Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/060209

Estimation of Population Mean in Presence of Non-Response in Double Sampling

G. N. Singh, A. Kumar and Gajendra K. Vishwakarma*

Department of Applied Mathematics, Indian Institute of Technology Dhanbad, Jharkhand-826004, India

Received: 9 Aug. 2016, Revised: 22 Apr. 2017, Accepted: 26 Apr. 2017
Published online: 1 Jul. 2017

Abstract: The work done in this article is concerned with the development and efficient estimation procedure of population mean, using information on auxiliary variables at the estimation stage in double sampling. We have proposed a general class of estimators and the properties of the proposed estimator are examined. Theoretical conditions have been made under which our proposed class of estimators are superior to the existing ones. Empirical studies are carried out to assess the behaviours of the proposed class of estimators with respect to the exiting estimator.

Keywords: Double sampling, study variable, auxiliary variable, bias, mean square error, non-response

1 Introduction

It is well established fact that auxiliary information in study of sample survey gives us an efficient estimate of population parameters like as population mean or total, under some crucial conditions. This information may be used for drawing a random sample using simple random sampling without replacement (SRSWOR / SRSWR) simple random sampling with replacement, to stratification, systematic or probability proportional to size sampling strategy or for estimating the population parameter or at both purposes. Auxiliary information gives us a variety of techniques by means of ratio, product, regression and other methods. Incorporating the knowledge of the auxiliary variables is very important for the construction of efficient estimators for the estimation of population parameters and increasing the efficiency of the estimators in different sampling design.

While conducting the sample surveys in the field of agriculture, social sciences and medical sciences, the problem of non response is very common in practice. An estimate obtained from such incomplete data may be misleading especially when the respondents differ from the non-respondents because the estimate can be biased. The problem of estimation of population mean using the technique of sub sampling from non respondents was first introduced by Hansen and Hurwitz [13]. It is well known fact that in sample surveys precision in estimating the population mean may be increased by using information on single or multiple auxiliary variables. Following Hansen and Hurwitz [13] technique, several authors including Cochran [23], Rao [14,15], Khare and Srivastava [5,6,7], Khare and Rehman [1], Okafor and Lee [8] and Tabasum and Khan [17,18] and Singh and Kumar [10] have studied the problem of the estimation procedure of population mean in presence of non-response using information on auxiliary variable. Olkin [12], Mohanty [19], Srivastava [21], Singh and Kumar [11], Khare and Sinha [4], Vishwakarma and Singh [9] and others have made the extension of the ratio method of estimation to the case where multiple auxiliary variables are used to increase the precision of estimates. However, in many situations of practical importance the problem of estimation of population mean of the study variable y assumes importance when the population mean of the auxiliary variable x is not known in presence of non-response. In such a situation the estimate of population mean of the x is furnished from a large first phase sample of size drawn from a population of units by simple random sampling without replacement (SRSWOR). A smaller second phase sample of size n (i.e. $n' < n$) is drawn from by SRSWOR and the variable y under investigation is measured on it. This technique is known as double sampling. Double sampling happens to be a powerful and cost effective (econo- mical) technique for obtaining the reliable estimate in first-phase (preliminary) sample for the unknown

* Corresponding author e-mail: amod.ism01@gmail.com

population parameters of the auxiliary variables. For example, Okafor and Lee [8] and Tabasum and Khan [17] have mentioned that the procedure of double sampling can be applied in a household survey where the household size is used as an auxiliary variable for the estimation of family expenditure. Information can be obtained completely on the family size, while there may be non-response on the household expenditure.

In the present paper, we have proposed a general class of estimators for estimating population mean using auxiliary variable with double sampling in presence of non-response. We have obtained the expressions for bias and mean square errors of the proposed class of estimators for the fixed value of and n, also for the optimum values of the constants. An illustration of the proposed class of estimators has been made with the relevant class of estimators.

2 Notations

The double sampling in presence of non-response sampling scheme is that, let a finite population $U = (U_1, U_2, \cdots, U_N)$ of N units y and x are the variables under study and auxiliary variable respectively with population means $\bar{Y}$ and $\bar{X}$. Let y_k and x_k be the values of y and x for the k-th $(k = 1, 2, \cdots, N)$ unit in the population. If the information on an auxiliary variable x whose population mean $\bar{X}$ is known and highly correlated to y is readily available for all the units of the population, it is well known that regression and ratio type estimators of population mean $\bar{Y}$ could be used for good performance. However, in certain practical situations when population mean $\bar{X}$ is not known, a priori in such case the technique of two-phase sampling is useful. If there is non-response in the second phase sample one may form an estimator by utilizing the information only from the respondents or take a sub-sample of the non-respondents and re-contact them. We assume that at the first phase sample of size n', all the units supplied information on the auxiliary variables x and at the second phase sample of size n, in which n_1 units supply information and n_2 units refuse to respond for study variable y and as well as auxiliary x. Following Hansen and Hurwitz [13] technique of sub-sampling the non-responding group, a sub-sample of size m units $(m = \frac{n_2}{k}, k > 1)$ is selected at random (without replacement) from the n_2 non- respondent units, where k is the inverse sampling rate at the second phase sample of size n. All the m units respond at this time now and the whole population (i.e. U) is supposed to be consisting of two non-overlapping strata of N_1 and N_2 units. Stratum of N_1 responding units (denoted by U_1) would respond on the first call at the second phase and the stratum of N_2 $(N_2 = N - N_1)$ non-responding units (denoted by U_2) would not respond on the first call at the second phase but will respond on the second call. Further, we assume that the strata sizes of N_1 and N_2 are not known well in advance, see Tripathi and Khare [22]. The stratum weights of responding and non-responding groups are given by $(W_1 = \frac{N_1}{N})$ and $(W_2 = \frac{N_2}{N})$ and their estimates are considered as $(\bar{W_1} = \frac{N_1}{N})$ and $(\bar{W_2} = \frac{N_2}{N})$ respectively. Let first and second phase sample be denoted by u' and u respectively and let $u_1 = u \cap U_1$ and $u_2 = u \cap U_2$. The sub-sample of u_2 will be denoted by u_{2m}.

The following are the list of notations, considered for their further use:

$\bar{Y} = \sum_{i=1}^{N} \frac{y_i}{N}$: The population mean of the study variable y.
$\bar{X} = \sum_{i=1}^{N} \frac{x_i}{N}$: The population mean of the auxiliary variable x.
$\bar{Y}_1 = \sum_{i=1}^{N_1} \frac{y_i}{N_1}$: The population mean of the study variable y_1 of the response group.
$\bar{X}_1 = \sum_{i=1}^{N_1} \frac{x_i}{N_1}$: The population mean of the auxiliary variable x_1 of the response group.
$\bar{Y}_2 = \sum_{i=N_1+1}^{N_1+N_2} \frac{y_i}{N_2}$: The population mean of the study variable y_2 of the non-response group.
$\bar{X}_2 = \sum_{i=N_1+1}^{N_1+N_2} \frac{x_i}{N_2}$: The population mean of the auxiliary variable x_2 of the non-response group.
$S_y^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (y_i - \bar{Y})^2$: The population variance of the study variable y.
$S_x^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (x_i - \bar{X})^2$: The population variance of the auxiliary variable x.
$S_{y_1}^2 = \frac{1}{(N_1-1)} \sum_{i=1}^{N_1} (y_i - \bar{Y}_1)^2$: The population variance of the study variable y_1 of the response group.
$S_{x_1}^2 = \frac{1}{(N_1-1)} \sum_{i=1}^{N_1} (x_i - \bar{X}_1)^2$: The population variance of the auxiliary variable x_1 of the response group.
$S_{y_2}^2 = \frac{1}{(N_2-1)} \sum_{i=N_1+1}^{N_1+N_2} (y_i - \bar{Y}_2)^2$: The population variance of the study variable y_2 of the non-response group.
$S_{x_2}^2 = \frac{1}{(N_2-1)} \sum_{i=N_1+1}^{N_1+N_2} (x_i - \bar{X}_2)^2$: The population variance of the auxiliary variable x_2 of the non-response group.
$\rho_{yx} = \frac{S_{yx}}{S_y S_x}$: Correlation coefficient between the variable y and x (i.e. U).

The sample mean $\bar{y}_1 = \sum_{i=1}^{n} \frac{y_i}{n}$ is unbiased for $\bar{Y}_1$, but has a bias equal to $W_2(\bar{Y}_1 - \bar{Y}_2)$ in estimating the population mean $\bar{Y}$.

The sample mean $\bar{y}_{2r} = \sum_{i=1}^{r} \frac{y_i}{r}$ is unbiased for $\bar{Y}_2$ of the n_2 units. Thus, an unbiased estimator for the population mean $\bar{Y}$ is given by:

$$\bar{y}^* = \frac{n_1\bar{y}_1 + n_2\bar{y}_{2m}}{n} \tag{1}$$

and $\bar{x}_{2r} = \sum_{i=1}^{r} \frac{x_i}{r}$ denoted the mean of sub-sample units. An unbiased estimator for the population mean $\bar{X}$ is

$$\bar{x}^* = \frac{n_1\bar{x}_1 + n_2\bar{x}_{2m}}{n} \tag{2}$$

3 Suggested class of estimators

Motivated with the above work, using unknown real constant W_1' and W_2' and one auxiliary variables x, we define a general class of estimators for estimating $\bar{Y}$ as follows.

$$T(w_1', w_2') = \bar{y}^* h_3 \psi(w_1', w_2', h_2(0,1)) \tag{3}$$

where

$\psi(w_1', w_2', h_2(0,1)) = w_1' + w_2' h_2(0,1), \sum_{i=1}^{2} w_i' = 1$

$h_1(\alpha,\beta) = (\frac{\bar{x}'}{\bar{x}^*})^{\alpha}(\frac{\bar{x}'}{\bar{x}})^{\beta}, h_2(\alpha,\beta) = (\frac{\bar{x}^*}{\bar{x}'})^{\alpha}(\frac{\bar{x}'}{\bar{x}})^{\beta}, h_3(\alpha,\beta) = (\frac{\bar{x}'}{\bar{x}^*})^{\alpha}(\frac{\bar{x}}{\bar{x}'})^{\beta}$

$p(\bar{x}', \bar{x}^*, \bar{y}^*) = \frac{\bar{x}' - \bar{x}^*}{\bar{y}^*}$

Now we identify some of the members of the proposed class of estimators present below

$$T_1 = \bar{y}^* h_2(0,1) = \bar{y}^* \frac{\bar{x}'}{\bar{x}} \tag{4}$$

$$T_2 = \bar{y}^* h_2(0,-1) = \bar{y}^* \frac{\bar{x}}{\bar{x}'} \tag{5}$$

$$T_3 = \bar{y}^* h_3(1,0) = \bar{y}^* \frac{\bar{x}'}{\bar{x}^*} \tag{6}$$

$$T_4 = \bar{y}^* h_3(-1,0) = \bar{y}^* \frac{\bar{x}^*}{\bar{x}'} \tag{7}$$

$$T_5 = \bar{y}^* h_3(1,0) h_2(0,1) = \bar{y}^* \frac{\bar{x}'}{\bar{x}^*}\frac{\bar{x}'}{\bar{x}} \tag{8}$$

$$T_6 = \bar{y}^*(1 + b^* p(\bar{x}', \bar{x}^*, \bar{y}^*)) = \bar{y}^* + b^*(\bar{x}' - \bar{x}^*) \tag{9}$$

where $b^* = \frac{s_{ys}^*}{s_x^{*2}}$ is an estimator of population regression coefficient $\beta = \frac{S_{yx}}{S_x^2}$ based on second phase and

$s_{yx}^* = \frac{1}{(n-1)}(\sum_{u_1} x_j y_j + m\sum_{u_{2m}} x_j y_j - n\overline{xy}^*), s_x^{*2} = \frac{1}{(n-1)}(\sum_{u_1} x_j^2 + m\sum_{u_{2m}} x_j^2 - n\overline{xx}^*)$ and

$S_{yx} = \frac{1}{(N-1)}(\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X}))$

It may be noted that the estimators $T_i (i = 1, 2, \cdots, 6)$ are well known ratio, product, regression and chain type estimators in two phase sampling. The estimators (T_1, T_2, T_3, T_6) are first proposed and studied by Khare and Srivastava [5]. The estimator T_1 is revisited by Okafor and Lee [8] and Tabasum and Khan [17]. Further, the estimator T_3 is reconsidered by Tabasum and khan [18] and the estimator T_6 is revisited by Okafaor and Lee [8]. The estimator T_5 is proposed by Singh and Kumar [10] in the presence of non-response.

4 Some estimators of the proposed class

It is also visible to note that a number of chain ratio type, regression type and other estimators fall under the proposed class of estimators $T(w_1', w_2')$ for different choice of weights (w_1', w_2').

(i)Ratio type estimator T_1 proposed by Khare and Srivastava [5], Okafor and Lee [8] and Tabasum and Khan [17]

$$T(1,0) = \bar{y}^* h_3(1,0)\psi(1,0,h_2(0,1)) \tag{10}$$

(ii)Chain-ratio type estimator T_5 proposed by Singh and Kumar [10]

$$T(0,1) = \bar{y}^* h_3(1,0)\psi(0,1,h_2(0,1)) \tag{11}$$

(iii)Motivated by Chakrabarty [16] estimator

$$T(1-\alpha,\alpha) = \bar{y}^* h_3(1,0)\psi(1-\alpha,\alpha,h_2(0,1)) \tag{12}$$

(iv)Suggested by Ray et al. [20]

$$T(1-\alpha,-\alpha) = \bar{y}^* h_3(1,0)\psi(1-\alpha,-\alpha,h_2(0,1)) \tag{13}$$

5 Properties of estimator

The bias and mean square errors (MSE) of the proposed class of estimators $T(w_1', w_2')$ to the first order of approximations are derived under large sample approximations using the following transformations:

$$\bar{y}^* = \bar{Y}(1+e_0), \bar{x}^* = \bar{X}(1+e_1), \bar{x} = \bar{X}(1+e_2), \bar{x}' = \bar{X}(1+e_3)$$

Such that $|e_i| < 1 (i = 0,1,\cdots,3)$. Further, we have the following expectations.

$$E(e_i) = 0, (i = 0,1,\cdots,3), E(e_0^2) = f_1 C_y^2 + W_2 \tfrac{(k-1)}{n} C_{y(2)}^2, E(e_1^2) = f_1 C_x^2 + W_2 \tfrac{(k-1)}{n} C_{x(2)}^2,$$
$$E(e_2^2) = E(e_1 e_2) = f_1 C_x^2, E(e_3^2) = E(e_1 e_3) = E(e_2 e_3) = f_2 C_x^2, E(e_0 e_2) = f_1 \rho_{yx} C_y C_x, E(e_0 e_3) = f_2 \rho_{yx} C_y C_x,$$
$$E(e_0 e_1) = f_1 \rho_{yx} C_y C_x + W_2 \tfrac{(k-1)}{n} \rho_{yx(2)} C_{y(2)} C_{x(2)}$$

where

$$f_1 = \left(\tfrac{1}{n} - \tfrac{1}{N}\right), f_2 = \left(\tfrac{1}{n'} - \tfrac{1}{N}\right), f_3 = (f_2 - f_3) = \left(\tfrac{1}{n} - \tfrac{1}{n'}\right)$$

C_y, C_x: Coefficient of variations of the variables y and x respectively based on the whole population.
$\rho_{yx(2)}$: Correlation coefficients between the variables shown in suffice in the non-response group of the population (i. e. U_2).

Thus, expressing $T(w_1', w_2')$ in terms of e's and neglecting the terms of e's having power greater than two we get

$$\begin{aligned} T(w_1', w_2') = \bar{Y}[R + 1 + W_1'(e_0 - e_1 + e_3 + e_1^2 - e_0 e_1 + e_0 e_3 - e_1 e_3) \\ + W_2'(e_0 - e_1 - e_2 + 2e_3 + e_1^2 + e_2^2 + e_3^2 - e_0 e_1 - e_0 e_2 + 2e_0 e_3 + e_1 e_2 - 2e_1 e_3 - 2e_2 e_3)] \end{aligned} \tag{14}$$

where $R = (W_1' + W_2' - 1)$

Taking expectations on both sides of the equation (14) and using the expectation values, we obtain the expressions for bias $B(T)$ and mean square errors $M(T)$ of the class of estimators to the first order of approximations as

$$\begin{aligned} B(T) = \bar{Y}[R + W_1'(f_3 C_x^2 - f_3 \rho_{yx} C_y C_x + W_2 \frac{(k-1)}{n}(C_{x(2)}^2 - \rho_{yx(2)} C_{y(2)} C_{x(2)})) \\ + W_2'(3 f_3 C_x^2 - 2 f_3 \rho_{yx} C_y C_x + W_2 \frac{(k-1)}{n}(C_{x(2)}^2 - \rho_{yx(2)} C_{y(2)} C_{x(2)}))] \end{aligned} \tag{15}$$

and

$$M(T) = \bar{Y}^2 [W_1'^2 A + W_2'^2 B + 2W_1' W_2' C + 2W_1' RD + 2W_2' RE] \tag{16}$$

where

$$A = (f_1C_y^2 + f_3C_x^2 - 2f_3\rho_{yx}C_yC_x + W_2\tfrac{(k-1)}{n}(C_{y(2)}^2 + C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)}))$$
$$B = (f_1C_y^2 + 4f_3C_x^2 - 4f_3\rho_{yx}C_yC_x + W_2\tfrac{(k-1)}{n}(C_{y(2)}^2 + C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)}))$$
$$C = (f_1C_y^2 + 2f_3C_x^2 - 3f_3\rho_{yx}C_yC_x + W_2\tfrac{(k-1)}{n}(C_{y(2)}^2 + C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)}))$$
$$D = (f_3C_x^2 - f_3\rho_{yx}C_yC_x + W_2\tfrac{(k-1)}{n}(C_{x(2)}^2 - \rho_{yx(2)}C_{y(2)}C_{x(2)}))$$
$$E = (3f_3C_x^2 - 2f_3\rho_{yx}C_yC_x + W_2\tfrac{(k-1)}{n}(C_{x(2)}^2 - \rho_{yx(2)}C_{y(2)}C_{x(2)}))$$

6 Minimum MSE of suggested class of estimators

Since $W_1'(i=1,2)$ are unknown weights and have a specific choice of these yields a particular member of the class $T(w_1', w_2')$, it is desirable to detect that member of the class which has minimum MSE. This can be achieved by minimizing MSE given in equation (16) with respect to the unknown constant $W_1'(i=1,2)$. Differentiating equation (16) with respect to W_1' and W_2' and equating them zero, we have the optimum value of $W_1'(i=1,2)$ as follows

$$(W_1')_{opt} = \tfrac{(1+D)(1+B+2E)-(1+E)(1+C+D+E)}{(1+A+2D)(1+B+2E)-(1+C+D+E)^2} = W_1''$$

and

$$(W_2')_{opt} = \tfrac{(1+E)(1+A+2D)-(1+D)(1+C+D+E)}{(1+A+2D)(1+B+2E)-(1+C+D+E)^2} = W_2''$$

Substituting these optimum values of $W_i''(i=1,2)$ in equation (16), we have minimum MSE of T as

$$min.M(T) = \bar{Y}^2[W_1''^2A + W_2''^2B + 2W_1''W_2''C + 2W_1''RD + 2W_2''RE] \tag{17}$$

7 Efficiencies comparison

In this section we investigate the situations under which our proposed class of estimators $T(w_1', w_2')$ are preferable over the existing estimators such as Hansen and Hurwitz [13] sample mean estimator $\bar{y}^*$, $T_i(i=1,2,\cdots,6)$. The variance $V(.)/MSE$ of these estimators to the first order of approximations are obtained as

$$V(\bar{y}^*) = \bar{Y}^2[f_1C_y^2 + W_2\frac{(k-1)}{n}C_{y(2)}^2] \tag{18}$$

$$M(T_1) = \bar{Y}^2[f_1C_y^2 + f_3C_x^2 - 2f_3\rho_{yx}C_yC_c + W_2\frac{(k-1)}{n}C_{y(2)}^2] \tag{19}$$

$$M(T_2) = \bar{Y}^2[f_1C_y^2 + f_3C_x^2 + 2f_3\rho_{yx}C_yC_c + W_2\frac{(k-1)}{n}C_{y(2)}^2] \tag{20}$$

$$M(T_3) = \bar{Y}^2[f_1C_y^2 + f_3C_x^2 - 2f_3\rho_{yx}C_yC_c + W_2\frac{(k-1)}{n}(C_{y(2)}^2 + C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)})] \tag{21}$$

$$M(T_4) = \bar{Y}^2[f_1C_y^2 + f_3C_x^2 + 2f_3\rho_{yx}C_yC_c + W_2\frac{(k-1)}{n}(C_{y(2)}^2 + C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)})] \tag{22}$$

$$M(T_5) = \bar{Y}^2[f_1C_y^2 + 4f_3C_x^2 - 4f_3\rho_{yx}C_yC_c + W_2\frac{(k-1)}{n}(C_{y(2)}^2 + C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)})] \tag{23}$$

and

$$M(T_6) = \bar{Y}^2[f_1C_y^2 - f_3\rho_{yx}^2C_x^2] + W_2\frac{(k-1)}{n}[S_{y(2)}^2 + \beta_{yx}S_{x(2)}^2(\beta_{yx} - 2\beta_{yx(2)})] \tag{24}$$

It may be noted that the expression of the MSE of T shown in equation (17) is quite difficult. However, the performance of the proposed class of estimators is examined through empirical study over different population which established the superiority over the traditional ones.

8 Empirical study

To see the performance of the proposed class of estimators of the population mean, we consider three natural dataset of the variables y and x and the values of the various parameters are given as follows.

Population I- Source: [Khare and Sinha [2]]
The data belongs to the data on physical growth of upper-socio-economic group of 95 school children of Varanasi under an ICMR study, Department of Pediatrics, BHU during 1983-1984 has been taken under study. The first %25 (i.e. 24 children) units have been considered as non-response units. The values of the parameters related to the study variable y (the weight in kg) and the auxiliary variable x (the skull circumference in cm) have been given below. It is to be noted that this population was also considered by several authors including Singh and Kumar [11].

$N = 25, \overline{Y} = 19.49, \overline{X} = 51.17, C_y = 0.15, C_X = 0.03, C_{y(2)} = 0.12, C_{x(2)} = 0.02,$
$\rho_{yx} = 0.32, \rho_{yx(2)} = 0.47, N_2 = 24, W_2 = 0.25.$

Population II- Source: [District Cencus Handbook, 1981, Orissa, Published by Govt. of India]
The 109 Village / Town / Ward wise population of urban area under Police-station-Baria, Tahasil-Champua, Orissa, India has been taken under study. The last %25 villages (i. e. 27 villages) have been considered as non-response group of the population. The study variable (y) is number of literate persons in the village while the number of main workers in the village is considered as auxiliary variable (x). This population was also considered as numerical evidence in the works of several authors including Khare and Sinha [4].

$N = 109, \overline{Y} = 145.30, \overline{X} = 165.26, C_y = 0.76, C_X = 0.68, C_{y(2)} = 0.68, C_{x(2)} = 0.057,$
$\rho_{yx} = 0.81, \rho_{yx(2)} = 0.78, N_2 = 28, W_2 = 0.25.$

Population III- Source: [District Cencus Handbook, 1981, West Bengal, Published by Govt. of India]
Ninety-six village wise population of rural area under Police-station-Singur, District-Hooghly, West Bengal has been taken under the study. The %25 villages (i.e. 24 villages) whose area is greater than 160 hectares have been considered as non-response group of the population. The number of agricultural labours in the village is taken as study variable (y) while the area (in hectares) of the village is taken as auxiliary variables (x). It is to be noted that this population was also considered by Khare and Sinha [3].

$N = 96, \overline{Y} = 137.92, \overline{X} = 144.87, C_y = 1.32, C_X = 0.81, C_{y(2)} = 2.08, C_{x(2)} = 0.094,$
$\rho_{yx} = 0.77, \rho_{yx(2)} = 0.72, N_2 = 24, W_2 = 0.25.$

Here we have computed the percentage relative efficiency (PRE) of propose estimator and other exiting Hansen and Hurwitz [13] sample mean estimator$\overline{y}^*, T_1, T_3$ and T_5 with respect to usual unbiased estimator $\overline{y}^*$.

$$PRE = \frac{V(\overline{y}^*)}{M(\iota)} \times 100 where \iota = (\overline{y}^*, T_1, T_3, T_5, T) \tag{25}$$

9 Interpretations of results

The following interpretations may be read out from Table 1

(i)For all population I, II and III, the PRE of estimator T_1 decreasing for different choice of the sub-sampling fraction $(\frac{1}{k})$ and sample size (n, n').

(ii)For increase value of the sub-sampling fraction $(\frac{1}{k})$ and sample size (n, n'), the PRE for population I, II and III of estimators T_3, T_5 and T are increasing.

(iii)It is also visible that the estimator T_5 in population-II do not gain efficiency for the value $k = 2$ at $n' = 85$.

10 Conclusions

From above analyses, it is clear that the proposed class of estimators T contribute significantly to handle the different realistic situations of non-responses while estimating population mean in double sampling. It is visible that the proposed class of estimators is more efficient than the other existing estimators under the similar realistic situations.

Table 1: PRE of the different estimators with respect to $\bar{y}^*$

Population I											
		$\bar{y}^*$	T_1	T_3	T_5	T	$\bar{y}^*$	T_1	T_3	T_5	T
n	k	n'=65					n'=75				
30	2	100	106.66	108.11	108.78	109.37	100	107.59	108.88	109.65	110.32
	3	100	106.04	108.67	109.29	109.84	100	106.95	109.32	110.03	110.67
	4	100	105.52	109.15	109.72	110.24	100	106.41	109.7	110.37	110.97
35	2	100	105.99	107.76	108.36	108.89	100	107.11	108.7	109.42	110.04
	3	100	105.32	108.47	109.02	109.5	100	106.39	109.25	109.91	110.49
	4	100	104.78	109.05	109.55	110	100	105.8	109.72	110.32	110.86
40	2	100	105.25	107.37	107.9	108.36	100	106.58	108.5	109.17	109.74
	3	100	104.56	108.26	108.73	109.14	100	105.79	109.18	109.78	110.3
	4	100	104.02	108.96	109.38	109.76	100	105.17	109.73	110.27	110.75
Population II											
		$\bar{y}^*$	T_1	T_3	T_5	T	$\bar{y}^*$	T_1	T_3	T_5	T
n	k	n'=75					n'=85				
55	2	100	136.88	168.84	101.24	170.39	100	157.85	196.8	*	199.06
	3	100	128.09	179.99	113.97	181.66	100	143.59	204.62	107.04	206.87
	4	100	122.68	188.54	124.72	190.33	100	134.97	210.53	116.93	212.82
60	2	100	127.04	159.17	106.31	160.38	100	147.71	187.86	*	189.72
	3	100	120.27	172.49	120.9	173.84	100	135.26	197.7	112.56	199.59
	4	100	116.22	182.51	132.85	184	100	127.96	204.97	123.68	206.91
65	2	100	117.63	149.64	112.58	150.55	100	137.79	178.75	103.96	180.24
	3	100	113.02	165.26	129.15	166.32	100	127.4	190.85	119.1	192.4
	4	100	110.33	176.78	142.23	178.01	100	121.5	199.56	131.44	201.19
Population III											
		$\bar{y}^*$	T_1	T_3	T_5	T	$\bar{y}^*$	T_1	T_3	T_5	T
n	k	n'=50					n'=60				
30	2	100	127.31	159.09	143.21	166.65	100	139.82	173.52	149.52	182.14
	3	100	118.86	164.19	151.37	172.59	100	127.75	175.17	155.89	184.02
	4	100	114.41	167.34	156.61	176.75	100	121.29	176.2	160.11	185.61
35	2	100	118.96	151.68	140.63	157.74	100	131.13	166.53	147.58	173.53
	3	100	112.78	159.05	150.25	166.12	100	121.09	170.08	155.15	177.49
	4	100	109.64	163.43	156.14	171.62	100	115.95	172.2	159.9	180.3
40	2	100	111.76	145.05	138.2	149.93	100	123.46	160.08	145.69	165.77
	3	100	107.77	154.66	149.25	160.69	100	115.51	165.62	154.46	171.9
	4	100	105.8	160.18	155.74	167.43	100	111.58	168.81	159.71	175.89

Acknowledgement

The authors are very much thankful to the Indian Institute of Technology(ISM), Dhanbad. Authors are also thankful to the anonymous referee for his valuable suggestions that improved this paper.

References

[1] B.B. Khare and H.U. Rehman, International Journal Statistics Economics **15**, 64-68 (2014).
[2] B.B. Khare and R.R. Sinha, Sampling Theory and Quality Control **1**, 63-171 (2007).
[3] B.B. Khare and R.R. Sinha, Statistics in Transition-new series **10**, 3-14 (2009).
[4] B.B. Khare and R.R. Sinha, Statistica **49**, 75-83 (2012).
[5] B.B. Khare and S.R. Srivastava, National Academy of Science Letters India **16**, 111-114 (1993).
[6] B.B. Khare and S.R. Srivastava, Proceedings of National Academy of Science India **65**, 195-203 (1995).
[7] B.B. Khare and S.R. Srivastava, Communication in Statistics- Theory Methods **26**, 1779-1791 (1997).
[8] F.C. Okafor and H. Lee, Survey Methodology **26**, 183-188 (2000).
[9] G.K. Vishwakarma and R. Singh, Journal of Computations and Mdelling **6**,135-149 (2016).
[10] H.P. Singh and S. Kumar, Statistical Papers **51**, 559-582 (2010 a).
[11] H.P. Singh and S. Kumar, Journal of Statistical Planning and Inference **140**, 2536-2550 (2010 b).
[12] I. Olkin, Biometrika **45**, 154-165 (1958).
[13] M.H. Hansen and W.N. Hurwitz, Journal of American Statistical Association **41**, 517-529 (1946).
[14] P.S.R.S. Rao, Survey Methodology **12**, 217-230 (1986).
[15] P.S.R.S. Rao, Paper presented at a special contributed session of the International Statistical Association Meeting **Sept. 2-16** Tokyo, Japan (1987).
[16] R.P. Chakrabarty, Contribution to the theory of ratio-type estimators, Unpublished Ph.D. Thesis, Texas A and M University, College Station, U. S. A, (1968).
[17] R. Tabasum and I.A. Khan, Journal of the Indian Society of Agricultural Statistics **58**, 300-306 (2004).
[18] R. Tabasum and I.A. Khan, Assam Statistical Review **20**, 73-83 (2006).
[19] S. Mohanty, Journal of Indian Statistical Association **5**, 1-14 (1967).
[20] S.K. Ray, A. Sahat and A. Sahai, Annals of the Institute of Mathematical Statistics **31**, 141-144 (1979).
[21] S.K. Srivastava, Journal of American Statistical Association **66**, 404-407 (1971).
[22] T.P. Tripathi and B.B. Khare, Communications in Statistics - Theory and Methods **26**, 2255-2269 (1997).
[23] W.G. Cochran, Sampling Techniques (3rd ed.), John Wiley and Sons, New York, (1977).

G. N. Singh is a Professor of Statistics in the Department of Applied Mathematics, Indian Institute of Technology (ISM) Dhanbad, India. He obtained his Ph.D. degree in 1990 from Banaras Hindu University, Varanasi, India. He has more than 27 years of teaching experience in the field of statistics. He served as faculty in Panjab University, Chandigarh and Kurukshetra University, India. He has more than 30 years of research experience in the various field of Statistic which covers Sample Surveys, Statistical Inference, Data Analysis, Data Mining etc. He has published number of research papers in Indian and Foreign journals of repute. He presented his research problems in international and national conferences and delivered various invited talks in academic forum. He has produced 12 Ph.D, 5 M.phil and 4 research projects.

Amod Kumar is a research scholar in the Department of Applied Mathematics, Indian Institute of Technology (ISM) Dhanbad, India. He is pursing Ph.D. in Applied Statistics. His research interest is in the areas of Sample Survey and Statistical Inference.

Gajendra Kumar Vishwakarma is an Assistant Professor of Statistics in the Depart of Applied Mathematics, Indian Institute of Technology (ISM) Dhanbad, India. Gajendra Kumar Vishwakarma is an Assistant Professor of Statistics in the Department of Applied Mathematics, Indian Institute of Technology (ISM) Dhanbad, India. He obtained his Ph.D. degree in 2007 from Vikram University, Ujjain, India. He has several years of academic as well as industrial research experience in the field of applied statistics. His research experience covers both applied as well as theoretical provinces. He served as visiting scientist cum faculty in Indian Statistical Institute, North-East Centre, Tezpur (Assam), India and as an Associate Scientist in Lupin Research Park, Pune, India. He is elected Fellow of the Society of Earth Scientists, India, an Elected Member of International Statistical Institute, Netherlands and member Fellow of Royal Statistical Society, UK. He is member of the advisory boards and editorial board member of several journals. He has published number of research papers in international journal reputes. He presented his research problems in international and national conferences and delivered various invited talks at industrial as well as academic forum. He received Young Scientist Award from Center for Advanced Research and Design, Chennai, India.