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# Estimation of $P(X > Y)$ for Inverted Exponential-Two Parameter Exponential Models (Generalized Variable Approach)

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**Abstract:** In this paper we estimate $R = P(X > Y)$ when $X$ and $Y$ are independent random variables from inverted exponential distribution and two parameter exponential distribution respectively. We find maximum likelihood estimator of $R$ and consider the problem of constructing confidence interval for this parameter. We use two confidence interval procedures based on the generalized variable and percentile bootstrap confidence interval methods. We compare these interval estimation procedures in terms of coverage probability and expected length. Simulation studies show that the generalized variable method is satisfactory for practical applications even for small sample setting to construct confidence interval for parameter $R$.

**Keywords:** Inverted exponential distribution, Two-Parameter exponential distribution, Stress-Strength, Coverage probability, Generalized variable.

## 1 Introduction

In reliability theory one of the main parameter is stress-strength parameter. Its estimation is of special importance in reliability literature. The stress-strength reliability model, the probability of this event is that strength of the system is greater than stress enters the system, which includes two random variables $X$ and $Y$, where $X$ represents the strength variable of the system or the component, and $Y$ represents the stress variable which is subjected to it. The system fails if at any time the applied stress is greater than its strength, so the probability $R = P(X > Y)$ is the stress-strength reliability function. However, there are some applications where stress and strength can have discrete distributions, this is the case when the stress is the number of shocks the product undergoes and the strength is the number of shocks the product can withstand. In recent years, the estimation of stress-strength parameter of the discrete and continuous distributions has attracted the attention of many researchers. The term stress-strength was first introduced by [1, 4, 24] studied the geometric case. The negative binomial distribution was considered by [3, 14, 28] examined the Poisson case. The estimation of $R$ when $X$ and $Y$ are normally distributed has been considered by [8, 10, 27, 29, 30, 34], considered the maximum-likelihood estimator $(MLE)$ and the minimum variance unbiased estimator $(UMVUE)$ of , when $X$ and $Y$ were exponentially distributed. The gamma case has been studied by [6, 7, 13, 26] considered this stress-strength reliability problem for the Weibull case, and presented an interval estimation procedure. Most of results about stress-strength reliability problem are collected in [5, 16]. obtained the exact distributions of the MLEs of the scale and location parameters of a two-parameter exponential distribution, when the data are Type-I hybrid censored. [2] presented a shrinkage estimatorof $R$ when $X$ and $Y$ are independent two-parameter exponential random variables with common location parameter. [21], [22] considered generalized exponential and Weibull Distributions cases, respectively. [20] considered generalized variable $(GV)$ inferences on reliability in the two-parameter exponential stress-strength model. [15] studied the exponentiated Gumbel case. In all mentioned papers both stress and strength come from the same type of distribution. In some cases $X$ and $Y$ follow different types of distribution such as [25] considered the

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maximum-likelihood estimator and the minimum variance unbiased estimator and Bayes estimator of , when $X$ and $Y$ are different types of distribution, namely geometric and Poisson random variables. In this paper we focus on the case when $X$ and $Y$ follow different types of distribution, namely the inverted exponential and two parameter exponential distributions. The inverted exponential and two-parameter exponential distributions are one of the most widely used distributions in the reliability and survival studies. The rest of the paper is as follows: In the following section, we study some preliminary results for inverted exponential and two-parameter exponential distributions and the $MLE$ of $R$. In Section 3, we use the concept of the generalized confidence intervals $(GCI)$ to arrive the exact confidence intervals $(CIs)$ for the parameter $R$. Also, the percentile bootstrap (Boot-p) method is examined. A simulation study is performed in Section 4 to evaluate and compare the coverage probability $(CP)$ and Expected Length $(EL)$ of these two approaches. In section 5, some concluding remarks are stated.

## 2 Preliminary results

A random variable $X$ has an inverted exponential distribution with scale parameter $\tau$ if its probability density function (pdf) is given by

$$f_X(x;\tau)=\frac{1}{\tau x^2}e^{-\frac{1}{\tau x}};x>0,\tau>0. \tag{1}$$

The inverted exponential distribution denoted by $IE(\tau)$ . The cumulative distribution function (cdf) of$IE$ distribution is given as follows:

$$F_X(x;\tau)=e^{-\frac{1}{\tau x}};x>0,\tau>0. \tag{2}$$

It has been used very effectively for analyzing lifetime data, particularly when the data are censored. A random variable $Y$ is said to have a two-parameter exponential distribution if its pdf is given by

$$f_Y(y;\mu,\theta)=\frac{1}{\theta}e^{-\frac{1}{\theta}(y-\mu)};y>\mu,\theta,\mu>0. \tag{3}$$

Where $\mu$ and $\theta$ are the location and scale parameters, respectively. The two-parameter exponential distribution denoted by $E(\mu,\theta)$. The cdf of $E(\mu,\theta)$distribution is given as follows:

$$F_Y(y;\mu,\theta)=1-e^{-\frac{1}{\theta}(y-\mu)};y>\mu,\theta,\mu>0. \tag{4}$$

Our goal is to estimate the parameter of $R=P(X>Y)$ , where $X\sim IE(\tau)$ and $Y\sim E(\mu,\theta)$. For our problem, the reliability parameter $R$ is given by

$$R=P(X>Y)=\frac{1}{\theta}\int_{\mu}^{\infty}e^{-(\frac{x-\mu}{\theta}+\frac{\tau}{x})}dx. \tag{5}$$

It is notable that, if $\mu=0$ i.e. random variable $Y$ is distributed as exponential distribution with parameter$\theta$ then the reliability parameter $R$ is given by

$$R=2\sqrt{\frac{\tau}{\theta}}\,\mathrm{K}\left(1,2\sqrt{\frac{\tau}{\theta}}\right), \tag{6}$$

where $K_\nu(z)=\left(\frac{\pi}{2}\right)^\nu\sum_{k=0}^{\infty}\frac{I_{-\nu}(z)-I_\nu(z)}{\sin(\nu\pi)}$, $\quad I_\nu(z)=\left(\frac{z}{2}\right)^\nu\sum_{k=0}^{\infty}\frac{\left(\frac{z^2}{2}\right)^k}{k!\Gamma(\nu+k+1)}$ is the modified Bessel function of the second kind. Let $X_1,X_2,\ldots,X_{n_1}$ and $Y_1,Y_2,\ldots,Y_{n_2}$ be two independent random samples from $IE(\tau)$ and $E(\mu,\theta)$, respectively. It can be easily shown that $\widehat{\tau}$,$\widehat{\mu}$ and $\widehat{\theta}$ the maximum likelihood estimators $(MLE)$ of parameter$\tau$ ,$\mu$ and $\theta$are as

$$\widehat{\tau}=\frac{1}{n_1}\sum_{i=1}^{n_1}\frac{1}{X_i} \tag{7}$$

$$\widehat{\mu}=Y_{(1)} \qquad \widehat{\theta}=\overline{Y}-Y_{(1)} \tag{8}$$

Where $Y_{(1)}$is the smallest of the$Y_i^{,}s$ and $\overline{Y}=\frac{\sum_{i=1}^{n_2}Y_i}{n_2}$. Then by using equation (7) and (8) the $MLE$ of parameter of interest $R$ can be obtained by replacing the parameters $\tau$,$\mu$ and $\theta$ in (6) by their MLEs. That is, the $MLE$ of $R$ is given by

$$\hat{R}=\frac{1}{\widehat{\theta}}\int_{\widehat{\mu}}^{\infty}e^{-\left(\frac{x-\widehat{\mu}}{\widehat{\theta}}+\frac{\widehat{\tau}}{x}\right)}dx. \tag{9}$$

**Lemma.**The following results are obvious:
i) If $X \sim IE(\tau)$then it can be shown that $\frac{2}{\tau X} \sim \chi^2_{(2)}$
ii) By using part (i), we can write

$$\frac{2n_1\widehat{\tau}}{\tau} \sim \chi^2_{(2n_1)}. \tag{10}$$

iii) It is well known that $\widehat{\mu} = Y_{(1)}$ and $\widehat{\theta} = \overline{Y} - Y_{(1)}$ are independent.
iv) It can be shown that

$$\frac{2n_2\,(\widehat{\mu} - \mu)}{\theta} \sim \chi^2_{(2)} \quad and \quad \frac{2n_2\,\widehat{\theta}}{\theta} \sim \chi^2_{(2n_2-2)} \tag{11}$$

## 3 Constructing *CIs* for *R*

In this section, we consider the problem of constructing a $(1-\alpha)\%$ confidence interval for parameter $R$. We use the generalized pivot variable and percentile bootstrap approaches for interval estimation for this parameter.

### *3.1 The Generalized Confidence Interval for the R*

In this section, we present the generalized variables for $\tau$,$\mu$ ,$\theta$ of the $R$ to arrive the exact $CIs$ for $R$. The $GV$ approach is useful to develop a so called generalized pivotal quantity which is used to construct confidence intervals for a parametric function of interest. The advantages of the generalized variables method are as follows: The coverage studies indicate that the $GV$ approach is satisfactory even for small samples, and it can be used for applications and also applicable for unequal sample sizes, the ease of computation and implementation. In fact, the procedures can be easily coded in a programming language for implementation. For more details see the books by [32], [33] and [17], [18], [23] and Chapters 4-6 of [19]. [12] noted that the generalized variable procedures are a special case of fiducially inference procedures,and are asymptotically exact in many situations. In this section, we use the concept of the generalized confidence intervals to arrive the exact $CIs$ for $R$. The concept of generalized confidence intervals was introduced by [31]. At first, we review the concept of generalized confidence intervals as follows. Let $X$ be a random variable whose distribution depends on $\theta$ and $\eta$, a scalar parameter of interest and a nuisance parameter (parameter that is not of direct inferential interest), respectively.Furthermore,let $x$ denote the observed value of $X$. To obtain a generalized confidence interval for $\theta$, we need a generalized pivotal quantity. The random variable $T(X;x,\theta,\eta)$ is called a generalized pivotal quantity if it satisfies in the following two conditions:
(i) Given $x$, the distribution of $T(X;x,\theta,\eta)$is free of the unknown parameters $\theta$ and $\eta$;
(ii) The observed value of $T(X;x,\theta,\eta)$, i.e., $T(x;x,\theta,\eta)$ is equal to $\theta$.
The $CIs$ for $\theta$ obtained using the percentiles of $T(X;x,\theta,\eta)$ are referred to as the $GCIs$.Therefore$T_\alpha(x)$, the is a $100(1-\alpha)\%$ generalized lower confidence limit for $\theta$ if $P(T(X;x,\theta,\eta) \geq T_\alpha(x)) = 1-\alpha$. The quantiles $T_\alpha(x)$ and $T_{1-\alpha}(x)$are the lower and upper $100(1-\alpha)\%$ GCLs for $\theta$, respectively, whereas $[T_{\frac{\alpha}{2}}(x), T_{1-\frac{\alpha}{2}}(x)]$ is the two-sided equal-tailed $100(1-\alpha)\%$ $GCI$ for $\theta$ based on $T(X;x,\theta,\eta)$. Using above approach, the generalized pivotal quantity for $\tau$,$\mu$ ,$\theta$ for our problem can be constructed as follows.

Let $\widehat{\tau}_0$ be an observed value of $\widehat{\tau}$ , then using (7) the generalized pivot variable for $\tau$is

$$G_\tau = \frac{\tau}{2n_1\,\widehat{\tau}} 2n_1\widehat{\tau}_0 = \frac{2n_1\widehat{\tau}_0}{V_1}, \tag{12}$$

where $V_1 \sim \chi^2_{(2n_1)}$. It is easy to see that the generalized pivot variable $G_\tau$ satisfies the properties given earlier, because the value of $G_\tau$ at $\widehat{\tau} = \widehat{\tau}_0$ is $\tau$ and also for given$\widehat{\tau}_0$ , the distribution of $G_\tau$ is independent of any unknown parameters. Also, by using the method of [20] , let $\widehat{\mu}_0$ and $\widehat{\theta}_0$ be an observed values of $\widehat{\mu}$ and $\widehat{\theta}$, respectively. Then the generalized pivot variable for $\mu$ and $\theta$ can be written as

$$G_\mu = \widehat{\mu}_0 - \frac{\chi^2_{(2)}\,\widehat{\theta}_0}{\chi^2_{(2n_2-2)}} \quad and \quad G_\theta = \frac{2n_2\,\widehat{\theta}_0}{\chi^2_{(2n_2-2)}} \tag{13}$$

where $V_2 \sim \chi^2_{(2)}$ and $V_3 \sim \chi^2_{(2n_2-2)}$ . So, a generalized pivot variable for$R$ can be obtained by replacing the parameters in the form of the $R$ by their generalized variables as below

$$G_R = \frac{1}{G_\theta}\int_{G_\mu}^{\infty} e^{-\left(\frac{x-G_\mu}{G_\theta}+\frac{G_\tau}{x}\right)} dx, \tag{14}$$

where $G_\tau$,$G_\mu$ and$G_\theta$ are as in (12) and (13), respectively.
It is easy to check that the generalized pivot variable $G_R$ satisfies the two properties of eneralized pivot variable as mentioned above. Using the following algorithm we can construct a $100(1-\alpha)\%$ generalized confidence interval for parameter of interest $R$.

**Algorithm1**
**Step1**. For given random samples $(x_1,x_2,\ldots,x_{n_1})$ and $(y_1,y_2,\ldots,y_{n_2})$, compute the MLEs$\widehat{\tau}_0$ , $\widehat{\mu}_0$and$\widehat{\theta}_0$ .
**Step2**. Generate $V_1 \sim \chi^2_{(2n_1)}$,$V_2 \sim \chi^2_{(2)}$ and$V_3 \sim \chi^2_{(2n_2-2)}$.
**Step 3**. Compute $G_R$ in (14) by using $G_\tau$ , $G_\mu$ and $G_\theta$ in (12) and (13), respectively.
**Step 4**. Repeat the steps 2 and 3 a large number of times, say, $B$ times. The 100 $\alpha$th percentiles of these $B$ generated$R(G_\tau,G_\mu,G_\theta)'$ is a $1-\alpha$ lower confidence limit for $R$. In order to get consistent results regardless of the values of seed used for random number generation, we recommend simulation consisting of at least $B=100000$.

### *3.2 Bootstrap Confidence Intervals*

In this section we propose percentile bootstrap confidence intervals of $R$. The goal of bootstrap confidence interval theory is to calculate dependable confidence limits for a parameter of interest from the bootstrap distribution of . It provides a better approximation to exactness in most situations. It is assumed that we have independent random samples $(X_1,X_2,\ldots,X_{n_1})$ and $(Y_1,Y_2,\ldots,Y_{n_2})$ obtained from the inverted exponential and the two parameter exponential distribution, respectively. First we propose to use the following Algorithm to generate parametric bootstrap samples such as percentile bootstrap method that suggested by Efron and Tibshirani (1998) and Hall (1988). [9] and [11].

**Algorithm2**
**Step 1**. Generate independent bootstrap samples $x_1^*,x_2^*,\ldots,x_{n_1}^*$ and $y_1^*,y_2^*,\ldots,y_{n_2}^*$ taken with replacement from the given samples $x_1,x_2,\ldots,x_{n_1}$ and $y_1,y_2,\ldots,y_{n_2}$, respectively.
**Step 2**. Based on the bootstrap samples, compute the MLE $(\widehat{\tau}^*,\widehat{\mu}^*,\widehat{\theta}^*)$ of $(\tau,\mu,\theta)$as well as $\widehat{R}^* = R(\widehat{\tau}^*,\widehat{\mu}^*,\widehat{\theta}^*)$.
**Step 3**.Repeat Step 1 and step 2, $B$ times to obtain a set of bootstrap samples of $R$, say$\left\{\widehat{R}, j=1,2,\ldots,B\right\}$.

Using the above bootstrap samples of $R$ we obtain percentile bootstrap confidence intervals of $R$. The ordered $\widehat{R}_j^*$ for $j=1,2,\ldots,B$ will be denoted as:

$$\widehat{R}_1^* < \widehat{R}_2^* < \cdots < \widehat{R}_B^* \tag{15}$$

Let $\widehat{R}^*_{(\gamma)}$ be the $\gamma$ percentile of $\left\{\widehat{R}_j, j=1,2,\ldots,B\right\}$ i.e. $\widehat{R}^*_{(\gamma)}$ is such that

$$\frac{1}{B}\sum_{j=1}^{B} I(\widehat{R}_j^* < \widehat{R}^*_{(\gamma)} = \gamma, \tag{16}$$

Where $I(.)$ is the indicator function. Then, a $100(1-\alpha)\%$ Boot-$p$ lower confidence interval for $R$ is $\widehat{R}^*_{(\alpha)}$ and a $100(1-\alpha)\%$ Boot-$p$ two-sided confidence interval of $R$ is given by $(\widehat{R}^*_{(\frac{\alpha}{2})},\widehat{R}^*_{(1-\frac{\alpha}{2})})$.

## 4 Simulation Study

A Monte Carlo simulation is performed to compare the coverage probabilities and Expected Length of the given approaches (i) generalized confidence interval and (ii) percentile bootstrap confidence interval. Using 10000 times runs for each configuration, we generate $n_1 = 12$ observations from inverted exponential distribution with parameter$\tau$ and $n_2 = 10$ observations from two parameter exponential distribution with parameters $\mu$ and $\theta$ . We then estimate the $CP$ and $EL$ of each approach to construct a one-sided (lower) confidence interval with confidence coefficient $1-\alpha = 0.95$ for parameter $R$. Furthermore, the values of $\mu$, $\theta$ and $\tau$have been chosen to be $\mu = 0.5, 1.5, 2.5$ , $\theta = 1, 2.5, 10$ and

**Table 1:** Coverage Probability$(CP)$ and Expected Length $(EL)$ of 95% lower confidence limits for $R$

| $\mu$ | $\theta$ | | $\tau$ | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 0.5 | | 2.0 | | 5.0 | |
| | | | $R$ | $CP(EL)$ | $R$ | $CP(EL)$ | $R$ | $CP(EL)$ |
| 0.5 | 1 | CIGV | 0.229 | 0.950(0.406) | 0.352 | 0.958(0.772) | 0.853 | 0.950(0.515) |
| | | CIPB | | 0.850(0.349) | | 0.895(0.746) | | 0.878(0.459 |
| 0.5 | 2.5 | CIGV | 0.404 | 0.946(0.582) | 0.241 | 0.947(0.847) | 0.891 | 0.937(0.933) |
| | | CIPB | | 0.888(0.551) | | 0.912(0.842) | | 0.931(0.932) |
| 0.5 | 10 | CIGV | 0.68 | 0.957(0.800) | 0.111 | 0.947 (0.936) | 0.951 | 0.935(0.972) |
| | | CIPB | | 0.953(0.811) | | 0.974(0.945) | | 0.967(0.976) |
| 1.5 | 1 | CIGV | 0.423 | 0.947(0.595) | 0.199 | 0.957(0.874) | 0.915 | 0.944(0.946) |
| | | CIPB | | 0.864(0.539) | | 0.892(0.852) | | 0.866 (0.936) |
| 1.5 | 2.5 | CIGV | 0.54 | 0.957(0.692) | 0.150 | 0.946(0.905) | 0.937 | 0.952(0.961) |
| | | CIPB | | 0.891(0.655) | | 0.895(0.894) | | 0.896(0.955) |
| 1.5 | 10 | CIGV | 0.738 | 0.946(0.835) | 0.080 | 0.940(0.952) | 0.967 | 0.942(0.980) |
| | | CIPB | | 0.933(0.834) | | 0.938(0.954) | | 0.954(0.980) |
| 2.5 | 1 | CIGV | 0.55 | 0.957(0.697) | 0.140 | 0.958(0.911) | 0.94 | 0.954(0.963) |
| | | CIPB | | 0.862(0.646) | | 0.879(0.894) | | 0.872(0.956) |
| 2.5 | 2.5 | CIGV | 0.63 | 0.935(0.753) | 0.112 | 0.963(0.931) | 0.954 | 0.945(0.971) |
| | | CIPB | | 0.858(0.717) | | 0.890(0.920) | | 0.874(0.966) |
| 2.5 | 10 | CIGV | 0.777 | 0.938(0.859) | 0.064 | 0.944(0.961) | 0.974 | 0.954(0.984) |
| | | CIPB | | 0.897(0.851) | | 0.922(0.959) | | 0.939(0.983) |

$\tau = 0.5, 2, 5$. The results are presented in Table 1. The following results are found from this Table:
(i). The first and important result is that the $CP$ of the $GV$ approach is approximately close to the confidence coefficient $1-\alpha = 0.95$, for all configurations regardless of the parameters values.
(ii). The $CP$ of the Boot-$p$ approach is in general smaller than the nominal level 0.95.
(iii). The $ELs$ of both approaches $GV$ and Boot-p increase as parameter $\theta$ increases.
So, we recommend using $GV$ approach for practical applications even for small sample setting. We also performed another simulation study (not reported here) for larger sample sizes, and we observed that the $CP$ of the Boot-$p$ approach is approximately near to the nominal level 0.95. So, the Boot-$p$ method is recommended for large sample size only.

## 5 Concluding remarks

In this paper we considered the problem of estimation of reliability parameter $(R)$ for the inverted exponential distribution and two parameter exponential distribution and derive the interval estimation of reliability parameter $R$. Toward this end, we obtained the $GCI$ on the basis of a generalized pivotal quantity for reliability parameter $(R)$. We compare this method with percentile bootstrap procedure in terms of $CP$ and $EL$. Simulation studies show that the generalized variable method is satisfactory for practical applications even for small sample setting to construct confidence interval for parameter $(R)$.

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