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Confidence Limits for Progressive Censored Burr Type-XII data under Constant-Partially ALT

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Abstract: In product life testing experiments, accelerated life testing is widely used since it provides significant reduction in time and cost of experiment. A constant-partially accelerated life test based on progressively censored Burr Type-XII data is considered in the present article. Approximate confidence intervals based on the normal approximation to the asymptotic distribution of Maximum Likelihood Equation, Bootstrap Confidence Interval, and One-Sample Bayes prediction bound lengths are obtained. The analysis of the present discussion has carried out by a real life example.

Keywords: Partially Accelerated Life Test (PALT), Progressive Type-II Censoring, Approximate Confidence Intervals (ACI) , Bootstrap Confidence Intervals, One-Sample Bayes Prediction Bound Length

1 Introduction

The Burr system of distributions includes twelve types of cumulative distribution functions that yield a variety of density shapes, and were listed in [1]. It has applied in business, chemical engineering, quality control, medical, and reliability studies. The probability density function and cumulative density function of Burr Type-XII distribution are given as

$$f\left(x;\alpha,\beta\right) = \beta\alpha x^{\beta-1}\left(1+x^{\beta}\right)^{-\alpha-1};\ \alpha>0,\beta>0,x\geq 0 \tag{1}$$

and

$$F\left(x;\alpha,\beta\right) = 1-\left(1+x^{\beta}\right)^{-\alpha};\ \alpha>0,\beta>0,x\geq 0. \tag{2}$$

Failure rate function and the reliability function of Burr Type-XII distribution are given as

$$\rho\left(x\right) = \beta\alpha\frac{x^{\beta-1}}{1+x^{\beta}};\ \alpha>0,\beta>0,x\geq 0 \tag{3}$$

and

$$R(x) = \left(1+x^{\beta}\right)^{-\alpha};\ \alpha>0,\beta>0,x\geq 0. \tag{4}$$

Here, the parameter α does not affect the shape of failure rate function $\rho(x)$ given in Eq. (3). The parameter α and β both are the shape parameter of Burr Type-XII distribution. Also, $\rho(x)$ has a unimodal curve when $\beta>1$ and it has decreasing failure rate function when $\beta\leq 1$. The parameter β plays an important role for the distribution. It covers a variety of curve shapes and provides a wide range of values of skewness and kurtosis that can used to model for any general lifetime data (biological, clinical, or other experimental data).

The present distribution is useful in failure time modeling, quality control, and reliability studies. Few important and

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resent references on the topic are including here. The empirical Bayes estimators of reliability performances based on LINEX loss function under progressively Type-II censored samples was study be [2]. Lee et al. [3] assessing the lifetime performance index of products from progressively Type-II right censored data based on Burr Type-XII model. Based on exponentiated Burr Type-XII population [4] provided a number of references on the applications of Burr model in different fields of applied statistics.

Soliman et al.[5] obtained some Bayes estimation from Burr Type-XII distribution by using progressive first-failure censored data. [6] discussed about the problem of estimating the parameters and reliability function of the Burr Type-III distribution based on Type-II Doubly censored sample. A Koziol-Green model of random censorship for estimating the Bayes estimator of unknown parameters was discussed by [7]. A multicomponent stress strength reliability by assuming Burr Type-XII distribution was studied by [8] recently.

The focus of the paper is to study about different confidence limits for the Burr Type-XII distribution under constant-partially accelerated life test. Based on the normal approximation to the asymptotic distribution of MLE, the approximate confidence intervals (ACI), Percentile Bootstrap confidence intervals (PBCI), and One-Sample Bayes prediction bound lengths are obtained in different sections. The analysis of the present discussion has carried out by a real life example in last section with conclusion.

2 Constant-Partially Accelerated Life Tests

The Type-I and Type-II censoring do not allow the units to be removed from the test at points other than the terminal point of the experiment when a compromise between reduced time of experimentation and the observations of at least some extreme lifetimes are sought. This makes the lifetime testing under normal conditions very costly and takes a long time. For this reason, accelerated life tests (ALT) are prefer to be used in manufacturing industries to obtain enough failure data, in a short period. In ALT, the test units are run at higher than the usual stress levels to induce early failures.

In ALT, the units are tested only at accelerated conditions. However, in PALT, the units are tested at both accelerated and normal conditions. When the acceleration factor cannot be, assume as a known value, PALT will be a good choice to perform the life test. The focus of this paper is based on the constant-stress PALT, in which runs each item at either use or in accelerated condition only. Several references have with ALT, including [9], [10], [11], [12] and [13].

Now, the lifetime of an item tested at use condition follows following probability density function, distribution function and failure rate

$$f_1(x_1;\alpha,\beta)=\beta\alpha x_1^{\beta-1}\left(1+x_1^{\beta}\right)^{-\alpha-1};\ \alpha>0,\beta>0,x_1\geq 0,$$

$$F_1(x_1;\alpha,\beta)=1-\left(1+x_1^{\beta}\right)^{-\alpha};\ \alpha>0,\beta>0,x_1\geq 0$$

and

$$\rho_1(x_1)=\beta\alpha\frac{x_1^{\beta-1}}{1+x_1^{\beta}};\ \alpha>0,\beta>0,x_1\geq 0.$$

If failure rate function $\rho_2(x_2)$ is denoted for an item tested at accelerated condition with acceleration factor $\lambda(>1)$, then it is defined as

$$\rho_2(x_2)=\lambda\rho_1(x_1).$$

Under accelerated condition, the failure rate, probability density function, and distribution function are obtained as

$$\rho_2(x_2)=\beta\alpha\lambda\frac{x_2^{\beta-1}}{1+x_2^{\beta}};\ \alpha>0,\beta>0,\lambda>0,x_2\geq 0,$$

$$F_2(x_2;\alpha,\beta,\lambda)=1-\left(1+x_2^{\beta}\right)^{-\alpha\lambda};\ \alpha>0,\beta>0,\lambda>1,x_2\geq 0$$

$$f_2(x_2;\alpha,\beta,\lambda)=\beta\alpha\lambda x_2^{\beta-1}\left(1+x_2^{\beta}\right)^{-\alpha\lambda-1};\ \alpha>0,\beta>0,\lambda>1,x_2\geq 0.$$

3 Approximate Confidence Intervals

Let n_1 items are randomly chosen among n test items which are allocated to use condition and $n_2 = n - n_1$ remaining items are subjected to an accelerated condition. The Progressive Type-II censoring is applied as usual (See [14]). Based on progressively Type-II censoring scheme the joint probability density function of order statistics $X_{1_j:m_j:n_j}^{(R_{j1},R_{j2},\ldots,R_{jm})}, X_{2_j:m_j:n_j}^{(R_{j1},R_{j2},\ldots,R_{jm})}, \ldots, X_{m_j:m_j:n_j}^{(R_{j1},R_{j2},\ldots,R_{jm})}$; $j = 1,2$ is defined as

$$L(\alpha,\beta,\lambda|\underline{x}) \propto \left\{\prod_{i=1}^{m_1} f_1\left(x_{(1i)};\alpha,\beta\right)\left(1-F_1\left(x_{(1i)};\alpha,\beta\right)\right)^{R_{1i}}\right\}$$

$$\cdot\left\{\prod_{i=1}^{m_2} f_2\left(x_{(2i)};\alpha,\beta,\lambda\right)\left(1-F_2\left(x_{(2i)};\alpha,\beta,\lambda\right)\right)^{R_{2i}}\right\} \tag{5}$$

$$L(\alpha,\beta,\lambda|\underline{x}) \propto \left\{\prod_{i=1}^{m_1} \beta\alpha x_{1i}^{\beta-1}\left(1+x_{1i}^{\beta}\right)^{-\alpha-1}\left(1+x_{1i}^{\beta}\right)^{-\alpha R_{1i}}\right\}$$

$$\cdot\left\{\prod_{i=1}^{m_2} \beta\alpha\lambda x_{2i}^{\beta-1}\left(1+x_{2i}^{\beta}\right)^{-\alpha\lambda-1}\left(1+x_{2i}^{\beta}\right)^{-\alpha\lambda R_{2i}}\right\}$$

$$\Rightarrow L(\alpha,\beta,\lambda|\underline{x}) \propto \beta^{m_1+m_2}\alpha^{m_1+m_2}\lambda^{m_2}e^{(\beta-1)T_0-T(\beta)}e^{-\alpha T_1(\beta)}e^{-\alpha\lambda T_2(\beta)} \tag{6}$$

where $T_j(\beta) = \sum_{i=1}^{m_j}(1+R_{ji})\log\left(1+x_{ji}^{\beta}\right)$; $j = 1,2$, $T_0 = \sum_{i=1}^{m_1}\log\ x_{1i} + \sum_{i=1}^{m_2}\log\ x_{2i}$ and $T(\beta) = \sum_{i=1}^{m_1}\log\left(1+x_{1i}^{\beta}\right) + \sum_{i=1}^{m_2}\log\left(1+x_{2i}^{\beta}\right)$.

Taking logarithm on Eq. (6), we get

$$Log\,L(\alpha,\beta,\lambda|\underline{x}) = l_M\,(say) = (m_1+m_2)\log\beta + (m_1+m_2)\log\alpha + m_2\log\lambda$$

$$-T(\beta) + (\beta-1)T_0 - \alpha T_1(\beta) - \alpha\lambda T_2(\beta). \tag{7}$$

The ML (maximum likelihood) estimator corresponding to parameter α is

$$\hat{\alpha}_{ML} = \frac{m_1+m_2}{T_1(\beta)+\lambda T_2(\beta)}. \tag{8}$$

Similarly the ML estimators corresponding to parameter β and λ are given as

$$\hat{\beta}_{ML} = \frac{m_1+m_2}{\sum_{i=1}^{m_1}(1+\alpha(1+R_{1i}))\left(\frac{x_{1i}^{\beta}log(x_{1i})}{1+x_{1i}^{\beta}}\right) + \sum_{i=1}^{m_2}(1+\alpha\lambda(1+R_{2i}))\left(\frac{x_{2i}^{\beta}log(x_{2i})}{1+x_{2i}^{\beta}}\right) - T_0} \tag{9}$$

and

$$\hat{\lambda}_{ML} = \frac{m_2}{\alpha T_2(\beta)}. \tag{10}$$

Further simplifications of ML estimators are not possible. Some suitable numerically method is applied here for obtaining the numerical values of the ML estimates.

The common method for obtaining the confidence bounds for the parameters is based on asymptotic normal distribution of ML estimators. The observed information matrix is now defined and obtained as

$$I = \begin{bmatrix} -\frac{\partial^2 l_M}{\partial\alpha^2} & -\frac{\partial^2 l_M}{\partial\alpha\partial\beta} & -\frac{\partial^2 l_M}{\partial\alpha\partial\lambda} \\ -\frac{\partial^2 l_M}{\partial\beta\partial\alpha} & -\frac{\partial^2 l_M}{\partial\beta^2} & -\frac{\partial^2 l_M}{\partial\beta\partial\lambda} \\ -\frac{\partial^2 l_M}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l_M}{\partial\lambda\partial\beta} & -\frac{\partial^2 l_M}{\partial\lambda^2} \end{bmatrix}. \tag{11}$$

The second order derivatives for the observed Information matrix with respect to parameters α, β and λ are given as

$$\frac{\partial^2 l_M}{\partial \alpha^2} = -\frac{m_1+m_2}{\alpha^2}$$

$$\frac{\partial^2 l_M}{\partial \beta^2} = -\frac{m_1+m_2}{\beta^2} - \sum_{i=1}^{m_1} (1+\alpha(1+R_{1i})) \left(\frac{x_{1i}^{\beta} log(x_{1i})}{1+x_{1i}^{\beta}} \right)^2 - \sum_{i=1}^{m_2} (1+\alpha\lambda(1+R_{2i})) \left(\frac{x_{2i}^{\beta} log(x_{2i})}{1+x_{2i}^{\beta}} \right)^2$$

$$\frac{\partial^2 l_M}{\partial \lambda^2} = -\frac{m_2}{\lambda^2}$$

$$\frac{\partial^2 l_M}{\partial \alpha \partial \beta} = \frac{\partial^2 l_M}{\partial \beta \partial \alpha} = -\sum_{i=1}^{m_1} (1+R_{1i}) \left(\frac{x_{1i}^{\beta} log(x_{1i})}{1+x_{1i}^{\beta}} \right) - \lambda \sum_{i=1}^{m_2} (1+R_{2i}) \left(\frac{x_{2i}^{\beta} log(x_{2i})}{1+x_{2i}^{\beta}} \right)$$

$$\frac{\partial^2 l_M}{\partial \beta \partial \lambda} = \frac{\partial^2 l_M}{\partial \lambda \partial \beta} = -\alpha \sum_{i=1}^{m_2} (1+R_{2i}) \left(\frac{x_{2i}^{\beta} log(x_{2i})}{1+x_{2i}^{\beta}} \right)$$

and

$$\frac{\partial^2 l_M}{\partial \alpha \partial \lambda} = \frac{\partial^2 l_M}{\partial \lambda \partial \alpha} = -\sum_{i=1}^{m_2} (1+R_{2i}) \, log\left(1+x_{2i}^{\beta}\right).$$

Using above values the observed information matrix I from Eq. (11) is obtained. Now, the variance covariance matrix V (say) is approximated as

$$V = I^{-1}. \tag{12}$$

The expression, V involves three unknown parameters α, β and λ. Hence, an estimate of $V\left(=\hat{V}\right)$ (say) is obtained by substituting its ML estimators respectively. Hence, $100(1-\varepsilon)\%$ ACI for the parameters α, β and λ are obtained respectively as

$$\hat{\alpha} \mp Z_{\varepsilon/2}\sqrt{V_{11}} \tag{13}$$

$$\hat{\beta} \mp Z_{\varepsilon/2}\sqrt{V_{22}} \tag{14}$$

and

$$\hat{\lambda} \mp Z_{\varepsilon/2}\sqrt{V_{33}}. \tag{15}$$

Here V_{11}, V_{22} and V_{33} are the main diagonal elements of the variance-covariance matrix $\hat{V}$ and $Z_{\varepsilon/2}$ is the percentile of the standard normal distribution with right-tail probability $\varepsilon/2$.

4 Bootstrap Confidence Intervals

In statistical inference, the bootstrap is a re-sampling method for estimating biases, variance of an estimator and confidence intervals ([15]). In the present section, the confidence limits based on parametric bootstrap method ([16]) are obtained for the parameters α, β and λ respectively.

Based on following steps, the bootstrap samples are obtained :

The ML estimate $\hat{\alpha}_{ML}$ of the parameter α from Eq. (8), is obtained by using original progressive Type-II censored data $X_{1_j:m_j:n_j}^{(R_{j1},R_{j2},...,R_{jm})}, X_{2_j:m_j:n_j}^{(R_{j1},R_{j2},...,R_{jm})}, ..., X_{m_j:m_j:n_j}^{(R_{j1},R_{j2},...,R_{jm})}$; $j = 1,2$. Similarly, the ML Estimates $\hat{\beta}_{ML}$ of parameter β from Eq. (9), and ML Estimates $\hat{\lambda}_{ML}$ of λ from Eq. (10), are also obtained respectively from original progressive Type-II censored data $X_{1_j:m_j:n_j}^{(R_{j1},R_{j2},...,R_{jm})}, X_{2_j:m_j:n_j}^{(R_{j1},R_{j2},...,R_{jm})}, ..., X_{m_j:m_j:n_j}^{(R_{j1},R_{j2},...,R_{jm})}$; $j = 1,2$.

Again, generate two independent progressive samples of sizes m_1 and m_2 from Burr Type-XII distribution based on considered censoring scheme $R_{ji}\,(i = 1,2,...,m_j, j = 1,2)$. Based on generated samples, compute the bootstrap sample estimates of ML estimators $\hat{\alpha}_{ML}, \hat{\beta}_{ML}$ and $\hat{\lambda}_{ML}$ say $\hat{\hat{\alpha}}_{ML}, \hat{\hat{\beta}}_{ML}$ and $\hat{\hat{\lambda}}_{ML}$ respectively.

Repeat the above step up to $N(= 1000)$ times to obtain $N(= 1000)$ different bootstrap samples. Arrange all these samples $\left(\hat{\hat{\alpha}}_{ML}, \hat{\hat{\beta}}_{ML} \text{ and } \hat{\hat{\lambda}}_{ML}\right)$ in ascending order to obtain final bootstrap sample of the form

$$\tau_{\alpha}^{1} \leq \tau_{\alpha}^{2} \leq ... \leq \tau_{\alpha}^{N} \quad for \;\; \hat{\hat{\alpha}}_{ML}$$

$$\tau_{\beta}^{1} \leq \tau_{\beta}^{2} \leq ... \leq \tau_{\beta}^{N} \quad for \;\; \hat{\hat{\beta}}_{ML}$$

and

$$\tau_{\lambda}^{1} \leq \tau_{\lambda}^{2} \leq ... \leq \tau_{\lambda}^{N} \quad for \;\; \hat{\hat{\lambda}}_{ML}.$$

If $G(y) = P\left(\tau_k^* \leq y\right)$ be the cumulative density function of τ_k^*. Where, $\tau_k^*; \forall k = \alpha, \beta, \lambda$ be the final bootstrap samples. Then the $100(1-\varepsilon)\%$ approximate bootstrap confidence limits is given by

$$\left[\tau_{k(B)}^*\left(\frac{\varepsilon}{2}\right), \tau_{k(B)}^*\left(\frac{2-\varepsilon}{2}\right)\right] \tag{16}$$

where $\tau_{k(B)}^* = G^{-1}(y)$ for given y. Here, the Eq.(16) represent the Percentile Bootstrap Confidence Limits.

5 One-Sample Bayes Prediction Limit

The Bayes predicative density of future observation Y is denoted by $h_{\Theta}\,(y|\underline{x})$ and obtained by simplifying

$$h_{\Theta}\,(y|\underline{x}) \propto \int_{\Theta} f\,(y;\alpha,\beta)\,\pi_{\Theta}^{*} d\Theta. \tag{17}$$

where, $\pi_{\Theta}^{*}; (\Theta = \alpha, \beta, \lambda)$ be the posterior density corresponding to parameter $\Theta\,(= \alpha, \beta, \lambda)$ respectively.

Let us assume the prior densities corresponding to parameters α, β and λ are given respectively as

$$\pi_{\alpha} \propto \alpha^{-1}; \alpha > 0,$$

$$\pi_{\beta} \propto \beta^{-1}; \beta > 0$$

and

$$\pi_{\lambda} \propto \lambda^{-1}; \lambda > 0.$$

Here, the considered prior are vague priors, so that the priors do not have any significant roles in the analyses that follow. One may use conjugate priors for the analysis. The joint prior is thus obtained as

$$\pi_{(\alpha,\beta,\lambda)} = \frac{1}{\alpha\beta\lambda}.$$

Hence, the joint posterior density is thus defined as

$$\pi^*_{(\alpha,\beta,\lambda)} = \frac{\pi_{(\alpha,\beta,\lambda)} \cdot L(\alpha,\beta,\lambda|\underline{x})}{\int_\beta \int_\lambda \int_\alpha \pi_{(\alpha,\beta,\lambda)} \cdot L(\alpha,\beta,\lambda|\underline{x})\, d\alpha\, d\lambda\, d\beta}.$$

Now, the marginal posterior density for parameter α is defined and obtained as

$$\pi^*_{(\alpha)} = \frac{\int_\beta \int_\lambda \pi_{(\alpha,\beta,\lambda)} \cdot L(\alpha,\beta,\lambda|\underline{x})\, d\lambda\, d\beta}{\int_\beta \int_\lambda \int_\alpha \pi_{(\alpha,\beta,\lambda)} \cdot L(\alpha,\beta,\lambda|\underline{x})\, d\alpha\, d\lambda\, d\beta}$$

$$\Rightarrow \pi^*_{(\alpha)} = \bar{\beta}\Gamma(m_2)\alpha^{m_1-1}\int_\beta \frac{\beta^{m_1+m_2-1}}{(T_2(\beta))^{m_2}} e^{(\beta-1)T_0-T(\beta)} e^{-\alpha T_1(\beta)} d\beta \tag{18}$$

where $\bar{\beta} = \left\{\Gamma(m_1)\Gamma(m_2)\int_\beta \frac{\beta^{m_1+m_2-1} e^{(\beta-1)T_0-T(\beta)}}{(T_1(\beta))^{m_1}(T_2(\beta))^{m_2}} d\beta\right\}^{-1}$.

Similarly, the marginal posterior densities corresponding to parameters β and λ are obtained as

$$\pi^*_{(\beta)} = \bar{\beta}\Gamma(m_1)\Gamma(m_2)\beta^{m_1+m_2-1} e^{(\beta-1)T_0-T(\beta)} (T_1(\beta))^{-m_1} (T_2(\beta))^{-m_2} \tag{19}$$

and

$$\pi^*_{(\lambda)} = \bar{\beta}\Gamma(m_1+m_2)\lambda^{m_2-1}\int_\beta \frac{\beta^{m_1+m_2-1}}{(T_1(\beta)+\lambda T_2(\beta))^{m_1+m_2}} e^{(\beta-1)T_0-T(\beta)} d\beta. \tag{20}$$

Using Eq. (1) and Eq. (18) in Eq. (17), the Bayes predictive density of future variable for the parameter α is obtained as

$$h_\alpha(y|\underline{x}) \propto \frac{\beta y^{\beta-1}}{(1+y^\beta)} \int_\alpha \frac{\alpha^{m_1}}{e^{\alpha \log(1+y^\beta)}} \int_\beta \frac{\beta^{m_1+m_2-1} e^{(\beta-1)T_0-T(\beta)} e^{-\alpha T_1(\beta)}}{(T_2(\beta))^{m_2}} d\beta\, d\alpha. \tag{21}$$

Similarly, the Bayes predictive density of future variable for the parameter β is obtained by using Eq. (1) and Eq. (19) in Eq. (17) as

$$h_\beta(y|\underline{x}) \propto \alpha \int_\beta \frac{y^{\beta-1}(1+y^\beta)^{-\alpha-1}\beta^{m_1+m_2}}{(T_1(\beta))^{m_1}(T_2(\beta))^{m_2}} e^{(\beta-1)T_0-T(\beta)} d\beta. \tag{22}$$

The Bayes predictive density corresponding to the parameter λ for future variable is obtained as by using Eq. (1) and Eq. (20) in Eq. (17)

$$h_\lambda(y|\underline{x}) \propto \beta\alpha \frac{y^{\beta-1}}{(1+y^\beta)^{\alpha+1}} \int_\lambda \lambda^{m_2-1} \int_\beta \frac{\beta^{m_1+m_2-1} e^{(\beta-1)T_0-T(\beta)}}{(T_1(\beta)+\lambda T_2(\beta))^{m_1+m_1}} d\beta\, d\lambda. \tag{23}$$

If l_1 and l_2 be the lower and upper Bayes prediction limits of the future observation and $(1-\varepsilon)$ be the confidence prediction coefficient, then the one-sided Bayes prediction bound limits are obtain by solving following equality

$$Pr(Y \le l_1) = \frac{\varepsilon}{2} = Pr(Y \ge l_2). \tag{24}$$

Using Eq. (21) & Eq. (24), the Bayes predictive bound limits for the parameter α are obtained by solving following equations

$$\frac{\varepsilon}{2} = \int_{Z=0}^{\log\left(1+l_1^\beta\right)} \int_\alpha \frac{\alpha^{m_1}}{e^{\alpha Z}} \int_\beta \beta^{m_1+m_2-1} \frac{e^{(\beta-1)T_0-T(\beta)} e^{-\alpha T_1(\beta)}}{(T_2(\beta))^{m_2}} d\beta\, d\alpha\, dZ \tag{25}$$

and

$$\frac{2-\varepsilon}{2} = \int_{Z=0}^{\log\left(1+l_2^\beta\right)} \int_\alpha \frac{\alpha^{m_1}}{e^{\alpha Z}} \int_\beta \beta^{m_1+m_2-1} \frac{e^{(\beta-1)T_0-T(\beta)} e^{-\alpha T_1(\beta)}}{(T_2(\beta))^{m_2}} d\beta\, d\alpha\, dZ \tag{26}$$

Table 1: Relief Time (in hours) for 24 Arthritic Patients

0.70	0.58	0.54	0.59	0.71	0.55	0.63	0.84
0.49	0.87	0.73	0.72	0.62	0.82	0.84	0.29
0.51	0.61	0.57	0.29	0.36	0.46	0.68	0.34

It is clear that, the nice close form of Eq. (25) and Eq. (26) do not exists. Some numerical technique is applied here for the numerical findings. Based on numerical findings of l_1 from Eq. (25) and l_2 from Eq. (26), the Bayes predictive bound length for the parameter α is obtained as

$$L_\alpha = l_2 - l_1.$$

Similarly, the Bayes predictive bound limits for the parameter β are obtained by solving following equations;

$$\frac{\varepsilon}{2} = \int_\beta \left(1-\left(1+l_1^\beta\right)^{-\alpha}\right) \frac{\beta^{m_1+m_2-1} e^{(\beta-1)T_0 - T(\beta)}}{\left(T_1(\beta)\right)^{m_1}\left(T_2(\beta)\right)^{m_2}} d\beta \tag{27}$$

and

$$\frac{2-\varepsilon}{2} = \int_\beta \left(1-\left(1+l_2^\beta\right)^{-\alpha}\right) \frac{\beta^{m_1+m_2-1} e^{(\beta-1)T_0 - T(\beta)}}{\left(T_1(\beta)\right)^{m_1}\left(T_2(\beta)\right)^{m_2}} d\beta \tag{28}$$

The Bayes predictive bound length for the parameter β is obtained by numerical findings of l_1 from Eq. (27) and l_2 from Eq. (28) as,

$$L_\beta = l_2 - l_1.$$

On similar line, the Bayes predictive bound limits and bound length for the parameter λ are obtained by using Eq. (23) and Eq. (24) as

$$\frac{\varepsilon}{2} = \left(1-\left(1+l_1^\beta\right)^{-\alpha}\right) \int_\lambda \lambda^{m_2-1} \int_\beta \frac{\beta^{m_1+m_2-1} e^{(\beta-1)T_0 - T(\beta)}}{\left(T_1(\beta)+\lambda T_2(\beta)\right)^{m_1+m_2}} d\beta d\lambda \tag{29}$$

$$\frac{2-\varepsilon}{2} = \left(1-\left(1+l_2^\beta\right)^{-\alpha}\right) \int_\lambda \lambda^{m_2-1} \int_\beta \frac{\beta^{m_1+m_2-1} e^{(\beta-1)T_0 - T(\beta)}}{\left(T_1(\beta)+\lambda T_2(\beta)\right)^{m_1+m_2}} d\beta d\lambda \tag{30}$$

and

$$L_\lambda = l_2 - l_1.$$

6 Numerical Analysis

For illustrative purposes, the performance of the proposed procedures is studied by a real data set on relief time (in hours) for 24 arthritic patients ([17]). Recently, [18], presents some analysis based on present data under Burr Type-XII distribution. The data are given in the Table 1.

The progressive censoring scheme for the censored sample size m_1 and m_2 are assume as

Based on above Progressive censoring scheme and data given in Table 1, the ML estimates for the parameters α, β and λ are obtained, and presented in the Table 3 for selected parametric values. It is observed from the table that, the ML estimate increases as the censored sample size increases. Similar properties also have seen when parametric values increases.

The approximate confidence limits (ACL), percentile Bootstrap confidence limits (PBCL) and Bayes predictive bound lengths (BPBL) are obtained for the parameters α, β and λ respectively and presented in Table 4-6, with confidence values $\varepsilon = 90\%, 95\%, 99\%$ and selected parametric values.

Table 2: Different Progressive Censoring Scheme

$m_j; j=1,2$	$R_{ji}; i=1,2,...,m_j; j=1,2$
10	1 2 0 1 0 1 0 0 3 1
15	1 0 1 3 0 2 1 0 3 1 0 0 3 1 1
20	1 0 2 0 4 1 0 2 3 0 2 1 0 0 1 0 1 0 2 1

Table 3: Different ML Estimates

$n=24, \lambda=2.00$		ML Estimators		
$(m_1,m_2)\downarrow$	$(\alpha,\beta)\downarrow$	$\hat{\alpha}_{ML}$	$\hat{\beta}_{ML}$	$\hat{\lambda}_{ML}$
(10, 10)	(0.25, 1.00)	0.6781	1.7556	1.3152
(15, 20)	(0.50, 3.00)	0.6918	1.7666	1.3655
(20, 20)	(1.00, 5.00)	0.6921	1.7679	1.3855

Table 4: ACL, PBCL & BPBL for the Parameter α

$n=24, \lambda=2.00$			$\leftarrow \alpha \rightarrow$		
$\varepsilon\downarrow$	$(m_1,m_2)\downarrow$	$(\alpha,\beta)\downarrow$	ACL	PBCL	BPBL
90%	(10, 10)	(0.25, 1.00)	0.8756	0.8654	0.8689
	(15, 20)	(0.50, 3.00)	0.9368	0.9219	0.9296
	(20, 20)	(1.00, 5.00)	1.0303	1.0191	1.0302
95%	(10, 10)	(0.25, 1.00)	0.8816	0.8754	0.8789
	(15, 20)	(0.50, 3.00)	0.9432	0.9316	0.9411
	(20, 20)	(1.00, 5.00)	1.0381	1.0218	1.0348
99%	(10, 10)	(0.25, 1.00)	0.9156	0.8881	0.8951
	(15, 20)	(0.50, 3.00)	0.9796	0.9576	0.8796
	(20, 20)	(1.00, 5.00)	1.0676	1.0436	1.0531

Table 5: ACL, PBCL & BPBL for the Parameter β

$n=24, \lambda=2.00$			$\leftarrow \beta \rightarrow$		
$\varepsilon\downarrow$	$(m_1,m_2)\downarrow$	$(\alpha,\beta)\downarrow$	ACL	PBCL	BPBL
90%	(10, 10)	(0.25, 1.00)	1.2158	1.2095	1.2141
	(15, 20)	(0.50, 3.00)	1.5776	1.5694	1.5754
	(20, 20)	(1.00, 5.00)	1.6917	1.6418	1.6842
95%	(10, 10)	(0.25, 1.00)	1.4538	1.4314	1.4318
	(15, 20)	(0.50, 3.00)	1.8735	1.8601	1.8609
	(20, 20)	(1.00, 5.00)	2.1009	1.8497	2.0101
99%	(10, 10)	(0.25, 1.00)	1.7091	1.6528	1.6833
	(15, 20)	(0.50, 3.00)	2.2260	2.1768	2.1977
	(20, 20)	(1.00, 5.00)	2.2699	2.1921	2.0032

It is observed from the Tables (4-6) that, the limits increase as ε increases. Similar, behavior also has seen when censoring scheme changed or censored sample size increases. Remarkable point is that, the percentile Bootstrap confidence limit (PBCL) has minimum length, whereas the approximate confidence limit (ACL) shows maximum lengths for all considered parametric values.

Table 06 :: ACL, PBCL & BPBL for the Parameter λ

$n=24, \lambda=2.00$			$\leftarrow \lambda \rightarrow$		
$\varepsilon \downarrow$	$(m_1, m_2) \downarrow$	$(\alpha, \beta) \downarrow$	ACL	PBCL	BPBL
90%	(10, 10)	(0.25, 1.00)	1.1121	1.1056	1.1856
	(15, 20)	(0.50, 3.00)	1.1898	1.1828	1.2684
	(20, 20)	(1.00, 5.00)	1.2967	1.2891	1.3824
95%	(10, 10)	(0.25, 1.00)	1.1851	1.1776	1.2701
	(15, 20)	(0.50, 3.00)	1.2749	1.2668	1.3638
	(20, 20)	(1.00, 5.00)	1.3985	1.3897	1.4975
99%	(10, 10)	(0.25, 1.00)	1.2577	1.2491	1.3155
	(15, 20)	(0.50, 3.00)	1.3605	1.3512	1.3647
	(20, 20)	(1.00, 5.00)	1.4021	1.3292	1.5155

7 Conclusions

In the present article, the Burr Type-XII distribution is taken here as the underlying model for the study about approximate confidence intervals (ACI), percentile Bootstrap CI, and One-Sample Bayes prediction bound lengths. The constant-partially accelerated life test based on progressive censored data is considered here for the discussion. It is observed from the numerical findings is that; the Percentile Bootstrap confidence limit has minimum length, whereas the approximate confidence limit shows maximum lengths for all considered parametric values.

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