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Experimental Analysis of Fractional PID Controller Parameters on Time Domain Specifications

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Abstract: A fractional PID ($PI^{\lambda}D^{\mu}$) controller is an extension of the classical PID controller, employing five tuning parameters rather than just three. General guidelines are available for the effect of classical controller parameters on the time domain specification. However, no guidelines are available for fractional PID controllers, particularly for the order of differentiation (μ) and integration (λ). To assist with fine tuning, the effect of the order of differentiation and integration parameters on the time domain specifications for various plants are investigated. The relationship with the time domain specification serves as general guideline for manual tuning, and the effect of parameters will also assist with auto-tuning. In this paper, five plants covering integer order as well as non-integer order are simulated. The relationship between time domain specifications is plotted by varying the order of differentiation and integration between 0 and 2. Simulation results have revealed an association between the order of differentiation (μ) and the maximum overshoot (M_P) for all plants. No other particular behavior was observed with other time domain specifications. However, some remarks on time domains specifications are made from the simulation results. Simulation results were validated using an experimental set up of the quadruple tank system.

Keywords: Fractional PID controller,fractional calculus, effect of parameters, fractional order controller, auto tuning of fractional PID controller.

1 Introduction

The prospects of fractional calculus continue to get brighter. The applications of fractional calculus in control systems include the designing of fractional PID controllers and the modeling of plants using fractional differentiation equations. Fractional PID controllers concern an area of research that has been receiving growing attention [1,2,3,4,5,6]. A fractional PID controller is an extension of the classical PID controller and encompasses two additional parameters, namely the order of differentiation (μ) and integration (λ), which are not found in the classical PID controllers. These two extra parameters enable the fractional PID controller to improve the performance of the system.

A fractional PID controller is recognized to provide robust performance [7,8,9,10,11]; secures five different types of objectives; and offers better results for fractional and integer-order plants [13]. Many real systems can be modelled more accurately using fractional order systems , for instance electrical circuits, electro-analytical chemical analysis, and nuclear reactors [12], as well as many physical phenomena [13].

The current work sought to study the relationship between different parameters of a fractional PID controller and the specifications related to the time domain for the order of differentiation and integration. Knowledge of such relationships will facilitate in tuning a fractional-order PID controller both manually and automatically. The relationships between different parameters based on tuning parameters in the case of the classical PID controller are shown in Table 1 [14,15], which serves as a general guideline for fine tuning and works with most plants.

Tuning of any controller is always a challenging task [16]. Although many auto-tune algorithms are currently available for designing classical and fractional PID controllers [17], yet it is necessary to fine-tune a controller. In practice, the tuning of any controller needs to be followed by fine tuning. Even in a model-based control design, the performance of the system depends on the accuracy of the model; if it is not accurate enough, fine tuning is necessary. For fine-tuning a classical

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Table 1: Effects of increasing a parameter independently.

Parameter	Rise Time T_r	Overshoot M_P	Settling Time T_{ss}	Steady State Error E_{ss}	Stability
K_P	Decrease	Increase	Small change	Decrease	Degrade
K_I	Decrease	Increase	Increase	Eliminate	Degrade
K_D	Minor change	Decrease	Decrease	No effect in theory	Improve if K_D is small

PID controller, general guideline regrading the effect of parameters is available (Table 1): but for fine-tuning a fractional PID controller, no such guidelines are available, although many heuristic methods for tuning have been developed for that purpose. The summary of different tuning methods for fractional PID controller was presented by D. Valerio and J. Costa in 2010 [18]. The current work was designed to find out the effect of fractional-order parameters on time-domain specifications in the case of a fractional PID controller.

The effect on the order of differentiation (μ) and integration (λ) (from 0 to 2) was ascertained experimentally as well as by using a simulation. The simulation studied five different plant order systems, namely first order, second order, higher order, fractional order and first order delay time, covering different dynamics of various plants. Initially, these plants were tuned based on the Nelder Mean optimization approach. The Nelder Mean optimization method is based on the concept of simplex approach (sort, reflection, expansion, contraction, shrink). Afterwards, the order of differentiation and integration was raised in steps of 0.1 and the results were plotted for all the five plants.

To validate the results of the simulation, the effect of the fractional-order parameters was analyzed experimentally using a quadruple tank system. This set-up was connected to MATLAB/Simulink by the Open Platform Communication (OPC) protocol. Only one controlled variable was considered. Using FOMCON, a *fractional order modeling and control tool*, a fractional PID controller was implemented in real time.

The simulation results reveal the specific relationship between maximum overshoot and the order of differentiation (μ). There exists no straightforward relationship with other time domain specifications. However, following points are observed in this study:

– There exists a particular relationship between μ and maximum overshoot (M_P).
– By changing the values of λ and μ, the time domain specifications can be further improved which is advantage of the fractional PID controller.
– For a fractional order model, the influence is almost same for different values of λ on time domain specifications.
– The settling time goes worst as μ approaches 2 for integer order system.

Maximum overshoot is an important characteristic of a control system. For many critical systems such as pressure, even a small overshoot can be dangerous. However, as shown in the current paper, this maximum overshoot can be varied using the order of differentiation of a fractional PID controller. For optimization of the controller, maximum overshoot can be used as a measure of performance.

This paper is organized as follows. In Section 2, basics of fractional calculus and fractional order controller are covered at the elementary level. The different types of tuning methods are also listed out in this section. In Section 3, design for simulation work is specified. This simulation work is validated using experimental set up by the quadruple tank system. Results and discussions are presented in Section 4. In Section 5, conclusions are presented for this work. Finally, references are given at the end.

2 Basics of Fractional Calculus and Fractional PID Controller

2.1 Fractional calculus

Fractional calculus, although it predates classical calculus by more than 300 years, is rarely appreciated in research [19]. However, over the last few decades, many researchers have explored the applications of fractional calculus in different areas including control systems, speech signal processing, process modeling, chaos, and fractals [7,10,20].

In fractional calculus, ${}_aD_t^\alpha$, the differentiation integration operator, is defined as follows [21]:

$$ {}_aD_t^\alpha = \begin{cases} \frac{\mathrm{d}^\alpha}{\mathrm{d}x^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_a^t (d\tau)^{-\alpha} & \alpha < 0 \end{cases} \tag{1} $$

where α is the order of the operator and $\alpha \in \mathbb{R}$. The theory of fractional calculus is dogged by some controversy, as a consequence of which fractional calculus is defined in many different ways. The relevant definitions are briefly described below.

2.1.1 Caputo definition

The Caputo definition is extensively used in engineering [1,22,23], as the definition offers a straightforward association between the type of initial conditions and fractional operator. A derivative of the constant is bounded in the case of the Caputo definition, which is given by

$$ {}_aD_t^\alpha = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{2} $$

where n is an integer number, which satisfies the condition $(n-1) \leq \alpha \leq n$, α is a real number, and a and t are the limits of integration. For example, if α is 0.8, then n would be 1 because $0 \leq 0.8 \leq 1$.

2.1.2 Riemann-Liouville definition

The Riemann Liouville (RL) fractional definition is given by the following equation

$$ {}_aD_t^\alpha = D^n J^{n-\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{3} $$

where n is an integer number, which satisfies the condition $(n-1) \leq \alpha \leq n$, α is a real number, J is the integral operator, and a and t are the limits of integration.

2.1.3 Grunwald-Letnikov definition

The Grunwald Letnikovs (GL) fractional definition is defined as

$$ {}_aD_t^\alpha = \lim_{h\to 0} \frac{1}{h^\alpha} \sum_{r=0}^{\left[\frac{t-a}{h}\right]} (-1)^r \binom{n}{r} f(t-rh) \tag{4} $$

where n is an integer number, which satisfies the condition $(n-1) \leq \alpha \leq n$, a and t are the limits of differentiation, h is the step size for differentiation, $[\frac{t-a}{h}]$ is integer part and $\binom{n}{r}$ is the binomial coefficient.

2.2 Fractional PID controller

The fractional-order controller was introduced by I. Podlubny for fractional-order systems [13,24,25]. I. Podlubny demonstrated that a fractional-order controller had a better response than an integer-order controller for a fractional-order plant. The beauty of well tuned fractional PID controller is that it is less sensitive to changes in the variables of the controlled system and the controller [11]. This type of controller makes it possible to adjust greater number of system dynamics. Many researchers confirm that fractional-order controllers outperform classical PID controllers in many applications [16,26,27].

A fractional PID controller has five parameters for tuning, as shown in Eq. (5). Fig. 1 shows a block diagram of the fractional PID controller, which has the following structure [28,27]:

$$C(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s^{\lambda}} + K_D s^{\mu}; (0 \leq \lambda, \mu \leq 2) \tag{5}$$

where $C(s)$ is the controller transfer function, $U(s)$ is the Laplace of control signal, $E(s)$ is the Laplace of error signal, K_P is the proportional constant gain, K_I is the integration constant gain, K_D is the derivative constant gain, λ is the order of integration and μ is the order of differentiator. A fractional PID controller becomes a PID controller if $\lambda = \mu = 1$ as shown in the Fig. 2.

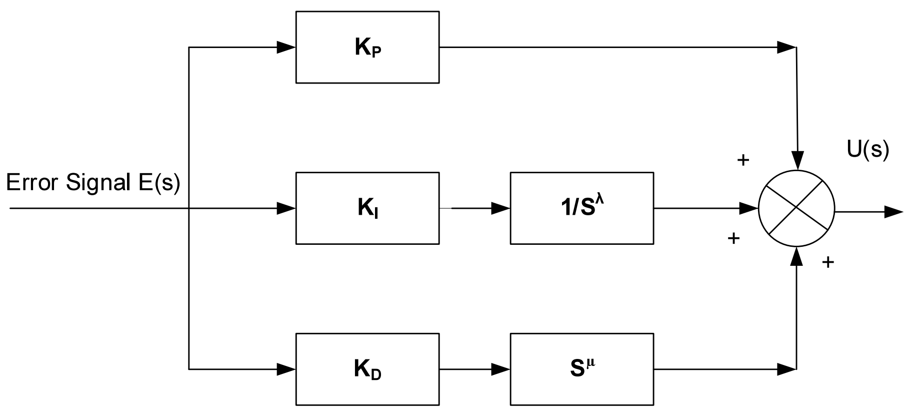


Fig. 1: Block diagram of fractional PID controller.

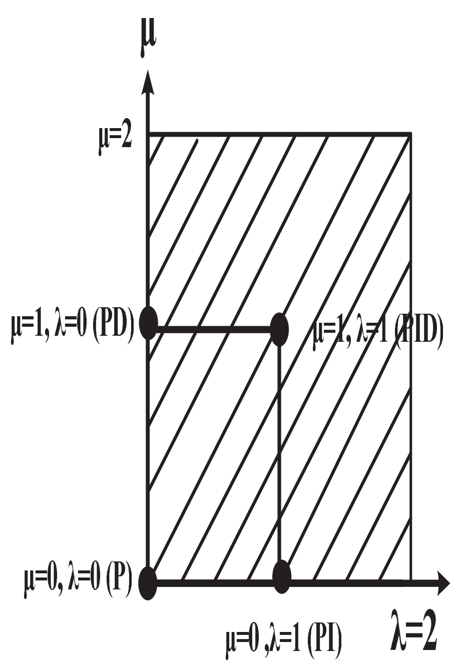


Fig. 2: The fractional PID controller plane.

A fractional PID controller is also implemented in real-time applications using analog as well as digital approximation methods. In most cases, the orders of the fractional PID controller lie between 0 and 2. Many variations of the fractional-order controller have been investigated, including $(PI)^{\lambda}$, non-liner fractional PID controller, TID (tilted proportional and integral), and CRONE controller (Commande Robuste d'Ordre Non Entier, meaning non-integer order robust control).

Tuning a fractional PID controller [29] is harder than that of a classical PID controller, because as the former offers more parameters. Tuning methods can be numerical, analytical, or rule based. Tuning methods based on optimization, such as genetic algorithm, adaptive genetic algorithm, enhanced particle swarm optimization, and M_S (peak value of sensitivity function) constrained integral optimization (MIGO) fall under the category of numerical methods. In analytical methods, the parameters of a controller are obtained by solving equations, which are calculated with the help of the desired specifications. In rule-based methods, a modified version of the Ziegler Nichols technique has been developed for tuning a fractional PID controller. Apart from the above methods, internal model based (IMC) and auto-tuning methods are also used for tuning a fractional-order controller [17]. The review of different tools associated to fractional calculus and control can be found in [30,31].

3 Simulation and Experiment Work

3.1 Simulation work

Five different systems were simulated to study the relationships between the order of fractional parameters and the time-domain specifications by varying the order of fractional parameters in the fractional PID controller. As mentioned earlier, the plants were of first order, second order, higher order, fractional order systems, and first order system with delay time system (FOPDT). The higher-order plant was described by H. Panagopoulos in 2002 [32], whereas the fractional-order plant was described by I. Podlubny in 1994 [24]. The general structure of the first order delay time and second-order systems was considered for the simulations.

$$P_1(s) = \frac{5}{4s+1} \tag{6}$$

$$P_2(s) = \frac{1}{(3s+1)(10s+1)} \tag{7}$$

$$P_3(s) = \frac{1}{s(s+1)^3} \tag{8}$$

$$P_4(s) = \frac{1}{0.8s^{2.2}+0.5s^{0.9}+1} \tag{9}$$

$$P_5(s) = \frac{2}{5s+1}e^{-3s} \tag{10}$$

Time-domain specifications can be divided into two categories, namely transient performance and steady-state performance. Both were covered in the simulation. Rise time, peak time, settling time, and maximum overshoot were perceived as performance parameters for evaluating the effects of the fractional-order parameters. Time-domain specifications are relevant parameters in designing control systems, and are are frequently considered as performance indices even for the optimization of controllers [16].

The optimization approach used for tuning the plants is shown in Fig. 3. The Nelder Mead method was used for the simulation [33] for optimizing the parameters of the fractional PID controller. This method finds out minimum of a function from more than one independent variables without using derivatives. A simplex has $n+1$ points in n dimensional space, which represents the number of independent variables. For tuning of fractional PID controller, the integrated square error (ISE) was chosen as the performance index. This measure is more useful because the range of error was large in most cases and was thus more appropriate for designing the controller. . The integrated square error is defined as follows,

$$ISE = \int_0^t e^2(t)\,\mathrm{dt} \tag{11}$$

where $e(t)$ is the error signal, and it is given for unity feedback system considering unit step input,

$$e(t) = 1 - L^{-1}\left(\frac{1}{s}\frac{G_P(s)G_C(s)}{1+G_P(s)G_C(s)}\right) \tag{12}$$

Note: $L^{-1}\{F(s)\}$ represents the inverse Laplace transform of $F(s)$. In this case, the number of independent variables (n) is five. Firstly, initial simplex is generated for six points. Now, the cost function (ISE) is calculated for all points. Then, all the points are sorted based on cost function. In this space, the first point is considered best solution and last point as the worst solution. Finally, the algorithm iteratively updates the worst point by four possible actions: reflection, expansion, contraction, and multiple contraction. The optimal solution could be found by iterating the above steps.

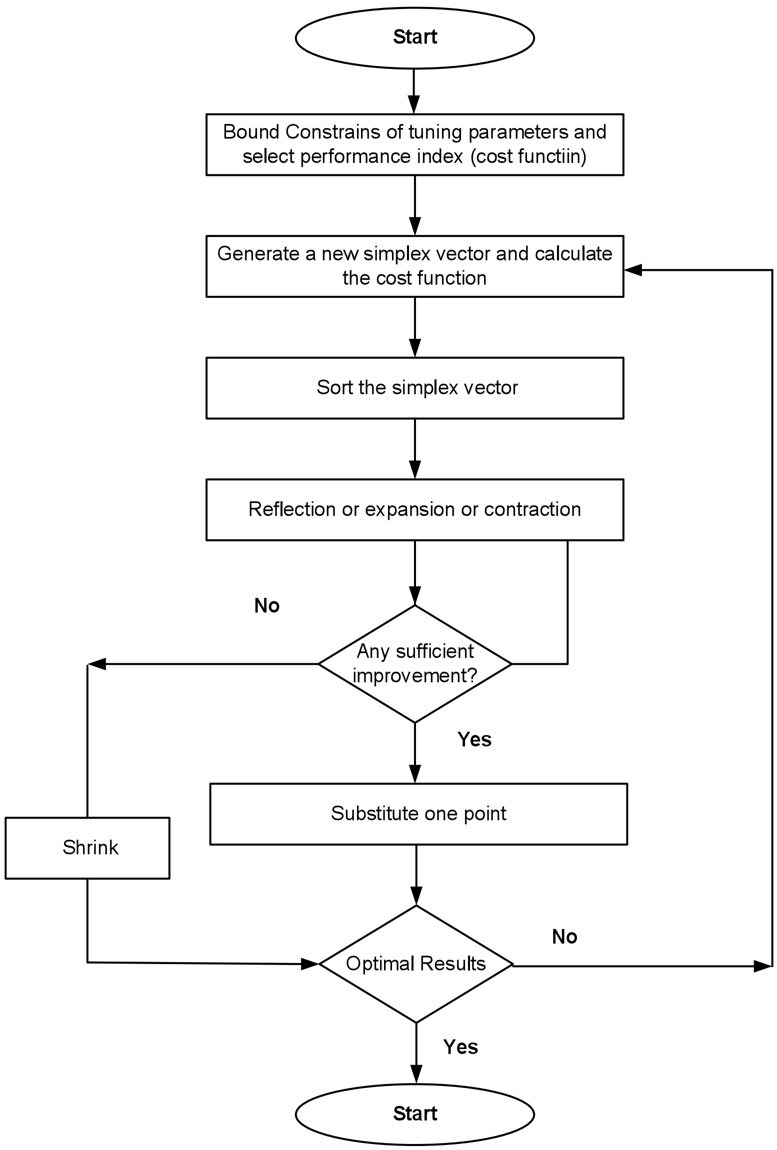


Fig. 3: Optimization approach for design of fractional PID controller.

Calculation of the time-domain specifications for a fractional-order system is a little tricky as the *stepinfo* function is valid only for integer systems. Here, the time-domain specifications for the fractional PID controller were calculated using FOMCON toolbox as follows.

–Define a plant transfer function in a fractional transfer function object ($G(s)$).

$$G(s) = \text{fotf}(BPOLY, APOLY)$$

For plant 4, $G(s) = \text{fotf}(\text{‘}1', \text{‘}0.8s^{2.2} + 0.5s^{0.9} + 1')$

–Based on the parameters of the fractional PID controller, create a fractional-transfer function of the controller ($C(s)$).

$$C(s) = \text{fracpid}(K_P, K_I, \lambda, K_D, \mu);$$

–Find out the closed-loop fractional-order transfer function.

$$\text{Closed loop transfer function } Gcl(s) = \frac{G(s)C(s)}{1+G(s)C(s)}$$

–Obtain an integer-order approximation of the fractional-order system using Oustaloups method (refer equation 13).

$$\text{sys\_int} = \text{ oustapp(Gcl, wb, wh, N)}$$

where *wb* and *wh* indicate the range of frequency for approximation, and N is the order of the approximation.

–Get the specifications of the time domain using the *stepinfo* function of MATLAB.

$$\text{S} = \text{stepinfo(sys\_int)}$$

The fractional-order system can be approximated by many methods [34,35,36], out of which the Oustaloup recursive approximation is the most popular [26,37,38], and the best approximation order (N) can be found by formula given by F. Merrikh-Bayat [39]:

$$s^{v} \approx K \prod_{k=-N}^{N} \frac{1+s/\omega_k}{1+s/\omega_k'} \tag{13}$$

For the above steps, the FOMCON toolbox was used for creating the fractional-order system. The toolbox, which is based on a fractional-order calculus, is used for designing control systems as well as modeling fractional-order systems [40,41,42].

3.2 Experiment validation using a quadruple tank system

A quadruple tank system is a non-linear as well as multivariable control system and contains four tanks and two pumps. Only one control variable, namely the level of lower tank h_2, was considered in the current experiment. This variable was controlled by adjusting the speed of the pump (υ_2). Pump 1 feeds tanks 1 and 4, and pump 2 feeds tanks 2 and 3 (Fig. 4). Fig. 5 exhibits a photograph of the plant. Different specifications of the quadruple tank system are shown in Table 2.

Table 2: Constants for experimental set-up.

Constant	Description	Value
A_i	Cross section area of tank i	196 cm^2
a_i	Cross section area of the outlet hole(for tank i)	0.64 cm^2
g	Acceleration due to gravity	981 cm/s^2
k_i	Pump flow constants	3.3 cm^3/sV

The quadruple tank system is connected to the OPC protocol. This protocol allows real-time plant data to be shared between control devices from different manufacturers of programmable logic controllers (PLC). Using the OPC protocol, data can be read and written in milliseconds. In Simulink, an OPC client can be configured with a local or a remote host, depending on the location of the OPC server. For reading and writing operations, the OPC read-and-write block of Simulink is used with an appropriate tag as configured in the OPC server. The schematic block diagram of the experimental setup is shown in Fig.6. The speed of motor (υ_2) is driven by output of the variable frequency drive (VFD). The output of level sensor is given to PLC, which is logged into OPC server and fetched to Simulink. Similarly, the output of the fractional PID controller is sent to PLC through OPC server and is given to manipulated variable VFD.

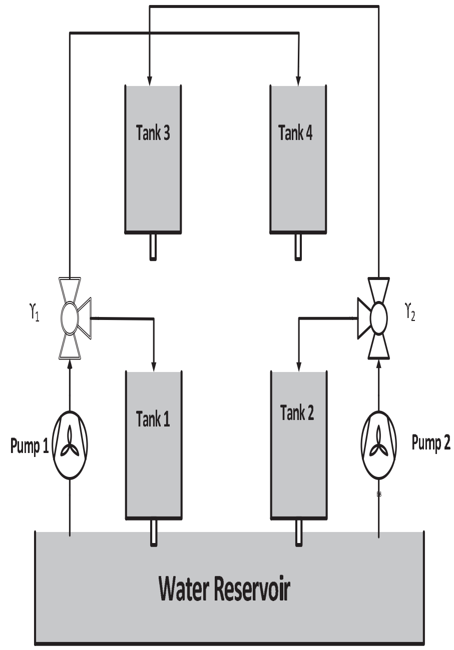


Fig. 4: A schematic diagram of the quadruple tank.

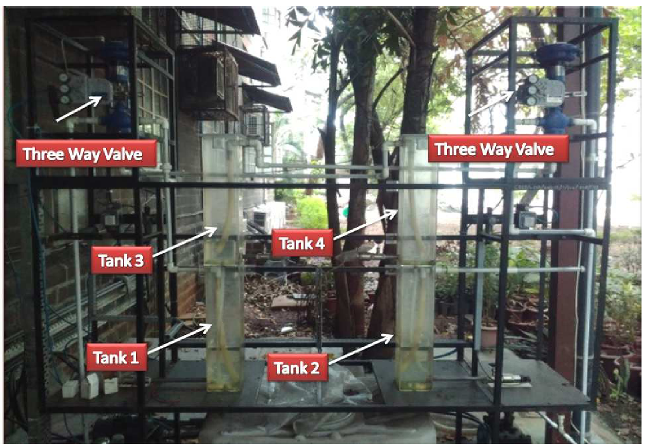


Fig. 5: Experiment set up quadruple tank plant.

4 Results and Discussions

4.1 Results of simulation

Initially, the plants were tuned using the Nelder Mean optimization method for fractional PID controller (the results are summarized in Table 3). The FOMCON toolbox was used for designing and tuning of fractional PID controller [43]. For each plant, the effect of λ and μ for time-domain specifications are plotted as bar plots to perceive the particular relationship with various time domain specifications. The following constraints were considered for the optimization:

$$0 \leq \lambda, \mu \leq 2 \text{ and } 0 \leq K_P, K_I, K_D \leq 1000 \tag{14}$$

The maximum overshoot was associated to the order of differentiation in all the plants as shown Figs. (d) of 8, 10, 12, 14 and 16. As the order of differentiation increases, the value of maximum overshoot decreases initially but starts

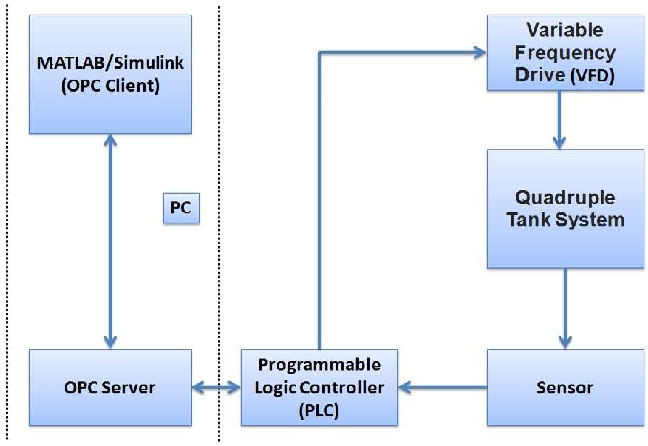


Fig. 6: Schematic block diagram of the experimental setup.

to increase after a certain point. If the objective of the controller is to minimize the overshoot, such a result will help in tuning fractional PID controllers.

Table 3: Fractional PID controller tuning parameters.

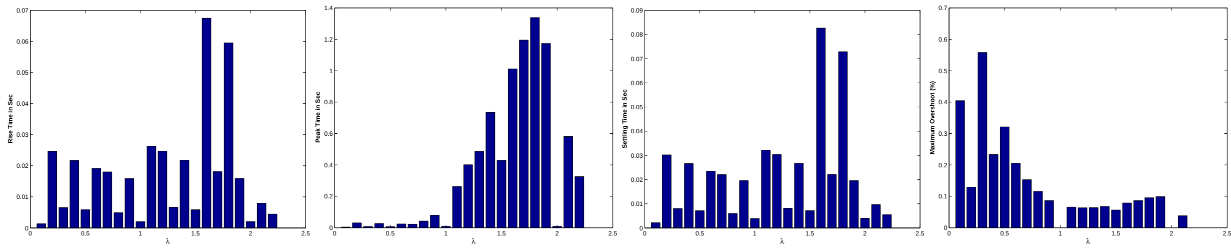
Plant	Fractional order parameters				
	K_P	K_I	λ	K_D	μ
$P_1(s)$	703.3	984.68	0.78455	84.544	0.11691
$P_2(s)$	987.87	43.702	0.18499	999.92	1.0637
$P_3(s)$	3.4415	0.1	1.0763	6.6299	1.7424
$P_4(s)$	92.141	549.14	0.84797	392.21	1.1767
$P_5(s)$	1000	1000	0.5273	1000	1.1279

Also, the settling time of a system approaching order 2 of the differentiation (μ) was longer for an integer order system, as shown in Figs. (c) of 8, 10, 12 and 16. For a fractional order system, the effect on time domain specifications by changing the order of integration (λ) is almost the same (refer Fig. 13). The effect on time domain specifications by varying the order of integration (λ) is shown in Figs. 7, 9, 11, 13 and 15. There exists no particular relationship between order of integration and time domain specifications. However, by changing the values of λ and μ, the time domain specifications can be further improved, which is advantage of the fractional PID controller.

4.2 Results of quadruple tank system

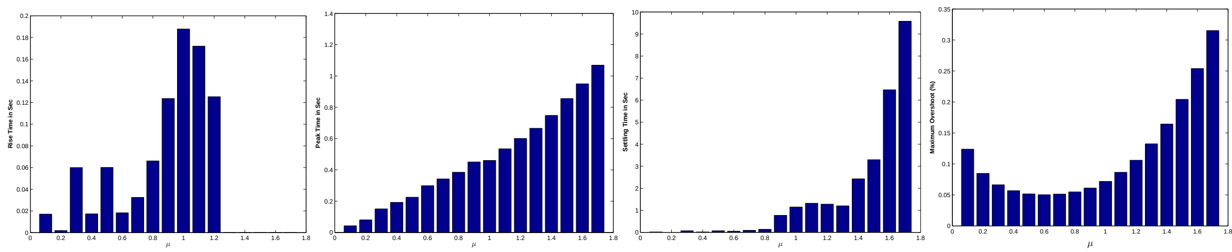
The effect of the order of differentiation on the quadruple tank system is shown in Fig. 17, and the step response for different values of μ is shown in Fig. 18 for set point of level 35 cm. The range of μ in the experimental set-up was from 0.1 to 1.2. The tuning parameters are shown in Table 4, which is obtained by process model of the experimental set-up, and is fine tunned for better responses. From Fig. 17, it is evident that the maximum overshoot is minimum for 0.4 order of differentiation.

The quadruple tank system also showed the same relationship between the order of differentiation and maximum overshoot as that obtained by the simulation. This result can be useful for automatic and manual tuning of fractional order controllers.



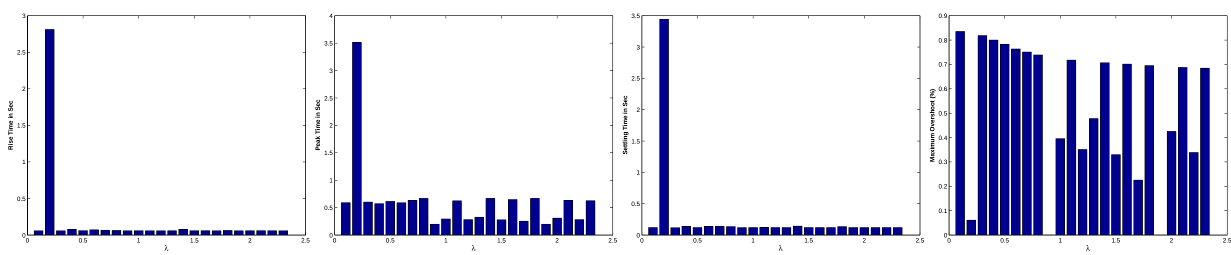
(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 7: Effect of λ on different specifications (Plant 1).



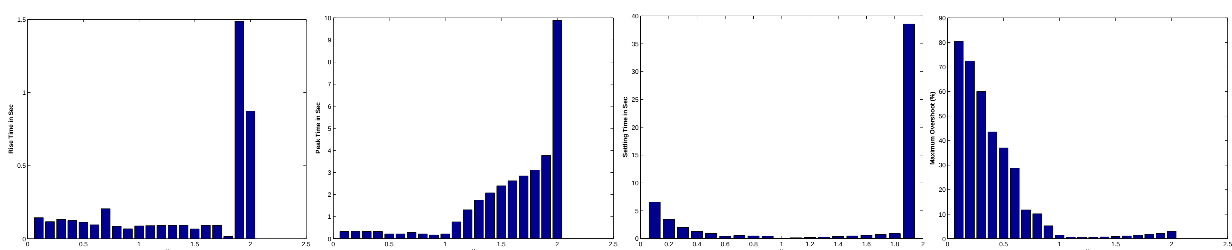
(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 8: Effect of μ on different specifications (Plant 1).



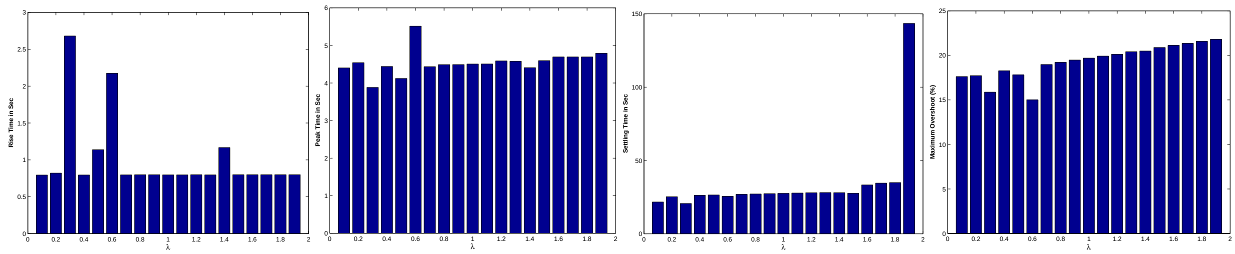
(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 9: Effect of λ on different specifications (Plant 2).



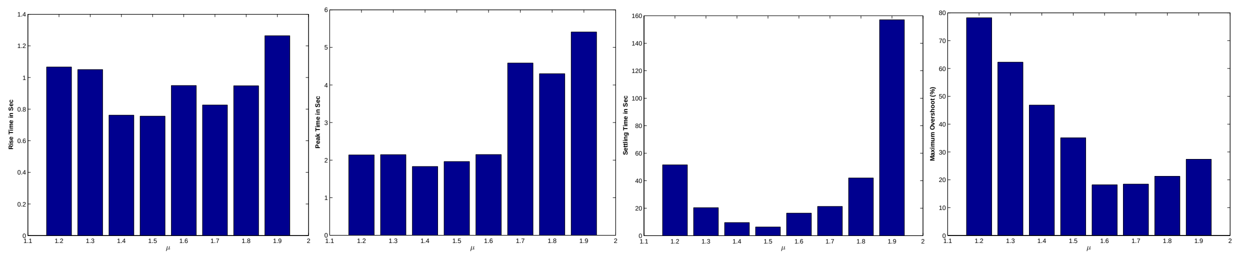
(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 10: Effect of μ on different specifications (Plant 2).



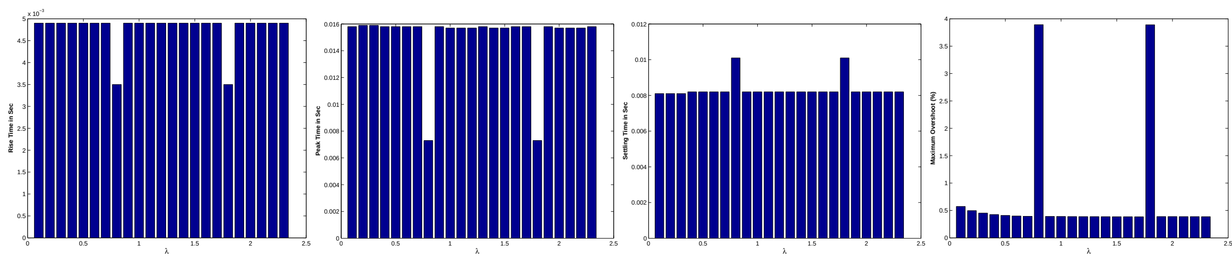
(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 11: Effect of λ on different specifications (Plant 3).



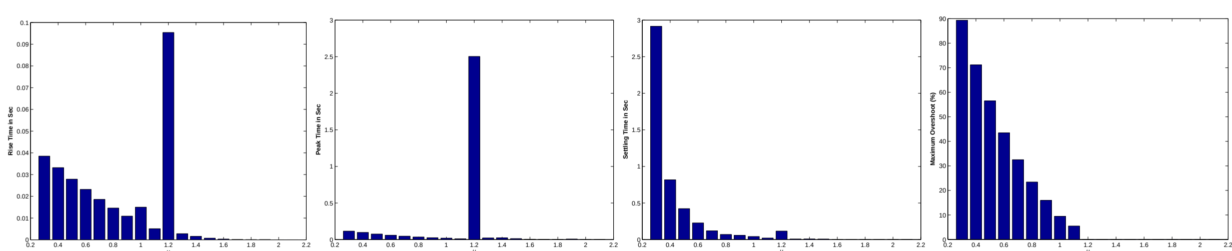
(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 12: Effect of μ on different specifications (Plant 3).



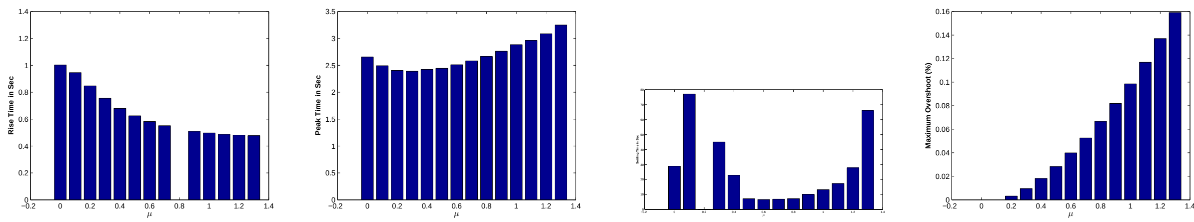
(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 13: Effect of λ on different specifications (Plant 4).



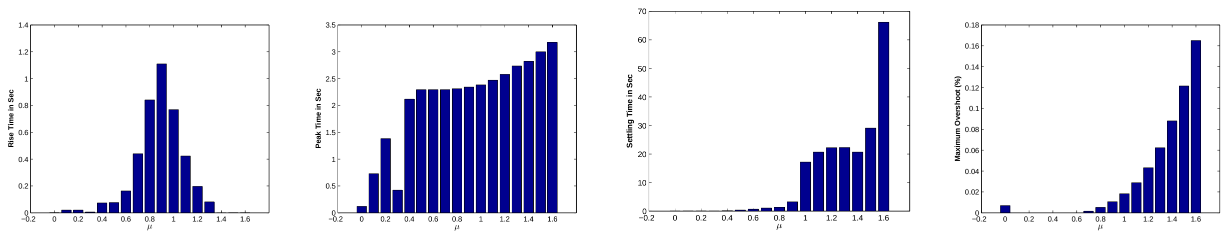
(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 14: Effect of μ on different specifications (Plant 4).



(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 15: Effect of λ on different specifications (Plant 5).



(a) Rise Time **(b)** Peak Time **(c)** Settling Time **(d)** Maximum Overshoot

Fig. 16: Effect of μ on different specifications (Plant 5).

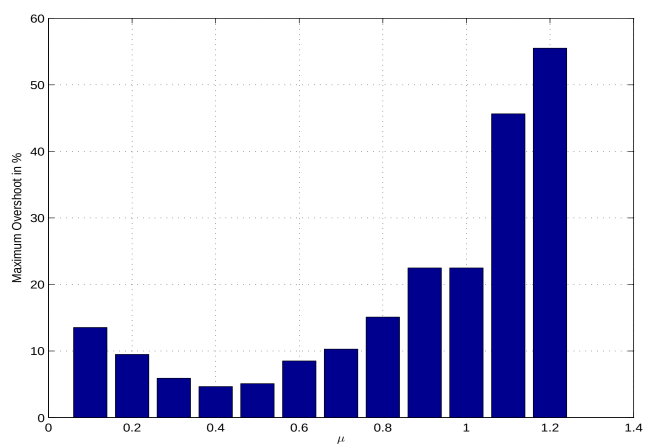


Fig. 17: Experimental results for effect of fractional order parameters.

Table 4: Fractional PID controller tuning parameters for experimental set-up.

Parameter	K_P	K_I	λ	K_D	μ
Value	3.85	3.99	0.48	16.25	0.4

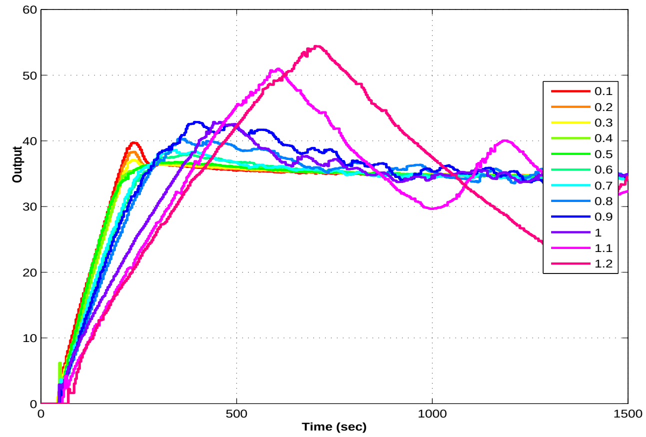


Fig. 18: Step response of the quadruple tank system for different values of of μ.

5 Conclusion

In this paper, the effect of differentiation (μ) and integrator (λ) order are investigated on various time domain specifications. Maximum overshoot has a particular characteristic of the order of differentiation from 0 to 2. Other specifications (rise, peak, and settling times) showed no particular pattern that matched the increase in the value of the parameters independently. However, following points are observed:

–There exists a particular relationship between μ and maximum overshoot (M_P).
–By changing the values of λ and μ, the time domain specifications can be further improved, which is advantage of the fractional PID controller.
–For a fractional order model, the influence is almost same for different values of λ on time domain specifications.
–The settling time goes worst as μ approaches 2 for integer order system.

The finding will facilitate in the tuning of fractional PID controllers, an especially useful feature for the plug-and-play type of controllers. The effect on the fractional-order parameters may be estimated for a given system, such as a first-order system or a second-order system.

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