Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/050214

Stochastic Analysis of a Two Unit Parallel System with Maximum Repair and Order Time for the New Unit

S. K. Singh and Kanti Sahu *

School of Studies in Statistics, Pt.Ravishankar Shukla University,Raipur, Chhattisgarh (India)-492010

Received: 13 Dec. 2015, Revised: 15 Apr. 2016, Accepted: 29 Apr. 2016
Published online: 1 Jul. 2016

Abstract: This paper deals with the cost analysis of a two identical units parallel system in which no unit is kept as standby. A single repair facility is available in the system to replace the failed one by the new ordered one, if it is not repaired up to a given prefixed time T. The failure, maximum repair, delivery and replacement time distributions of a unit are taken to be negative exponential while as repair time distribution is arbitrary. Using regeneration point technique several characteristics of the system effectiveness are obtained to carry out the profit analysis. At last some particular cases are also discussed.

Keywords: MTSF, Availability, Busy period analysis.

1 Introduction

Various authors including , Goel and Gupta [1], Gopalan et al [2], Kumar et al [3] ,Murari and Goel [4], Nakagawa et al [5] , Papageorgiou and Kokolakis [7] and Singh et al [8] have studied two unit parallel / standby systems under different sets of assumptions using the theory of Semi - Markov process, Regenerative process and Markov Renewal Process. In all the models of two unit standby redundant systems considered so far, it has been assumed that whenever operating unit fails standby unit operates immediately. Practice reveals that to keep an unit in standby, increases the inventory cost of the system. So, if there is no place for inventory, and if failed unit is not repaired up to a prefixed maximum time T then it would be beneficial to replace the failed unit by new ordered one. Recently Nakagawa and Osaki [5] have analyzed an one unit system under the assumption that as soon as an operating unit fails before a prefixed time T, an order is placed immediately for a new unit to replace the failed one. Okumoto, Kazu [6] has obtained the availability of a two component repairable system using bivariate exponential failure and repair time distribution with the assumption that whenever both components fail simultaneously, an order for two new units is placed to replace the failed ones. If the new units arrive before the completion of the repair, the failed components are rejected and replaced by the new ones; otherwise, the order is cancelled. Qingtai et al [9] has studied a class of multi-unit cold standby systems subject to Poisson shocks. However very few attempts have been made in this direction. The purpose of the present paper is to study a two unit parallel system in which an order is placed to replace the failed unit if it is not repaired up to a fixed time T. A single repair facility is continuously available in the system which serves the dual role of repair and replacement of a failed unit by the new ordered one. Using regenerative point technique following measures of system effectiveness are obtained:
(i) Mean time to system failure (MTSF).
(ii) Point wise availability of the system in (0, t] and in steady state.
(iii) Busy period of the repair facility in repair in (0, t].
(iv) Busy period of the repair facility in replacement of the failed unit with new order one in in (0, t].
(v)Expected number of orders for the new unit in (0, t].
(vi) Expected profit earned by the system in (0, t] and in steady state.

* Corresponding author e-mail: kanti.tej@gmail.com

2 System Description

1)The system comprises of two identical parallel units. Each unit has two modes- normal (N) and total failure (F).
2) Whenever repair time of the failed unit exceeds the given maximum time, then that unit is rejected and an order is placed for a new unit to replace the failed one.
3) There is a single repair facility which serves the dual role of repair and replacement of the failed unit.
4) Priority is given to replacement over the repair of the failed unit.
5) During the ordering time of a unit, if a unit fails and it is not repaired up to maximum repairing time, then this failed unit waits for ordered unit until the replacement of the first failed unit is not completed.
6) Failure, delivery, replacement and maximum repair time distributions are negative exponential whereas repair time distribution is arbitrary.
7) After repair, unit acts like a new one.

3 Notations and Stats of the systems

N_O :unit is in operating mode.
$F_r/F_{wr}/F_R$:unit is in failure mode and under repair/waiting for repair/ continues in repair.
F_{wO} : unit in F mode and waiting for ordered new unit.
R: unit is in failure mode and under replacement.
U_O : a new unit is in under order. Considering these symbols the system may be in any one of the following states:
$S_0 : (N_O, N_O)$,$S_1 : (N_O, F_r)$,$S_2 : (F_{wr}, F_R)$,$S_3 : (N_O, U_O)$,$S_4 : (F_r, U_O)$, $S_5 : (N_O, R)$,$S_6 : (F_{wr}, R)$,$S_7 : (F_{wO}, UO)$,$S_8 : (F_{wO}, R)$

4 Other Symbols

α: constant failure rate of a normal unit.
$g_1(t)$: pdf of repair rate of a failed unit.
γ: maximum repair time of failed unit.
β: constant delivery rate of an ordered unit.
δ: constant replacement rate of a failed unit.
$*, \sim$: Laplace transform/Laplace Stieltjes transform.
$[s]$: Laplace Stieltjes convolution.
Ⓒ: Laplace convolution
$q_{ij}(t), Q_{ij}(t)$: pdf and cdf from state i to j.
$g_1(\cdot), G(\cdot)$: pdf and cdf of repair time of a failed unit.
E: set of regenerative states $S_i\{i = 0-8\}$

Possible transition among different states, along with the transition rates, are shown in figure.

5 Transition Probabilities and Sojourn Times

Simple probabilistic considerations yield the following expressions for distribution function times:

$$Q_{01}(t) = [1-e^{-2\alpha}t], \quad Q_{10}(t) = \int_0^t g_1(t)e^{-(\alpha+\gamma)t}dt, \quad Q_{11}^{(2)}(t) = \int_0^t g_1(t)e^{-\gamma t}[1-e^{-2\alpha}t]dt,$$

$$Q_{13}(t) = \int_0^t \gamma e^{-(\alpha+\gamma)t}\bar{G_1}(t)dt, \quad Q_{14}^{(2)}(t) = \gamma\int_0^t e^{-(\gamma)t}[1-e^{-\alpha t}]\bar{G_1}(t)dt, \quad Q_{34}(t) = \alpha\int_0^t e^{-(\alpha+\beta)t}dt,$$

$$Q_{35}(t)=\beta\int_0^t e^{-(\alpha+\beta)t}dt,\quad Q_{43}(t)=\int_0^t g_1(t)e^{-(\gamma+\beta)t}dt,\quad Q_{46}(t)=\beta\int_0^t e^{-(\gamma+\beta)t}\bar{G_1}(t)dt,$$

$$Q_{47}(t)=\gamma\int_0^t e^{-(\gamma+\beta)t}\bar{G_1}(t)dt,\quad Q_{50}(t)=\delta\int_0^t e^{-(\alpha+\delta)t}(t)dt,\quad Q_{56}(t)=\alpha\int_0^t e^{-(\alpha+\delta)t}dt,$$

$$Q_{78}(t)=[1-e^{-\beta}t],Q_{61}(t)=[1-e^{-\delta}t]=Q_{83}(t) \qquad (1-14)$$

The non-zero elements p_{ij} obtained by letting $t\to\infty$ in (1-14) are
$p_{01}=p_{61}=p_{78}=p_{83}=1, p_{10}=g_1^*(\gamma+\alpha), p_{11}^{(2)}=g_1^*(\gamma)-g_1^*(\gamma+2\alpha),$
$p_{13}=\gamma[1-g_1^*(\gamma+\alpha)]/(\gamma+\alpha), p_{14}^{(2)}=[1-G_1^*(\alpha+\gamma)], p_{34}=\alpha/(\alpha+\beta)$
$p_{35}=\beta/(\alpha+\beta), p_{43}=g_1^*(\gamma+\beta)/(\alpha+\beta), p_{46}=\beta[1-G_1^*(\alpha+\beta)]/(\alpha+\beta),$
$p_{47}=\gamma[1-g_1^*(\gamma+\beta)]/(\gamma+\beta), p_{50}=\delta/(\alpha+\delta), p_{56}=\alpha/(\alpha+\delta)$ [15-26]

The mean sojourn times μ_i in states S_i are
$\mu_0=1/2\alpha, \mu_1=[1-G_1^*(\gamma+\alpha)], \mu_3=1/(\alpha+\beta), \mu_4=[1-G_1^*(\gamma+\alpha)]/(\gamma+\alpha)$
$\mu_5=1/(\alpha+\delta), \mu_6=\mu_8=1/\delta, \mu_7=1/\beta$ [27-33]

6 Time to System failure

To obtained the distribution function $\pi_i(t)$ of the limit to system failure with starting state$S_i\in E(i=0,1,3,5)$, we regard the down states S_2,S_4,S_6,S_7 andS_8 as absorbing. Using arguments as for the regenerative process we obtain the following recursive relations for $\pi_i(t)$:
$\pi_0(t)=Q_{01}(t)[s]\pi_1(t)$
$\pi_1(t)=Q_{10}(t)[s]\pi_0(t)+Q_{12}(t)+Q_{13}(t)[s]\pi_3(t)$
$\pi_3(t)=Q_{34}(t)+Q_{35}[s]\pi_5(t)$
$\pi_5(t)=Q_{50}(t)[s]\pi_0(t)+Q_{56}(t)$ [34-37]
Taking Laplace-Stieljes transform of [34-37] and solving for $\tilde{\pi}_0(s)$, we have
$\tilde{\pi}_0(s)=N_1(s)/D_1(s)$ [38]
where
$N_1(s)=\tilde{Q}_{01}(s)[\tilde{Q}_{12}(s)+\tilde{Q}_{13}(s)\{\tilde{Q}_{34}(s)+\tilde{Q}_{35}(s)\tilde{Q}_{56}(s)\}]$
$D_1(s)=1-\tilde{Q}_{01}(s)[\tilde{Q}_{10}(s)+\tilde{Q}_{13}(s)\tilde{Q}_{13}(s)\tilde{Q}_{50}(s)$
(Where we have omitted the arguments for brevity)
Hence, Starting with stateS_0, MTSF is
$$E(t)=-\frac{d}{ds}\tilde{\pi}_0(s)|_{s=0}=D_1'(0)-N_1'(0)/D_1(0)$$
$$=[\mu_0+\mu_1+p_{13}(\mu_3+\mu_5p_{35})]/[1-p_{10}-p_{13}p_{35}] \qquad [39]$$

7 Availability analysis

Let $A_i(t)$ be the probability that the system initially in state $S_i\in E$ is up at epoch t. The recursive relations for point wise availability $A_i(t)$ are
$A_0(t)=e^{-2\alpha t}+q_{01}(t)©A_1(t)$
$A_1(t)=e^{-(\gamma+\alpha)t}\bar{G_1}(t)+q_{10}(t)©A_0(t)+q_{11}^{(2)}(t)©A_1(t)+q_{13}(t)©A_3(t)$
$\quad +q_{14}^{(2)}(t)©A_4(t)$
$A_3(t)=e^{-(\alpha+\beta)t}+q_{34}(t)©A_4(t)+q_{35}(t)©A_5(t)$
$A_4(t)=q_{43}(t)©A_3(t)+q_{46}(t)©A_6(t)+q_{47}(t)©A_7(t)$
$A_5(t)=e^{-(\alpha+\delta)t}+q_{50}(t)©A_0(t)+q_{56}(t)©A_6(t)$
$A_6(t)=q_{61}(t)©A_1(t)$
$A_7(t)=q_{78}(t)©A_8(t)$
$A_8(t)=q_{83}(t)©A_3(t)$ [40-47]
where

$M_0(t)=e^{-2\alpha t}, M_1(t)=e^{-(\gamma+\alpha)t}, M_3(t)=e^{-(\alpha+\beta)t}, M_5(t)=e^{-(\alpha+\delta)t}$ [48-51]
Taking Laplace transforms of [40]-[47] and solving for $A_0^*(s)$ we have
$A_0^*(s)=N_2(s)/D_2(s)$ [52]
where

$$\begin{aligned}N_2(s)=&\,q_{01}^*(s)[q_{13}^*(s)+q_{14}^{*(2)}(s)\{q_{43}^*(s)+q_{47}^*(s)q_{83}^*(s)\}][M_3^*+M_5^*q_{45}^*(s)]\\&-[q_{43}^*(s)+q_{47}^*(s)q_{78}^*(s)q_{83}^*(s)]\{M_0^*[q_{34}^*(s)\{1-q_{11}^{*(2)}(s)\}+q_{14}^{*(2)}(s)q_{35}^*(s)q_{56}^*(s)q_{61}^*(s)]\\&+M_1^*q_{01}^*(s)q_{34}^*(s)\}+M_0^*[1-q_{11}^{*(2)}(s)-q_{61}^*(s)\{q_{14}^{*(2)}(s)q_{46}^*(s)+q_{13}^*(s)[q_{34}^*(s)q_{46}^*(s)\\&+q_{35}^*(s)q_{56}^*(s)]\}]+M_1^*q_{01}^*(s)\end{aligned}$$

$$\begin{aligned}D_2(s)=&\,1-q_{11}^{*(2)}(s)-q_{61}^*(s)[q_{13}^*(s)\{q_{34}^*(s)q_{46}^*(s)+q_{35}^*(s)q_{56}^*(s)\}]-[q_{43}^*(s)+q_{47}^*(s)q_{78}^*(s)q_{83}^*(s)]\\&[q_{34}^*(s)\{1-q_{11}^{*(2)}(s)\}+q_{14}^{*(2)}(s)q_{35}^*(s)q_{56}^*(s)q_{61}^*(s)-q_{01}^*(s)\{q_{10}^*(s)q_{34}^*(s)-q_{14}^{*(2)}(s)q_{35}^*(s)q_{56}^*(s)\}\\&+q_{01}^*(s)[q_{10}^*(s)+q_{13}^*(s)q_{35}^*(s)q_{50}^*(s)]\end{aligned}$$

Hence, starting from state S_0, the steady state availability of the system is
$A_0(\infty)=\lim_{s\to 0} sA_0^*(s)=\dfrac{N_2(0)}{D_2'(0)}$ [53]
where;

$$\begin{aligned}N_2(0)=&\,\mu_0[p_{35}(1-p_{11}^{(2)})-p_{14}^{(2)}p_{46}(1-p_{35}p_{56})+p_{34}p_{46}(p_{10}+p_{14}^{(2)})-p_{35}p_{56}(p_{13}+p_{14}^{(2)})]\\&+[p_{13}+p_{14}^{(2)}(1-p_{46})](\mu_3+\mu_5p_{35})+\mu_4[1-p_{34}(1-p_{46})]\end{aligned}$$

$$\begin{aligned}D_2'(0)=&\,(p_{13}p_{34}+p_{14}^{(2)})[\mu_4+\mu_6(p_{46}+p_{47})+\mu_7p_{47}]+[1-p_{34}(1-p_{46})](\mu_1+\mu_0p_{10})\\&+[p_{13}+p_{14}^{(2)}(1-p_{46})][\mu_3+p_{35}(\mu_5+\mu_0p_{50}+\mu_6p_{56})]\end{aligned}$$

The expected uptime of the system in $(0,t]$ is
$\mu_{up}(t)=\int_0^t A_0(u)du$ [54]
So that
$\mu_{up}^*(s)=A_0^*(s)/s$
The expected down time in $(0,t]$ is
$\mu_d(t)=t-\mu_{up}(t)$
So that
$\mu_d^*(s)=\mu_{up}^*(s)/s^2$

8 Busy Period Analysis

8.1 Expected busy period of the repairman in repair in $(0,t)$

Let $W_i(t)$ denote the probability that the system initially under repair in state $S_i\in E$ remains in the same state at least time t or passes to non-regenerative state and then continues to remain there under repair without visiting to any regenerative state including itself. By Probabilistic consideration, we have
$W_1(t)=e^{-\alpha t}\overline{G}_1(t), W_4(t)=e^{-(\alpha+\beta)t}\overline{G}_1(t)$ [55-56]
Recursive relations $B_i(t)$, the probability that the system starting from state S_i is busy at time t, are
$B_0(t)=q_{01}(t)©B_1(t)$
$B_1(t)=W_1(t)+q_{10}(t)©B_0(t)+q_{11}^{(2)}(t)©B_1(t)+q_{13}(t)©B_3(t)+q_{14}^{(2)}(t)©B_4(t)$
$B_3(t)=q_{34}(t)©B_4(t)+q_{35}(t)©B_5(t)$
$B_4(t)=W_4(t)+q_{43}(t)©B_3(t)+q_{46}(t)©B_6(t)+q_{47}(t)©B_7(t)$
$B_5(t)=q_{50}(t)©B_0(t)+q_{56}(t)©B_6(t)$
$B_6(t)=q_{61}(t)©B_1(t)$
$B_7(t)=q_{78}(t)©B_8(t)$
$B_8(t)=q_{83}(t)©B_8(t)$ [57-64]

Taking Laplace transforms of [57-64] and computing the relevant elements of the inverse matrix, we have

$$B_0^*(s) = N_3(s)/D_2(s) \qquad [65]$$

where;
$N_3(s) = q_{01}^*(s)[W_1\{1 - q_{34}^*(s)[q_{43}^*(s) + q_{47}^*(s)q_{78}^*(s)q_{83}^*(s)]\} + W_4\{q_{14}^{*(2)}(s) + q_{13}^*(s)q_{34}^*(s)\}]$

In the long run, the fraction of time for which the system under repair is given by
$B_0(\infty) = \lim_{0\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s) = N_3(0)/D_2'(0)$ [66]
The expected busy period of the repairman in repair in $(0,t]$ is
$\mu_b(t) = \int_0^t B_0(u)du$ [67]
so that
$\mu_b^*(s) = B_0^*(s)/s$

8.2 Expected busy period of the repairman in replacement in$(0,t)$

Let $W_i(t)$ denote the probability that the repairman busy with replacement of the unit initially in regenerative state S_i and remains busy in replacement at epoch t without transiting to any other regenerative state. By probabilistic arguments, we have
$W_5(t) = e^{-(\alpha+\delta)}, W_6(t) = e^{-\delta} = W_8(t)$
We define $R_i(t)$, the probability that at time t the server is busy with replacement of the operative unit by the newly delivered unit given that the system starting from regenerative state S_i at $t = 0$. By probabilistic arguments, we have the following recursive relations for $R_i(t)$
$R_0(t) = q_{01}(t)©R_1(t)$
$R_1(t) = q_{10}(t)©R_0(t) + q_{11}^{(2)}(t)©R_1(t) + q_{13}(t)©R_3(t) + q_{14}^{(2)}(t)©R_4(t)$
$R_4(t) = q_{43}(t)©R_3(t) + q_{46}(t)©R_6(t) + q_{47}(t)©R_7(t)$
$R_5(t) = W_5(t) + q_{50}(t)©R_0(t) + q_{56}(t)©R_6(t)$
$R_6(t) = W_6 + q_{61}(t)©R_1(t)$
$R_7(t) = q_{78}(t)©R_8(t)$
$R_8(t) = W_8(t) + q_{83}(t)©R_3(t)$ [68-74]
Taking Laplace transforms of [68-74] and computing the relevant elements of the inverse matrix, the Laplace transform of $R_0(t)$ is seen to be
$R_0^*(s) = N_4(s)/D_2(s)$ [75]
where
$N_4(s) = q_{01}^*(s)q_{35}^*(s)[q_{13}^*(s) + q_{14}^{*(2)}(s)\{+q_{43}^*(s) + q_{47}^*(s)q_{78}^*(s)q_{83}^*(s)\}][W_5 + W_6q_{56}^*(s)] + q_{01}^*(s)[q_{14}^{*(2)}(s) + q_{14}^*(s) + q_{13}^*(s)][W_6(s)q_{46}^*(s) + W_8(s)q_{47}^*(s)q_{78}^*(s)]$
The expected busy period of the repairman in repair in $(0,t]$ is
$\mu_R(t) = \int_0^t R_0(u)du$ [76]
so that
$\mu_R^*(s) = R_0^*(s)/s$

8.3 (c) Expected number of orders for the new unit in $(0,t)$

Let $V_i(t)$ be the expected number of orders for the new unit in $(0,t]$ given that the system entered regenerative state $S_i(t)$ at $t = 0$.By probabilistic arguments, we have
$V_0(t) = q_{01}(t)©V_1(t)$
$V_1(t) = q_{10}(t)©V_0(t) + q_{11}^{(2)}(t)©V_1(t) + q_{13}(t)©(1 + V_3(t)) + q_{14}^{(2)}(t)©V_4(t)$
$V_3(t) = q_{34}(t)©V_4(t) + q_{35}(t)©V_5(t)$
$V_4(t) = q_{43}(t)©(1 + V_3(t)) + q_{46}(t)©V_6(t) + q_{47}(t)©(1 + V_7(t))$
$V_5(t) = q_{50}(t)©V_0(t) + q_{56}(t)©V_6(t)$

$V_6(t) = q_{61}(t) © V_1(t)$
$V_7(t) = q_{78}(t) © V_8(t)$
$V_8(t) = q_{83}(t) © (1+V_3)(t)$ [77-84]
we have
$\tilde{V}_5(s) = N_5(s)/D_3(s)$ [85]
where

$$N_5(s) = \tilde{Q}_{01}(s)\{\tilde{Q}_{13}(s)[1-\tilde{Q}_{34}(s)-\tilde{Q}_{46}(s)-\tilde{Q}_{34}(s)\tilde{Q}_{47}(s)\tilde{Q}_{78}(s)\tilde{Q}_{83}(s)+\tilde{Q}_{47}(s)\tilde{Q}_{78}(s)\tilde{Q}_{83}(s)] + \tilde{Q}_{14}^{(2)}(s)[1-\tilde{Q}_{46}(s)+\tilde{Q}_{47}(s)\tilde{Q}_{78}(s)\tilde{Q}_{83}(s)]\}$$

$$D_3(s) = \{[1-\tilde{Q}_{11}^{(2)}(s)][1-\tilde{Q}_{34}(s)\tilde{Q}_{43}(s)-\tilde{Q}_{34}(s)\tilde{Q}_{47}(s)\tilde{Q}_{78}(s)\tilde{Q}_{83}(s)]-\tilde{Q}_{13}(s)\tilde{Q}_{34}(s)\tilde{Q}_{46}(s)\tilde{Q}_{61}(s) -\tilde{Q}_{13}(s)\tilde{Q}_{35}(s)\tilde{Q}_{56}(s)\tilde{Q}_{61}(s)+\tilde{Q}_{14}(s)\tilde{Q}_{46}(s)+\tilde{Q}_{14}^{(2)}(s)\tilde{Q}_{35}(s)[\tilde{Q}_{43}(s)\tilde{Q}_{56}(s) +\tilde{Q}_{47}(s)\tilde{Q}_{56}(s)\tilde{Q}_{78}(s)\tilde{Q}_{83}(s)]\}-\tilde{Q}_{01}(s)\{\tilde{Q}_{10}(s)[1-\tilde{Q}_{34}(s)\tilde{Q}_{43}(s)+\tilde{Q}_{34}(s)\tilde{Q}_{47}(s)\tilde{Q}_{78}(s)\tilde{Q}_{83}(s)]\} +\tilde{Q}_{01}(s)\{\tilde{Q}_{13}(s)\tilde{Q}_{35}(s)\tilde{Q}_{50}(s)+\tilde{Q}_{14}^{(2)}(s)\tilde{Q}_{35}(s)[\tilde{Q}_{43}(s)+\tilde{Q}_{47}(s)\tilde{Q}_{78}(s)\tilde{Q}_{83}(s)]\}$$

Not typed- In steady state, number of orders for the new unit per unit of time is given by

$$V_0(\infty) = \lim_{t\to\infty}\left[\frac{V_0(t)}{t}\right] = \lim_{s\to 0} s\tilde{V}(s) = \frac{N_5(0)}{D_3'(0)} \qquad [86]$$

Where;

$$N_5(0) = p_{01}\{p_{13}[1-p_{34}-p_{46}-p_{34}p_{47}p_{78}p_{83}+p_{47}p_{78}p_{83}]+p_{14}^{(2)}[1-p_{46}+p_{47}p_{78}p_{83}]\}$$

$$D_3'(0) = \mu_{01}[p_{01}+p_{10}+p_{47}+p_{78}p_{83}-p_{13}p_{35}p_{50}-p_{14}^{(2)}p_{35}(p_{43}+p_{47}p_{78}p_{83})] +\mu_{10}[p_{01}-p_{10}p_{34}p_{43}+p_{01}p_{34}p_{47}p_{78}p_{83}]+\mu_{11}^{(2)}[1-p_{34}p_{43}+p_{34}p_{47}p_{78}p_{83}] +\mu_{13}[p_{34}p_{46}p_{61}+p_{35}p_{56}p_{61}-p_{01}p_{35}p_{50}]-\mu_{14}[p_{46}+p_{35}p_{56}+p_{35}p_{47}p_{56}p_{78}p_{83}] -\mu_{14}^{(2)}p_{01}p_{35}[p_{43}+p_{47}p_{78}p_{83}]-\mu_{34}[p_{11}^{(2)}p_{43}+p_{13}p_{46}p_{61}-p_{01}p_{10}(p_{43}-p_{47}p_{78}p_{83}] +\mu_{35}[p_{13}p_{56}p_{61}-p_{14}p_{56}(p_{43}+p_{47}p_{78}p_{83})-p_{01}(p_{13}p_{50}+p_{14}^{(2)}p_{43}]-\mu_{43}[p_{11}^{(2)}p_{34} +p_{14}p_{35}p_{56}+p_{01}(p_{14}^{(2)}p_{35}+p_{10}p_{34})]+\mu_{46}[p_{13}p_{34}p_{61}-p_{14}]+\mu_{47}[p_{11}^{(2)}p_{34}p_{78}p_{83} -p_{14}p_{35}p_{56}p_{78}p_{83}+p_{01}(p_{10}p_{34}p_{78}p_{83}-p_{14}^{(2)}p_{35}p_{78}p_{83})]-\mu_{50}[p_{01}p_{13}p_{35}] +\mu_{56}[p_{13}p_{35}p_{61}-p_{14}p_{35}p_{47}p_{78}p_{83}-p_{14}p_{35}p_{43}]+\mu_{61}[p_{13}p_{34}p_{46}+p_{13}p_{35}p_{56}] +\mu_{78}[(p_{11}^{(2)}p_{34}-p_{14}p_{35}p_{56})p_{47}p_{83}+p_{01}p_{47}p_{83}(p_{10}p_{34}-p_{14}^{(2)}p_{35})] +\mu_{83}[p_{11}^{(2)}p_{34}p_{47}-p_{14}p_{35}p_{56}p_{78}+p_{01}p_{47}(p_{10}p_{34}-p_{14}^{(2)}p_{35}p_{78})]$$

Particular Case

Case1. When repair time distribution is taken to be negative exponential i.e. $g_1(t) = r_1e^{-r_1t}$ then the expression for $E(T), A_0(\infty), B_0(\infty), R_0(\infty)$ *and* $V_0(\infty)$ *become*

$$E(T) = \frac{L_1}{D_2};\ A_0(\infty) = \frac{L_2}{D_3};\ B_0(\infty) = \frac{L_3}{D_3};\ R_0(\infty) = \frac{L_4}{D_3};\ V_0(\infty) = \frac{L_5}{D_3} [87-91]$$

Where

$$L_1 = (\alpha+\beta)(\alpha+\delta)(\alpha+\gamma+r_1)+2\alpha[(\alpha+\beta)(\alpha+\delta)\gamma(\alpha+\beta+\delta)]$$
$$L_2 = \beta\delta(\gamma+r_1)(\gamma+\alpha+\beta+r_1)[\beta(\alpha+\delta)(\gamma+2\alpha+r_1)+\gamma\alpha(2\alpha+2\delta+\beta)]$$
$$L_3 = 2\alpha\beta\delta(\alpha+\delta)(\alpha+\gamma+\beta+r_1)[\beta(\alpha+\gamma+r_1)+\gamma\alpha]$$
$$L_4 = 2\gamma\alpha\beta(\alpha+\beta)(\alpha+\delta)(\gamma+r_1)(\gamma+\alpha+\beta+r_1)$$
$$L_5 = 2\gamma\alpha\beta\delta(\alpha+\delta)(\gamma+\alpha+\beta+r_1)[\beta(\alpha+\gamma+r_1)+\gamma\alpha]$$
$$D_2 = 2\alpha[(\alpha+\gamma+r_1)(\alpha+\beta)(\alpha+\delta)-\{r_1(\alpha+\delta)(\alpha+\beta)+\gamma\delta\beta]$$
$$D_3 = (\alpha+\delta)(\alpha+\beta+\gamma+r_1)[\beta^2\delta\{2\alpha(\alpha+\gamma+r_1)+r_1(\gamma+r_1)\}+2\gamma\alpha^2\{\beta(\gamma+\delta)+\gamma\delta\}] +\gamma\beta(\gamma+r_1)(\gamma+\alpha+\beta+r_1)[2\alpha(\alpha+\delta)(\beta+\delta)\beta\delta^2]$$

Case2. If we assume that whenever an operating unit fail, it is rejected and an order is placed immediately for a new unit to replace the failed unit i.e. $G_1(t)=0$ and $\alpha = \infty$, then we have

$$E(T) = [(\alpha+\beta)(3\alpha+\delta)+2\alpha\delta]/2\alpha^2(\alpha+\beta+\delta)$$
$$A_0(\infty) = \beta\delta[(\alpha+\delta)(2\alpha+\delta)+\alpha\beta]/D_4$$
$$R_0(\infty) = 2\alpha\beta(\alpha+\beta)(\alpha+\delta)/D_4 \qquad [92-96]$$
$$V_0(\infty) = 2\alpha\beta\delta(\alpha+\beta)(\alpha+\delta)/D_4$$
$$D_4(\infty) = 2\alpha(\alpha+\beta)(\alpha+\delta)(\beta+\delta)+\beta^2\delta^2$$

Profit Analysis

We are now in position to obtain the profit function of the system considering mean up time of the system and expected busy period of the server in repair and replacement. The expected total profit function incurred in (0,t) is
G(t)=expected total revenue in (0,t]-expected total service cost in (0,t]

$$= C_1\mu_{up}(t) - C_2\mu_b(t) - C_3\mu_R(t) - C_4V_0(t) \qquad [97]$$

The expected total profit per unit time in steady state is

$$G = \lim \frac{G(t)}{t} = \lim s^2 G^*(s)[98]$$

Where C_1 is the revenue per unit up time, C_2, C_3 are the costs per unit time in repair, replacement of failed unit by the new ordered unit and $C_4$is the cost per order for a new unit to replace the old one.

9 Conclusion

This paper analyzes the mean time to system failure, system availability and expected profit earned by the system. Also two particular cases are discussed- (i) When repair time distribution is taken to be negative exponential and (ii) whenever an operating unit fail, it is rejected and an order is placed immediately for a new unit to replace the failed unit.

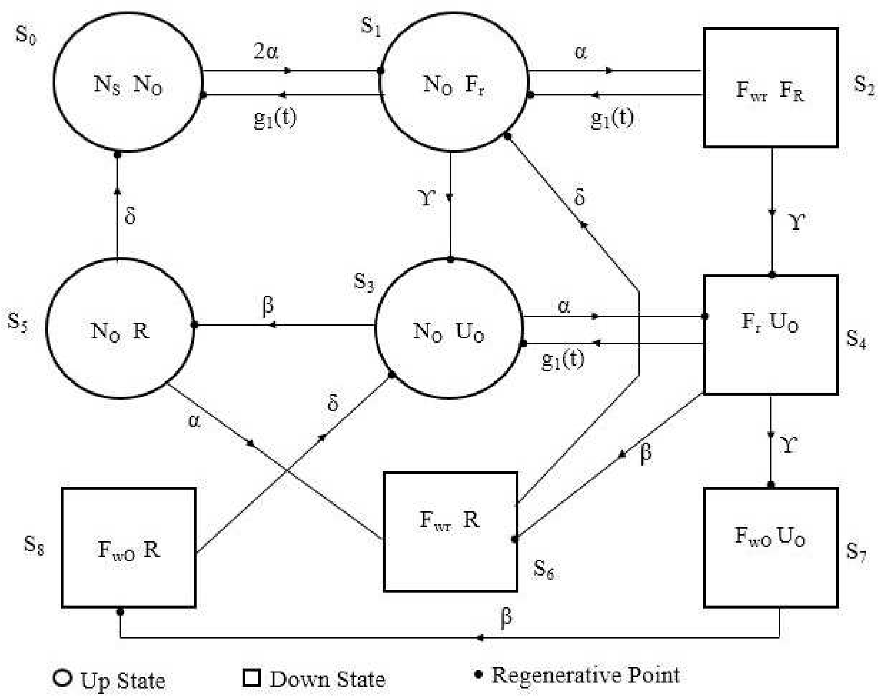

Fig. 1: State Transition Diagram

References

[1] Goel, L.R. and Gupta, Rakesh,Analysis of a two unit standby system with three modes and imperfect switching device, Microelectronics and Reliability, 24(3) (1984), 425-429.

[2] Gopalan, M.N. and Radhakrishnan, R. , Cost-benefit analysis of one-server two unit system subject to slow switch and random service, Microelectronics and Reliability ,25(1) (1985),183-194.

[3] Kumar,Jitendra,Kadyam,M.S.,Malik,S.C. Cost benefit analysis of a two unit parallel system subject to degradation after repair , Applied Mathematical Science, 4 (56) (2010), 2749-2758.
[4] Murari, K. and Goyal ,Vibha, Comparision of two unit cold standby system reliability models with three types of repair facilities, Microelectronics and Reliability,24(1) (1984), 35-49.
[5] Nakagawa, T. and S.Osaki, Markov renewal process with some non-regenerative points and their applications to reliability, Microelectronics Reliability, 15(6) (1976), 633-636.
[6] (6)Okumoto, Kazu, Availability of 2-component dependent system, IEEE Trans. On Reliability, R-30(2) (1985) 205.
[7] Papageorgiou Effii and Kokolakis George , A Two unit parallel system supported by (n-20 standbys with general and non-identical life times , International Journal of System Science,35(1) (2004), 1-12.
[8] Singh,S.N. and Singh,S.K., A study on a two unit parallel system with Erlangian repair time, Statistica (Bologona),67 (1) (2007), 69-77.
[9] Wu,Qingtai, Zhang,Jin and ang,Jiashan, Reliability evaluation for a class of multiunit cold standby systems under Poisson shocks, Comm.Statist.Theory Methods , 44(15) (2015), 3278-3288.

S. K. Singh
is Professor in School of Studies in Statistics, Pt. Ravishankar Shukla University, Raipur, Chhattisgarh India. He received his master degree in Statistics in 1977, Master of Philosophy in 1982 and Doctor of Philosophy in 1985. His research areas include Reliability Theory, Queuing Theory and Survival Analysis.

Kanti Sahu
is Assistant Professor in Statistics, Pt. Ravishankar Shukla University, Raipur, Chhattisgarh India. She received her master degree in Statistics in 2006 and Doctor of Philosophy in 2011. Her research area is Reliability Theory.