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Solving Singular Integral Equations by using Collocation Method

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Abstract: The aim of this work is to use the power series expansion with collocation method to approximate the solution of integral equations (IE) of the second kind on real axis. The technique of this method is based on transforming the IE to a matrix equation which corresponds to a system of linear equations with unknown coefficients. Two examples are presented to illustrate the performance of this method.

Keywords: Collocation methods, IE, Power series.

1 Introduction

Integral equation is an equation in which the unknown function appears within an integral sign. Such equations occur widely in diverse areas of applied mathematics, physics and engineering, and also it has a powerful technique to solve more practical problems. Singular integral equation is an integral equation, if either the limits of integration are infinite, or the kernel has singularity, in other words the kernel is unbounded within its domain of definition and which has a great role in variety fields for different applications, for example, mathematical physics, engineering, hydromechanics, etc.,(see [3] amd [4]).

In 2002, approximate methods for multi-dimensional weakly singular integral operators with operator-valued kernels are investigated and a polynomial collocation method also used for finding the numerical solution of a singular integral equation over the interval by [7]. Hassan [3] studied Cauchy-type singular integral equation by different numerical techniques. In 2009, the numerical schemes of collocation methods and mechanical quadratic methods to approximate the solutions of the singular integral equations are investigated by [1].

In this paper we try to solve singular integral equations of the second kind of the form

$$u(x) = f(x) + \int_{-\infty}^{\infty} k(x,t)u(t)dt \quad x \in (-\infty, \infty) \tag{1}$$

where f and k are given continuous functions, and u is unknown to be determined by collocation method. Throughout the paper, we assume that the equation (1) is integrable on $(-\infty, \infty)$, that is

$$\sup_{x\in\mathscr{R}} \int_{-\infty}^{\infty} |k(x,t)u(t)|\, dt < \infty$$

where $\mathscr{R}$ denoted to the set of all real numbers, see ([2,4, 6]).

This paper is organized as follows: in section 2 the technique of the approximate solution is considered. Section 3 consists some illustrate examples. Conclusions of the presented work are given in the final section.

2 Approximate Technique

In equation (1) we assume that $u(x) = w(x)g(x)$, where $w(x) = e^{-x^2}$ is a weight function defined on $(-\infty, \infty)$ [5], then equation 1 becomes

$$w(x)g(x) = f(x) + \int_{-\infty}^{\infty} w(x)k(x,t)g(t)dt. \tag{2}$$

Now, the unknown function $g(x)$ can be represents by a power series as follows:

$$g(x) = \sum_{r=0}^{\infty} c_r x^r. \tag{3}$$

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If the infinite series in (3) is truncated, then (3) can be written as

$$g(x) \approx g_n(x) = \sum_{r=0}^{n} c_r x^r. \tag{4}$$

where $c_r(r = 0, 1, ..., n)$ are unknown coefficients to be determined. By substituting (4) in (2) we get:

$$w(x)\sum_{r=0}^{n} c_r x^r = f(x) + \int_{-\infty}^{\infty} w(t)k(x,t)\sum_{r=0}^{n} c_r t^r dt,$$

and this implies that

$$\sum_{r=0}^{n} c_r(w(x)x^r - \Delta_r(x)) = f(x), \tag{5}$$

where the integral operator Δ is defined by $\Delta_r(x) = \int_{-\infty}^{\infty} w(t)k(x,t)t^r dt$. Choose $\{x_i\}_{i=0}^{n}$ to be a sequence of distinct points and collocate it in equation (5) we obtain the following:

$$\sum_{r=0}^{n} c_r(w(x_i)x_i^r - \Delta_r(x_i)) = f(x_i), \tag{6}$$

where $i = 0, 1, ..., n$, then (6) is a system of $n+1$ linear equations with $n+1$ unknowns $c_0, c_1, ..., c_n$, which can be represents as a matrix notation as follows:

$$AC = F, \tag{7}$$

where A is a matrix whose elements are defined by $a_{i,j} = w(x_i)x_i^j - \Delta_j(x_i)$, $C = (c_0, c_1, c_2, ..., c_n)^T$ and $F = (f(x_0), f(x_1), ..., f(x_n))^T$.

3 Illustrate Examples

In the following example, we try to apply the above technique to obtain an approximate solution on $(-\infty, \infty)$.

Example 1: We conceder the IE as the follows:

$$u(x) = x^3 e^{-x^2} - \frac{3\sqrt{\pi}}{4} + \int_{-\infty}^{\infty} (x+t)u(t)dt.$$

The theoretical solution is $g(x) = x^3$, and the approximate solution obtained by proposed method for different values of and the results showed in Figures 1, 2 and 3.

Example 2: We conceder the IE as the follows:

$$u(x) = x^2 e^{-x^2} - \frac{\sqrt{\pi}}{2} + \int_{-\infty}^{\infty} u(t)d(t).$$

The theoretical solution is $g(x) = x^2$, and the approximate solution obtained by proposed method and the results which is showed in Figures 4 and 5.

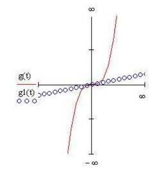

Fig. 1: presents the exact and approximate results $g_1(x) = 0.09246 + 1.60433x$ for n=1.

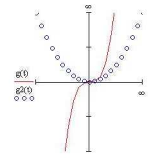

Fig. 2: presents the exact and approximate results $g_2(x) = 0.06386 + 1.57206x0.10974x^2$ for n=2.

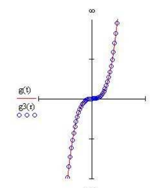

Fig. 3: presents the exact and approximate results $g_3(x) = x^3$ for n=3, which is coincide to the exact solution.

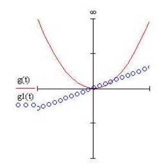

Fig. 4: presents the exact and approximate results $g_1(x) = 0.1472 + 2.9713x$ for n=1.

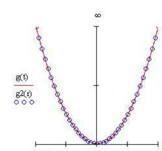

Fig. 5: presents the exact and approximate results $g_2(x) = x^2$ for n=2, which is coincide to the exact solution.

4 Conclusion

Power series with the collocation method were applied to find approximate solution of equation (1), the method was tested by two examples. A good approximation results depends on increase the number of terms taken in a power series (2), and sometimes, coinciding with the exact solution.

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