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A New Improved Class of Estimators For The Population Variance

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Abstract: In this paper by utilizing the information on the population mean of auxiliary variable, we proposed a new improved class of estimators for the population variance of the study variable. The large sample properties of proposed estimator have been studied up to the first order of approximation that is the mathematical expressions for the bias and mean square error (MSE) of the proposed class of estimators have been obtained up to the first order of approximation. The optimum values of the characterizing scalars, which minimize the MSE of proposed estimator, have been obtained. For these optimum values of characterizing scalars, the minimum MSE of proposed estimator has been obtained. Further a numerical study is also carried out. It has been shown that the proposed estimator is more efficient than sample variance, traditional ratio estimator due to Isaki [3], Singh et al. [6] exponential ratio estimator, estimator based on Kadilar and Cingi [4] ratio estimator, Upadhyaya and Singh [8] estimator and Asghar et al. [1] estimator for the population variance under optimum conditions.

Keywords: Ratio estimator, quartiles, bias, mean squared error, efficiency

1 Introduction

It is well established through various practices that variance is the most suitable measure of dispersion. It is also well known that the most appropriate estimator for the estimation of population variance is the corresponding statistic that is sample variance. Although it is an unbiased estimator of population variance but has large variance and our aim is to find the estimator with minimum variance or even biased but with minimum mean squared error. The use of auxiliary information fulfills this aim of minimizing the mean squared error. This information is obtained through auxiliary variable. The auxiliary variable is highly (positively or negatively) correlated with the main variable under study. When it is positively correlated with the study variable and the line of regression of y on x passes through origin, ratio type estimators are used for the estimation of population parameters. When it is negatively correlated with main variable, product type estimators are used otherwise regression type estimators are used. It may be used at both the stages of designing and estimation. We have used it at estimation stage only. It is well known that when it is used at the estimation stage, the ratio, product and regression methods of estimation are extensively used in many situations. Till now various ratio type, product type, difference and regression type estimators have been proposed by various authors in the literature. In the present paper, we have proposed a new improved class of estimators for the estimation of the population variance utilizing information on population mean of auxiliary variable. Our main purpose of this paper is to develop new estimators for improved and efficient estimation of the population variance.

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2 Review of Estimators in Literature

Let the finite population U consists of N distinct and identifiable units $U_1, U_2, \ldots\ldots, U_N$ and a sample of n units is drawn from this population using the simple random sampling without replacement (SRSWOR) technique. Let Y and X be the study and the auxiliary variables, respectively, with the assumption that these variables are highly (positively or negatively) correlated to each other.

Isaki [3] used the auxiliary information on population variance of auxiliary variable and proposed the ratio type estimator for population variance of the study variable as,

$$t_1 = s_y^2 \left(\frac{S_x^2}{s_x^2}\right) \tag{1}$$

where $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2$, $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$, $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i$, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n}y_i$, $\bar{X} = \frac{1}{N}\sum_{i=1}^{N}X_i$, $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N}Y_i$

The mean square error (MSE) of the estimator in (1), up to the first order of approximation, is given by

$$MSE(t_1) = \gamma S_y^4 \left[(\lambda_{40}-1)+(\lambda_{04}-1)-2(\lambda_{22}-1)\right] \tag{2}$$

where $\lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$, $\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N-1}(Y_i-\bar{Y})^r(X_i-\bar{X})^s$, $\gamma = \frac{1-f}{n}$ and $f = \frac{n}{N}$

The traditional product type estimator for the population variance may be defined as,

$$t_2 = s_y^2 \left(\frac{s_x^2}{S_x^2}\right) \tag{3}$$

The MSE of the estimator of above estimator, up to the first order of approximation, is given by

$$MSE(t_2) = \gamma S_y^4 \left[(\lambda_{40}-1)+(\lambda_{04}-1)+2(\lambda_{22}-1)\right] \tag{4}$$

Singh et al. [6] proposed the exponential ratio estimator for the population variance as,

$$t_3 = s_y^2\, exp\left[\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right] \tag{5}$$

The MSE of the estimator in (5), up to the first order of approximation, is

$$MSE(t_3) = \gamma S_y^4 \left[(\lambda_{40}-1)+\frac{(\lambda_{04}-1)}{4}-(\lambda_{22}-1)\right] \tag{6}$$

The exponential product type estimator for the population variance may be defined as,

$$t_4 = s_y^2\, exp\left[\frac{s_x^2-S_x^2}{s_x^2+S_x^2}\right] \tag{7}$$

The MSE of the estimator in (7), up to the first order of approximation, is

$$MSE(t_4) = \gamma S_y^4 \left[(\lambda_{40}-1)+\frac{(\lambda_{04}-1)}{4}+(\lambda_{22}-1)\right] \tag{8}$$

Adapting Kadilar and Cingi [4] ratio estimator for the population mean, the ratio type estimator for the population variance can be defined as,

$$t_5 = s_y^2 \left[\frac{(S_x^2)^2}{(s_x^2)^2}\right] \tag{9}$$

The MSE of the above estimator, up to the first order of approximation, is

$$MSE(t_5) = \gamma S_y^4 \left[(\lambda_{40}-1)+4(\lambda_{04}-1)-4(\lambda_{22}-1)\right] \tag{10}$$

The product type estimator for the population variance based on the estimator in (9) can be defined as,

$$t_6 = s_y^2 \left[\frac{(s_x^2)^2}{(S_x^2)^2} \right] \tag{11}$$

The MSE of the above estimator, up to the first order of approximation, is

$$MSE\,(t_6) = \gamma\, S_y^4 \left[(\lambda_{40} - 1) + 4(\lambda_{04} - 1) + 4(\lambda_{22} - 1) \right] \tag{12}$$

Upadhyaya and Singh [8] proposed a modified ratio estimator of population variance using the population mean of the auxiliary variable as,

$$t_7 = s_y^2 \left[\frac{\bar{X}}{\bar{x}} \right] \tag{13}$$

The bias and the mean squared error of above estimator up to the first order of approximation respectively are,

$$B(t_7) = \gamma\, S_y^2 [C_x^2 - \lambda_{21} C_x] \tag{14}$$

$$MSE\,(t_7) = \gamma\, S_y^4 \left[(\lambda_{40} - 1) + C_x^2 - 2\lambda_{21} C_x \right], \;\; where \;\; \lambda_{21} = \frac{\mu_{21}}{\mu_{20}\;\mu_{02}^{1/2}} \tag{15}$$

The product type estimator of population variance based on Upadhyaya and Singh [8] estimator may be given by,

$$t_8 = s_y^2 \left[\frac{\bar{x}}{\bar{X}} \right] \tag{16}$$

The bias and the mean squared error of above estimator up to the first order of approximation respectively are,

$$B(t_8) = \gamma\, S_y^2 [C_x^2 + \lambda_{21} C_x] \tag{17}$$

$$MSE\,(t_8) = \gamma\, S_y^4 \left[(\lambda_{40} - 1) + C_x^2 + 2\lambda_{21} C_x \right] \tag{18}$$

Asghar et al. [1] proposed an improved ratio and product type estimators of population variance using population mean of auxiliary variable respectively as,

$$t_9 = s_y^2 \; exp \left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right] \tag{19}$$

$$t_{10} = s_y^2 \; exp \left[\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right] \tag{20}$$

The bias and mean squared error of above estimators up to the first order of approximation respectively are,

$$B(t_9) = \gamma\, S_y^2 \left[\frac{C_x^2}{8} - \frac{\lambda_{21}}{2} C_x \right] \tag{21}$$

$$MSE\,(t_9) = \gamma\, S_y^4 \left[(\lambda_{40} - 1) + \frac{C_x^2}{4} - \lambda_{21} C_x \right] \tag{22}$$

$$B(t_{10}) = \gamma\, S_y^2 \left[\frac{C_x^2}{8} + \frac{\lambda_{21}}{2} C_x \right] \tag{23}$$

$$MSE\,(t_{10}) = \gamma\, S_y^4 \left[(\lambda_{40} - 1) + \frac{C_x^2}{4} + \lambda_{21} C_x \right] \tag{24}$$

Various authors in the literature have proposed different estimators by utilizing auxiliary information in the form of different parameters of auxiliary variable for estimating the population variance of the main variable under study.

3 Proposed Class of Estimators

Motivated by Solanki et al. [7] and Upadhyaya and Singh [8] and adapting Solanki et al. [7] estimator of population mean for the estimation of population variance using population mean of auxiliary variable, we propose the estimator of the population variance as

$$t_{(\alpha,\delta)} = s_y^2 \left[2 - \left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} exp\left(\frac{\delta(\bar{x}-\bar{X})}{\bar{x}+X}\right)\right] \tag{25}$$

where (α,δ) are suitably chosen scalars which minimizes the mean squared error of $t_{(\alpha,\delta)}$.

For $\delta = 0$, the proposed class of estimators reduces to the class of estimators as

$$t_{(\alpha,0)} = s_y^2 \left[2 - \left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}\right] \tag{26}$$

While for $\alpha = 0$, the proposed estimator $t_{(\alpha,\delta)}$, reduces to a new class of estimators as

$$t_{(0,\delta)} = s_y^2 \left[2 - exp\left(\frac{\delta(\bar{x}-\bar{X})}{\bar{x}+X}\right)\right] \tag{27}$$

In order to study the large sample properties of the proposed class of estimators $t_{(\alpha,\delta)}$, we define

$s_y^2 = S_y^2(1+\varepsilon_0)$ and $\bar{x} = \bar{X}(1+\varepsilon_1)$ such that $E(\varepsilon_i) = 0$ for $(i = 0,1)$ and $E(\varepsilon_0^2) = \gamma(\lambda_{40}-1), E(\varepsilon_1^2) = \gamma C_x^2, E(\varepsilon_0,\varepsilon_1) = \gamma\lambda_{21}C_x$

Expressing (25) in terms of ε_i's we have

$$\begin{aligned} t_{(\alpha,\delta)} &= S_y^2(1+\varepsilon_0)\left[2-(1+\varepsilon_1)^{\alpha}\, exp\left(\frac{\delta\varepsilon_1}{2+\varepsilon_1}\right)\right] \\ &= S_y^2(1+\varepsilon_0)\left[2-(1+\varepsilon_1)^{\alpha}\, exp\left(\frac{\delta\varepsilon_1}{2}\left(1+\frac{\varepsilon_1}{2}\right)^{-1}\right)\right] \end{aligned} \tag{28}$$

We assume that $|\varepsilon_1| < 1$, so that $(1+\varepsilon_1)^{\alpha}$ and $\left(1+\frac{\varepsilon_1}{2}\right)^{-1}$ may be expanded. Now expanding the right-hand side of (28), we have,

$$\begin{aligned} t_{(\alpha,\delta)} &= S_y^2(1+\varepsilon_0)\left[2-\left(1+\alpha\varepsilon_1+\frac{\alpha(\alpha-1)}{2}\varepsilon_1^2+.....\right)\left(1+\frac{\delta\varepsilon_1}{2}\left(1+\frac{\varepsilon_1}{2}\right)^{-1}+\frac{\delta^2\varepsilon_1^2}{8}\left(1+\frac{\varepsilon_1}{2}\right)^{-2}+.....\right)\right] \\ &= S_y^2(1+\varepsilon_0)\left[2-\left(1+\alpha\varepsilon_1+\frac{\alpha(\alpha-1)}{2}\varepsilon_1^2+.....\right)\left(1+\frac{\delta\varepsilon_1}{2}-\frac{\delta\varepsilon_1^2}{4}+\frac{\delta^2\varepsilon_1^2}{8}-.....\right)\right] \\ &= S_y^2(1+\varepsilon_0)\left[2-\left(1+\alpha\varepsilon_1+\frac{\alpha(\alpha-1)}{2}\varepsilon_1^2+\frac{\delta\varepsilon_1}{2}+\frac{\alpha\delta\varepsilon_1^2}{2}-\frac{\delta\varepsilon_1^2}{4}+\frac{\delta^2\varepsilon_1^2}{8}-.....\right)\right] \\ &= S_y^2(1+\varepsilon_0)\left[2-\left(1+\frac{(2\alpha+\delta)}{2}\varepsilon_1+\frac{(2\alpha+\delta)(2\alpha+\delta-2)}{8}\varepsilon_1+.....\right)\right] \\ &= S_y^2\left[1+\varepsilon_0-\frac{(2\alpha+\delta)}{2}(\varepsilon_1+\varepsilon_0\varepsilon_1)-\frac{(2\alpha+\delta)(2\alpha+\delta-2)}{8}(\varepsilon_1^2+\varepsilon_0\varepsilon_1^2)-....\right] \end{aligned}$$

Retaining the terms up to the first order of approximation, we have

$$t_{(\alpha,\delta)} = S_y^2\left[1+\varepsilon_0-\frac{(2\alpha+\delta)}{2}(\varepsilon_1+\varepsilon_0\varepsilon_1)-\frac{(2\alpha+\delta)(2\alpha+\delta-2)}{8}\varepsilon_1^2\right] \tag{29}$$

Subtracting S_y^2 from both sides of (29), we get

$$\left[t_{(\alpha,\delta)}-S_y^2\right] = S_y^2\left[\varepsilon_0-\frac{(2\alpha+\delta)}{2}(\varepsilon_1+\varepsilon_0\varepsilon_1)-\frac{(2\alpha+\delta)(2\alpha+\delta-2)}{8}\varepsilon_1^2\right] \tag{30}$$

Taking expectation both sides of (30), we get the bias of $t_{(\alpha,\delta)}$ as

$$B\left[t_{(\alpha,\delta)}-S_y^2\right] = \gamma S_y^2\left[-\frac{(2\alpha+\delta)}{2}\lambda_{21}C_x-\frac{(2\alpha+\delta)(2\alpha+\delta-2)}{8}C_x^2\right] \tag{31}$$

Squaring both sides of (30) and retaining the terms up to the first order of approximation, we have

$$\left[t_{(\alpha,\delta)} - S_y^2\right]^2 = S_y^4\left[\varepsilon_0^2 - \frac{(2\alpha+\delta)^2}{4}\varepsilon_1^2 - (2\alpha+\delta)\varepsilon_0\varepsilon_1\right] \tag{32}$$

Taking expectation both sides of (32), we get the mean square error of $t_{(\alpha,\delta)}$, up to the first order of approximation, as

$$MSE\left[t_{(\alpha,\delta)}\right] = \gamma S_y^4\left[(\lambda_{40}-1) + \frac{(2\alpha+\delta)^2}{4}C_x^2 - (2\alpha+\delta)\lambda_{21}C_x\right] \tag{33}$$

which is minimum if,

$$(2\alpha+\delta) = 2\frac{\lambda_{21}}{C_x} = 2K,\ where\ \ K = \frac{\lambda_{21}}{C_x} \tag{34}$$

Thus the minimum MSE of $t_{(\alpha,\delta)}$ is,

$$MSE_{min}\left[t_{(\alpha,\delta)}\right] = \gamma S_y^4\left[(\lambda_{40}-1) - \lambda_{21}^2\right] \tag{35}$$

Now, for different values of α and δ, we can form different estimators. For $\delta = 0$ and $\alpha = 0$, the proposed estimator $t_{(\alpha,\delta)}$ reduces to estimators (26) and (27) respectively. The MSE of (26) and (27), up to the first order of approximation respectively are,

$$MSE\left[t_{(\alpha,0)}\right] = \gamma S_y^4\left[(\lambda_{40}-1) + \alpha^2C_x^2 - 2\alpha\lambda_{21}C_x\right] \tag{36}$$

$$MSE\left[t_{(0,\delta)}\right] = \gamma S_y^4\left[(\lambda_{40}-1) + \frac{\delta^2}{4}C_x^2 - \delta\lambda_{21}C_x\right] \tag{37}$$

For $\alpha = 1$ and $\delta = 1$, the proposed estimator, $t_{(\alpha,\delta)}$ reduces to a new estimator $t_{(1,1)}$ as

$$t_{(1,1)} = s_y^2\left[2 - \left(\frac{\bar{x}}{\bar{X}}\right) exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right)\right] \tag{38}$$

The MSE of above estimator, up to the first order of approximation, is

$$MSE\left[t_{(1,1)}\right] = \gamma S_y^4\left[(\lambda_{40}-1) + \frac{9}{4}C_x^2 - 3\lambda_{21}C_x\right] \tag{39}$$

It would be worth notable to say that naturally, many more ratio and product estimators can be developed by putting various values of α and δ.

4 Efficiency Comparison

The most suitable estimator for population variance is the sample variance $t_0 = s_y^2$ and its variance is given by

$$V(t_0) = \gamma S_y^4(\lambda_{40}-1) \tag{40}$$

From (40) and (35), we have

$$V(t_0) - MSE_{min}\left[t_{(\alpha,\delta)}\right] = \gamma S_y^4\left[\lambda_{21}\right] > 0 \tag{41}$$

From (2) and (35), we have

$$MSE(t_1) - MSE_{min}\left[t_{(\alpha,\delta)}\right] = \gamma S_y^4[(\lambda_{04}-1) - 2(\lambda_{22}-1) + \lambda_{21}^2] > 0\ \ if$$

$$(\lambda_{04}-1) + \lambda_{21}^2 - 2(\lambda_{22}-1) > 0 \tag{42}$$

From (4) and (35), we have

$$MSE(t_2) - MSE_{min}\left[t_{(\alpha,\delta)}\right] = \gamma S_y^4\left[(\lambda_{04}-1) + 2(\lambda_{22}-1) + \lambda_{21}^2\right] > 0\ \ if$$

$$(\lambda_{04}-1)+\lambda_{21}^2+2(\lambda_{22}-1)>0 \tag{43}$$

From (6) and (35), we have

$$MSE(t_3)-MSE_{min}\left[t_{(\alpha,\delta)}\right]=\gamma S_y^4\left[\frac{(\lambda_{04}-1)}{4}-(\lambda_{22}-1)+\lambda_{21}^2\right]>0\ \ if$$

$$\frac{(\lambda_{04}-1)}{4}+\lambda_{21}^2-(\lambda_{22}-1)>0 \tag{44}$$

From (8) and (35), we have

$$MSE(t_4)-MSE_{min}\left[t_{(\alpha,\delta)}\right]=\gamma S_y^4\left[\frac{(\lambda_{04}-1)}{4}+2(\lambda_{22}-1)+\lambda_{21}^2\right]>0\ \ if$$

$$\frac{(\lambda_{04}-1)}{4}+\lambda_{21}^2+2(\lambda_{22}-1)>0 \tag{45}$$

From (10) and (35), we have

$$MSE(t_5)-MSE_{min}\left[t_{(\alpha,\delta)}\right]=\gamma S_y^4\left[4(\lambda_{04}-1)-4(\lambda_{22}-1)+\lambda_{21}^2\right]>0\ \ if$$

$$4(\lambda_{04}-1)+\lambda_{21}^2-4(\lambda_{22}-1)>0 \tag{46}$$

From (12) and (35), we have

$$MSE(t_6)-MSE_{min}\left[t_{(\alpha,\delta)}\right]=\gamma S_y^4\left[4(\lambda_{04}-1)+4(\lambda_{22}-1)+\lambda_{21}^2\right]>0\ \ if$$

$$4(\lambda_{04}-1)+\lambda_{21}^2+4(\lambda_{22}-1)>0 \tag{47}$$

From (15) and (35), we have

$$MSE(t_7)-MSE_{min}\left[t_{(\alpha,\delta)}\right]=\gamma S_y^4[C_x-\lambda_{21}]^2>0\ \ if \tag{48}$$

From (18) and (35), we have

$$MSE(t_8)-MSE_{min}\left[t_{(\alpha,\delta)}\right]=\gamma S_y^4[C_x+\lambda_{21}]^2>0 \tag{49}$$

From (22) and (35), we have

$$MSE(t_9)-MSE_{min}\left[t_{(\alpha,\delta)}\right]=\gamma S_y^4\left[\frac{C_x}{2}-\lambda_{21}\right]^2>0 \tag{50}$$

From (24) and (35), we have

$$MSE(t_{10})-MSE_{min}\left[t_{(\alpha,\delta)}\right]=\gamma S_y^4\left[\frac{C_x}{2}+\lambda_{21}\right]^2>0 \tag{51}$$

5 Numerical Examples

To examine the performance of the proposed estimator along with the other estimators of population variance, the empirical study has been carried out using two real populations.

Population I: Murthy [5]
Y: Number of workers, X: Output
$N=25, n=25, \bar{Y}=33.8465, \bar{X}=283.875, \rho_{yx}=0.9136, C_y=0.352, C_x=0.746, \lambda_{04}=3.65, \lambda_{40}=2.2667, \lambda_{22}=2.3377,$ $\lambda_{21}=1.0475,$

Population II: Gujarati [2]
Y: Top Speed (miles per hour), X :Average (miles per gallon)
$N=81,\ n=21,\ \bar{Y}=2137.068,\ \bar{X}=112.4568,\ \rho_{yx}=-0.6911,\ C_y=0.1248,\ C_x=0.48,\ \lambda_{04}=6.82,\ \lambda_{40}=3.59,\ \lambda_{22}=$ $2.110,\ \lambda_{21}=1.4137,$

Table 1: Percent relative efficiency of different estimators with respect to proposed estimator $t_{(\alpha,\delta)}$

Estimator	Population-I	Population-II
t_0	747.58	437.91
t_1	732.59	1046.58
t_2	3890.52	1797.28
t_3	496.63	496.24
t_4	1928.06	871.59
t_5	3845.55	3623.29
t_6	10161.41	5124.69
t_7	153.64	247.39
t_8	1998.41	706.33
t_9	368.50	332.91
t_{10}	1290.88	562.38
$t_{(\alpha,\delta)}$	100	100

6 Conclusion

In this paper we have proposed a class of estimators of population variance using auxiliary information on single variable. This estimator was proposed through the motivation of Solanki et al. [7] and Upadhyaya and Singh [8]. We have studied its large sample properties that bias and mean squared error up to the first order of approximation. Further we have compared this with the other estimators of population variance theoretically. From the theoretical discussions in section-4 and the empirical study given in Table-1, it is inferred that the proposed estimator $t_{(\alpha,\delta)}$, for estimating the population variance of the study variable under the optimum condition performs better than the sample variance estimator $t_0 = s_y^2$, traditional ratio type estimator t_1 and other estimators of population variance given in Table-1.

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