examples present that the class of cone metric spaces contains the class of metric spaces.

Definition 2.3. [19] Let (X,d) be a cone metric space. We say that $\{x_n\}$ is

(i) a Cauchy sequence if for every ε in E with $0 \ll \varepsilon$, then there is an $\mathbb{N}$ such that for all $n,m > \mathbb{N}$, $d(x_n,x_m) \ll \varepsilon$;

(ii) a convergent sequence if for every ε in E with $0 \ll \varepsilon$, then there is an $\mathbb{N}$ such that for all $n > \mathbb{N}$, $d(x_n,x) \ll \varepsilon$ for some fixed x in X.

A cone metric space X is said to be complete if every Cauchy sequence in X is convergent in X.

The following definitions are given in [27].

Definition 2.4. (Measurable function) Let (Ω,Σ) be a measurable space with $\Sigma-$a sigma algebra of subsets of Ω and V be a nonempty subset of a metric space $X = (X,d)$. Let 2^V be the family of nonempty subsets of V and $C(V)$ the family of all nonempty closed subsets of V. A mapping $G : \Omega \to 2^V$ is called measurable if for each open subset U of V, $G^{-1}(U) \in \Sigma$, where $G^{-1}(U) = \{\omega \in \Omega : G(\omega) \cap U \neq \varnothing\}$.

Definition 2.5. (Measurable selector) A mapping $\xi : \Omega \to V$ is called measurable selector of a measurable mappings $G : \Omega \to 2^V$ if ξ is measurable and $\xi(\omega) \in G(\omega)$ for each $\omega \in \Omega$.

Definition 2.6. (Random operator) The mapping $T : \Omega \times V \to X$ is called a random operator iff for each fixed $x \in V$, the mapping $T(.,x) : \Omega \to X$ is measurable.

Definition 2.7. (Continuous random mapping) A random operator $T : \Omega \times V \to X$ is called continuous random operator if for each fixed $x \in V$ and $\omega \in \Omega$, the mapping $T(\omega,.) : \Omega \to X$ is continuous.

Definition 2.8. (Random fixed point) A measurable mapping $\xi : \Omega \to V$ is a random fixed point of a random operator $T : \Omega \times V \to X$ iff $T(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

Definition 2.9. (Commuting random mappings) A random operators $T, S : \Omega \times V \to X$ are called commuting random operators if $T(\omega, S(\omega, x)) = S(\omega, T(\omega, x))$ for any given $\omega \in \Omega$ and $x \in V$.

Definition 2.10. (Cone random metric space) Let V be a nonempty set and the mapping $d : \Omega \times V \to P$, where P is a cone, $\omega \in \Omega$ be a selector, satisfy the following conditions

(i) $d(x(\omega), y(\omega)) \geq 0$ and $d(x(\omega), y(\omega)) = 0 \Leftrightarrow x(\omega) = y(\omega)$ for all $x(\omega), y(\omega) \in \Omega \times V$,

(ii) $d(x(\omega), y(\omega)) = d(y(\omega), x(\omega))$ for all $x, y \in V$, $\omega \in \Omega$ and $x(\omega), y(\omega) \in \Omega \times V$,

(iii) $d(x(\omega), y(\omega)) \leq d(x(\omega), z(\omega)) + d(z(\omega), y(\omega))$ for all $x, y, z \in V$ and $\omega \in \Omega$ be a selector,

(iv) for any $x, y \in V$, $\omega \in \Omega$, $d(x(\omega), y(\omega))$ is non-increasing and left continuous.

Then d is called cone random metric on V and (V, d) is called a cone random metric space.

3 Main Results

In this section we shall prove a common random fixed point theorems under generalized M_J-contraction and generalized $M_{J\psi}-$contraction condition for two mappings in the setting of cone random metric spaces.

Theorem 3.1. Let (X, d) be a complete cone random metric space with respect to a cone P and let V be a nonempty separable closed subset of X. Let S and T be two commuting random mappings defined on V. If $T(X) \subseteq S(X)$ and S is continuous satisfying the following generalized M_J-contraction condition

$$d(T(x(\omega)), T(y(\omega))) \leq \alpha \begin{pmatrix} d(S(x(\omega)), S(y(\omega))), d(S(x(\omega)), T(x(\omega))), d(S(y(\omega)), T(y(\omega))), \\ [d(S(x(\omega)), T(x(\omega)))]^r [d(S(y(\omega)), T(x(\omega)))]^s d(S(x(\omega)), T(y(\omega))), \\ d(S(y(\omega)), T(x(\omega)))[d(S(x(\omega)), T(x(\omega)))]^m \end{pmatrix}, \tag{4}$$

for all $x(\omega), y(\omega) \in \Omega \times X$, $r, s, m \in \mathbb{R}_+$ and for some $\alpha \in M_J$. Then the two random mappings have a unique common random fixed point in X.

Proof. For each $x_\circ(\omega) \in \Omega \times X$ and $n = 0, 1, 2, ..$ we choose $y_1(\omega) \in \Omega \times X$ such that $y_1(\omega) = T(x_\circ(\omega)) = S(x_1(\omega))$. In general we define sequence of elements of V such that $y_n(\omega) = T(x_n(\omega)) = S(x_{n+1}(\omega))$. Then from (4), we get

$$\begin{aligned} d(y_n(\omega), y_{n+1}(\omega)) &= d(T(x_n(\omega)), T(x_{n+1}(\omega))) \\ &\leq \alpha \begin{pmatrix} d(S(x_n(\omega)), S(x_{n+1}(\omega))), d(S(x_n(\omega)), T(x_n(\omega))), \\ d(S(x_{n+1}(\omega)), T(x_{n+1}(\omega))), [d(S(x_n(\omega)), T(x_n(\omega)))]^r \\ .[d(S(x_{n+1}(\omega)), T(x_n(\omega)))]^s.d(S(x_n(\omega)), T(x_{n+1}(\omega))), \\ d(S(x_{n+1}(\omega)), T(x_n(\omega)))[d(S(x_n(\omega)), T(x_n(\omega)))]^m \end{pmatrix} \\ &= \alpha \begin{pmatrix} d(y_{n-1}(\omega), y_n(\omega)), d(y_{n-1}(\omega), y_n(\omega)), d(y_n(\omega), y_{n+1}(\omega)), \\ [d(y_{n-1}(\omega), y_n(\omega))]^r [d(y_n(\omega), y_n(\omega))]^s.d(y_{n-1}(\omega), y_n(\omega))), \\ d(y_n(\omega), y_n(\omega))[d(y_{n-1}(\omega), y_n(\omega))]^m \end{pmatrix} \\ &= \alpha \left(d(y_{n-1}(\omega), y_n(\omega)), d(y_{n-1}(\omega), y_n(\omega)), d(y_n(\omega), y_{n+1}(\omega)), 0, 0 \right). \end{aligned}$$

Then by axiom (M_{J2}) of the function α, we get

$$d(y_n(\omega), y_{n+1}(\omega)) \leq kd(y_{n-1}(\omega), y_n(\omega)). \tag{5}$$

For some $k \in [0, 1)$. In this fashion, one can obtain

$$d(y_n(\omega), y_{n+1}(\omega)) \leq kd(y_{n-1}(\omega), y_n(\omega)) \leq k^2 d(y_{n-2}(\omega), y_{n-1}(\omega)) \leq .... \leq k^n d(y_\circ(\omega), y_1(\omega)).$$

Also, for $n > m$, we get

$$\begin{aligned} d(y_n(\omega), y_m(\omega)) &\leq d(y_n(\omega), y_{n-1}(\omega)) + d(y_{n-1}(\omega), y_{n-2}(\omega)) + ... + d(y_{m+1}(\omega), y_m(\omega)) \\ &\leq (k^{n-1} + k^{n-2} + ... + k^m) d(y_1(\omega), y_\circ(\omega)) \leq \left(\frac{k^m}{1-k} \right) d(y_1(\omega), y_\circ(\omega)). \end{aligned}$$

Let $0 \ll \varepsilon$ is given. Choose a natural number $\mathbb{N}$ such that $\left(\frac{k^m}{1-k}\right) d(y_1(\omega), y_\circ(\omega)) \ll \varepsilon$ for every $m \geq \mathbb{N}$, hence

$$d(y_n(\omega), y_m(\omega)) \leq \left(\frac{k^m}{1-k}\right) d(y_1(\omega), y_\circ(\omega)) \ll \varepsilon,$$

this implies that $\{y_n(\omega)\}$ is a Cauchy sequence in $\Omega \times X$.
Since (X,d) is complete, then there exists $z(\omega) \in \Omega \times X$ such that $y_n(\omega) \to z(\omega)$ as $n \to \infty$. It follows that $\lim_{n\to\infty} T(x_n(\omega)) = \lim_{n\to\infty} S(x_{n+1}(\omega)) = z(\omega)$. Since S is continuous and S,T are random commuting mappings i.e. $T(\omega, S(\omega,x)) = S(\omega, T(\omega,x))$ for any given $\omega \in \Omega$. Then we have

$$S(z(\omega)) = S(\lim_{n\to\infty} S(x_{n+1}(\omega))) = \lim_{n\to\infty} S(S(x_{n+1}(\omega))), \tag{6}$$

$$S(z(\omega)) = S(\lim_{n\to\infty} T(x_n(\omega))) = \lim_{n\to\infty} S(T(x_n(\omega))) = \lim_{n\to\infty} T(S(x_n(\omega))). \tag{7}$$

Also from (4), we obtain

$$d(T(S(x_n(\omega))), T(z(\omega))) \leq \alpha \begin{pmatrix} d(S(S(x_n(\omega))), S(z(\omega))), d(S(S(x_n(\omega))), T(S(x_n(\omega)))), d(S(z(\omega)), T(z(\omega))), \\ [d(S(S(x_n(\omega))), T(S(x_n(\omega))))]^r [d(S(z(\omega)), T(S(x_n(\omega))))]^s d(S(S(x_n(\omega))), T(z(\omega))), \\ d(S(z(\omega)), T(S(x_n(\omega))))[d(S(S(x_n(\omega))), T(S(x_n(\omega))))]^m \end{pmatrix}.$$

Taking the limit as $n \to \infty$, applying (6) and (7) in above inequality and from condition (M_{J2}), we have

$$d(S(z(\omega)), T(z(\omega))) \leq \alpha(0, 0, d(S(z(\omega)), T(z(\omega))), 0, 0) \leq k.0 = 0, \tag{8}$$

this implies that $d(S(z(\omega)), T(z(\omega))) \leq 0$, thus $-d(S(z(\omega)), T(z(\omega))) \in P$, but $d(S(z(\omega)), T(z(\omega))) \in P$, therefore by Definition 2.1 (c_3), we have $d(S(z(\omega)), T(z(\omega))) = 0$ and so

$$S(z(\omega)) = T(z(\omega)). \tag{9}$$

Again, by using generalized M_J-contraction condition, we obtain

$$d(T(x_n(\omega)), T(z(\omega))) \leq \alpha \begin{pmatrix} d(S(x_n(\omega)), S(z(\omega))), d(S(x_n(\omega)), T(x_n(\omega))), d(S(z(\omega)), T(z(\omega))), \\ [d(S(x_n(\omega)), T(x_n(\omega)))]^r [d(S(z(\omega)), T(x_n(\omega)))]^s d(S(x_n(\omega)), T(z(\omega))), \\ d(S(z(\omega)), T(x_n(\omega)))[d(S(x_n(\omega)), T(x_n(\omega)))]^m \end{pmatrix}. \tag{10}$$

Taking limits in (10) and using (9), gives

$$d(z(\omega), T(z(\omega))) \leq \alpha(d(z(\omega), S(z(\omega))), 0, 0, 0, 0) \leq k.0 = 0, \tag{11}$$

this leads to $d(z(\omega), T(z(\omega))) \leq 0$, thus $-d(z(\omega), T(z(\omega))) \in P$, but $d(z(\omega), T(z(\omega))) \in P$, therefore by Definition 2.1 (c_3), we have $d(z(\omega), T(z(\omega))) = 0$ and so $z(\omega) = T(z(\omega))$.
From (9), we obtain

$$z(\omega) = T(z(\omega)) = S(z(\omega)).$$

Hence $z(\omega)$ is a common random fixed point of T and S.

Now, we show the uniqueness. Let $q(\omega) \neq z(\omega)$ be another common random fixed point of T and S, then from (4), one can write

$$d(z(\omega), q(\omega)) = d(T(z(\omega)), T(q(\omega))) \leq \alpha \begin{pmatrix} d(S(z(\omega)), S(q(\omega))), d(S(z(\omega)), T(z(\omega))), d(S(q(\omega)), T(q(\omega))), \\ [d(S(z(\omega)), T(z(\omega)))]^r [d(S(q(\omega)), T(z(\omega)))]^s d(S(z(\omega)), T(q(\omega))), \\ d(S(q(\omega)), T(z(\omega)))[d(S(z(\omega)), T(z(\omega)))]^m \end{pmatrix}.$$

By condition (M_{J2}), we get

$$d(z(\omega), q(\omega)) \leq \alpha(d(z(\omega), q(\omega)), 0, 0, 0, 0) \leq k.0 = 0, \tag{12}$$

which gives $z(\omega) = q(\omega)$, hence $z(\omega)$ is a unique common random fixed point of T and S.

Theorem 3.2. Let (X,d) be a complete cone random metric space with respect to a cone P and let V be a nonempty separable closed subset of X. Let S and T be a commuting random mappings satisfying generalized $M_{J\psi}$-contraction condition under same condition (3). If $T(X) \subseteq S(X)$ and S is continuous. Then S and T have a unique common random fixed point in X.

Proof. In a manner similar step by step in the proof Theorem 3.1, by (4), (5) and axiom $(M_{J\psi 2})$ of the function α, we get

$$d(y_n(\omega), y_{n+1}(\omega)) \leq \psi(d(y_{n-1}(\omega), y_n(\omega))). \tag{13}$$

We have inductively from (13) that

$d(y_n(\omega), y_{n+1}(\omega)) \leq \psi(d(y_{n-1}(\omega), y_n(\omega))) \leq \psi^2(d(y_{n-2}(\omega), y_{n-1}(\omega))) \leq \ldots\ldots\ldots\ldots \leq \psi^n(d(y_\circ(\omega), y_1(\omega))),$

this gives

$$d(y_n(\omega), y_{n+1}(\omega)) \leq \psi^n(d(y_\circ(\omega), y_1(\omega))). \tag{14}$$

For $l \in \mathbb{N}$, using (14) inductively in the repeated application of triangle inequality yields

$$\begin{aligned} d(y_n(\omega), y_{n+l}(\omega)) &\leq \sum_{k=n}^{n+l-1} \psi^k(d(y_\circ(\omega), y_1(\omega))) = \sum_{j=0}^{l-1} \psi^{n+j}(d(y_\circ(\omega), y_1(\omega))) \\ &= \sum_{k=0}^{n+l-1} \psi^k(d(y_\circ(\omega), y_1(\omega))) - \sum_{k=0}^{l-1} \psi^k(d(y_\circ(\omega), y_1(\omega))). \end{aligned}$$

Let $0 \ll \varepsilon$ is given. Choose a natural number $\mathbb{N}$ such that

$$\sum_{k=0}^{n+l-1} \psi^k(d(y_\circ(\omega), y_1(\omega))) - \sum_{k=0}^{l-1} \psi^k(d(y_\circ(\omega), y_1(\omega))) \ll \varepsilon,$$

for every $l \in \mathbb{N}$, since ψ is a (c)-comparison function, hence

$$d(y_n(\omega), y_{n+l}(\omega)) \leq \sum_{k=0}^{n+l-1} \psi^k(d(y_\circ(\omega), y_1(\omega))) - \sum_{k=0}^{l-1} \psi^k(d(y_\circ(\omega), y_1(\omega))) \ll \varepsilon,$$

this implies that $\{y_n(\omega)\}$ is a Cauchy sequence in $\Omega \times X$.

Since (X,d) is complete, then there exists $z(\omega) \in \Omega \times X$ such that $y_n(\omega) \to z(\omega)$ as $n \to \infty$. It follows that $\lim_{n\to\infty} T(x_n(\omega)) = \lim_{n\to\infty} S(x_{n+1}(\omega)) = z(\omega)$.

Using (6), (7) and $(M_{J\psi 2})$ condition in (4) then (8) yields

$$d(S(z(\omega)), T(z(\omega))) \leq \alpha(0,0,d(S(z(\omega)), T(z(\omega))),0,0) \leq \psi(0) = 0,$$

which implies that (9).

Again, by using generalized $M_{J\psi}-$contraction condition and condition $(M_{J\psi 2})$, (11) becomes

$$d(z(\omega), T(z(\omega))) \leq \alpha(d(z(\omega), S(z(\omega))),0,0,0,0) \leq \psi(0) = 0,$$

it follows that

$$z(\omega) = T(z(\omega)) = S(z(\omega)).$$

Hence $z(\omega)$ is a common random fixed point of T and S.

For uniqueness, from (12), we have

$$d(z(\omega), q(\omega)) \leq \alpha(d(z(\omega), q(\omega)),0,0,0,0) \leq \psi(0) = 0,$$

which gives $z(\omega) = q(\omega)$, hence $z(\omega)$ is a unique common random fixed point of T and S.

Finally, we present an example to verify the requirements of Theorem 3.1 as follows:

Example 3.1. Let $P = \mathbb{R}$ and $p = \{x \in P : x \geq 0\}$, $\Omega = [0,1]$ and $\sum$ be the sigma algebra of Lebesgue's measurable subset of $[0,1]$. Let $X = [0,\infty)$ and define a mapping $d : (\Omega \times X) \times (\Omega \times X) \to P$ by $d(x(\omega), y(\omega)) = |x(\omega) - y(\omega)|$. It's clearly (X,d) is a cone random metric space. Define random operators $S,T : (\Omega \times X) \to X$ as $S(\omega,x) = \frac{1-\omega^2+2x}{3}$ and $T(\omega,x) = \frac{1-\omega^2+x}{2}$. Also sequence of mapping $\xi_n : \Omega \to X$ is defined by $\xi_n(\omega) = (1 + \frac{1}{n} - \omega^2)$ for every $\omega \in \Omega$. Hence $T(\omega,x) \subseteq S(\omega,x)$ and

$$T(\omega, S(\omega,x)) = T(\omega, \frac{1-\omega^2+2x}{3}) = \frac{2(1-\omega^2)+x}{3} = \frac{1-\omega^2+2\left(\frac{1-\omega^2+x}{2}\right)}{3} = S(\omega, \frac{1-\omega^2+x}{2}) = S(\omega, T(\omega,x)),$$

this show that S and T are commuting random mappings, also

$$d(T(x(\omega)), T(y(\omega))) = \frac{1}{2}|x(\omega) - y(\omega)| \leq \frac{2}{3}|x(\omega) - y(\omega)| = d(S(x(\omega)), S(y(\omega))),$$

which show that S and T satisfy the generalized M_J-contraction condition. Therefore all requirements of Theorem 3.1 are satisfied and $(1-\omega^2)$ is a unique common random fixed point of S and T.

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